Bottom Up Parsers

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Bottom Up Parsing

- They are powerful compared with TD parsers.
- Understandably more complex.
- Left recursion is not a problem for BU parsers.
- Right recursion is a bit of problem but not serious ?
- *Not suitable for hand coding.*

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Bottom-up Parsing:

- LR (1)
- LR (0)
- SLR (1)
- LALR (1)

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Bottom Up Parsing

- Basic operation is to *shift* terminals from the input to the stack until the right-hand side of an appropriate grammar rule is seen, and then to *reduce* the stuff on the stack that matches the right-hand side to the single non-terminal of the rule. Hence, bottom-up parsers are often called *shift-reduce parsers.*
- Stack can be viewed as containing both terminals and non-terminals.
- Table-driven using an explicit stack.

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Example Grammar 1

$E' \rightarrow E$

$E \rightarrow E + n \mid n$

Input: 2 + 3, or $n + n$

Parse: ($ is EOF in input, also bottom of stack)

	Parsing stack	Input	Action
1	$	n + n $	shift
2	$ n	+ n $	reduce $E \rightarrow n$
3	$ E	+ n $	shift
4	$ E +	n $	shift
5	$ E + n	$	reduce $E \rightarrow E + n$
6	$ E	$	reduce $E' \rightarrow E$
7	$ E'	$	Accept

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Example Grammar 2

$S' \rightarrow S$

$S \rightarrow (S)\ S \mid \epsilon$

Input String: ( )

	Parsing Stack	Input	Action
1	$	( ) $	shift
2	$ (	) $	reduce $S \rightarrow \epsilon$
3	$ (S	) $	shift
4	$ (S)	$	reduce $S \rightarrow \epsilon$
5	$ (S) S	$	reduce S → (S) S
6	$ S	$	reduce $S' \rightarrow S$
7	$ S'	$	Accept

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Right Sentential Form (RSF) & Viable Prefixes

1. E' → E → E + n → n + n
2. S' → S → (S) S → (S) → ( )

- RSF splits the stack and the input during parsing.
- E, E + , E + n are all viable prefixes of RSF of E + n.

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Handle

- Parser keeps shifting the terminals on the stack till the time it can perform reduction, that occurs when the RHS of a production rule matches. This string, position where it occurs and the production rule is called handle.
- Main task of a SR parser is to determine the next handle in the string.

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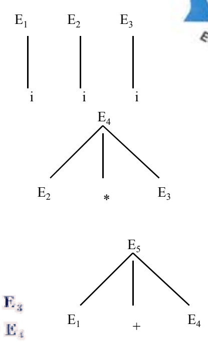
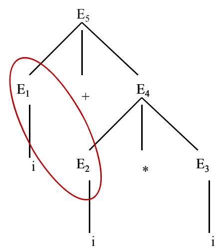
Derivation Construction: $E \rightarrow E + E \mid E * E \mid i$
String = i + i * i

Stack	Input	Production
	i + i * i	shift
i	+ i * i	reduce $E_1 \rightarrow i$
E_1	+ i * i	shift +
E_1 +	i * i	shift i
E_1 + i	* i	reduce $E_2 \rightarrow i$
E_1 + E_2	* i	shift *
E_1 + E_2 *	i	shift i
E_1 + E_2 * i		reduce $E_3 \rightarrow i$
E_1 + E_2 * E_3		reduce $E_4 \rightarrow E_2 * E_3$
E_1 + E_4		reduce $E_5 \rightarrow E_1 + E_4$
E_5		





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Item

- An *item* is a grammar rule option with a distinguished position (indicated by a period or other symbol):

$$A \rightarrow \alpha \,.\, \beta$$

- The position in an item indicates that a parse has reached to the place in recognizing that rule α is then on the stack, and a β may be coming in the input (α is called a *viable prefix*).

- A stack state consists of the set of items which have "compatible" viable prefixes.

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Item Example 1

E' → E
E → E + n | n

E' → .E
E' → E.
E → .E + n
E → E. + n
E → E + .n
E → E + n.
E → .n
E → n.

Initial Items

Final Items

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Item Example 2

S' → S
S → (S) S | e

S' → .S
S' → S.
S → .(S) S
S → (.S) S
S → (S.) S
S → (S). S
S → (S) S.
S → .

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FA of LR (0) Items

LR(0) items can be used as the states of FA which maintains the information about the parsing stack and the progress of shift-reduce parse.

Typically it starts with NFA and from this one can construct DFA using Subset construction or directly.

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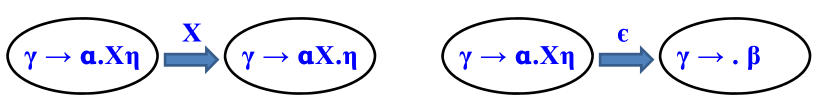
NFA Construction

Suppose $S \rightarrow \alpha.\gamma$ and imagine γ begins with a symbol X

$\gamma \rightarrow \alpha.X\eta$

If X is terminal it is simple just make a transition to that terminal.

But if X is not a terminal, suppose $X \rightarrow \beta$, then a process starts by recognizing $.\beta$, so for each such case we need a transition of ϵ.



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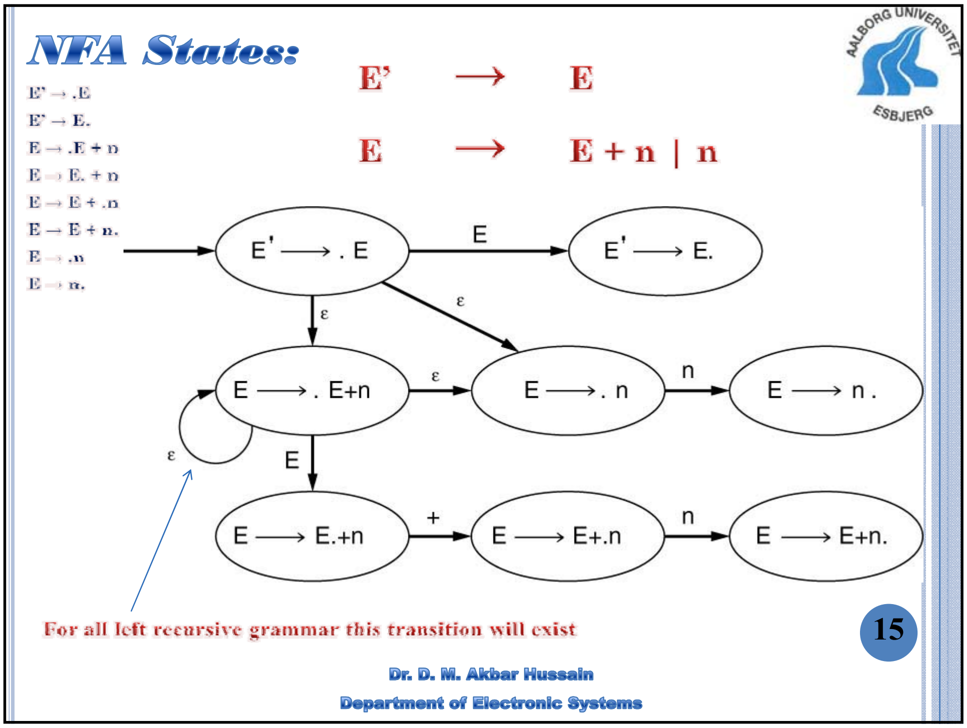
NFA States:
E' → .E
E' → E.
E → .E + n
E → E. + n
E → E + .n
E → E + n.
E → .n
E → n.
E' → E
E → E + n | n
E' ⟶ . E
E
E' ⟶ E.
ε
ε
E ⟶ . E+n
ε
E ⟶ . n
n
E ⟶ n .
ε
E
E ⟶ E.+n
+
E ⟶ E+.n
n
E ⟶ E+n.
For all left recursive grammar this transition will exist
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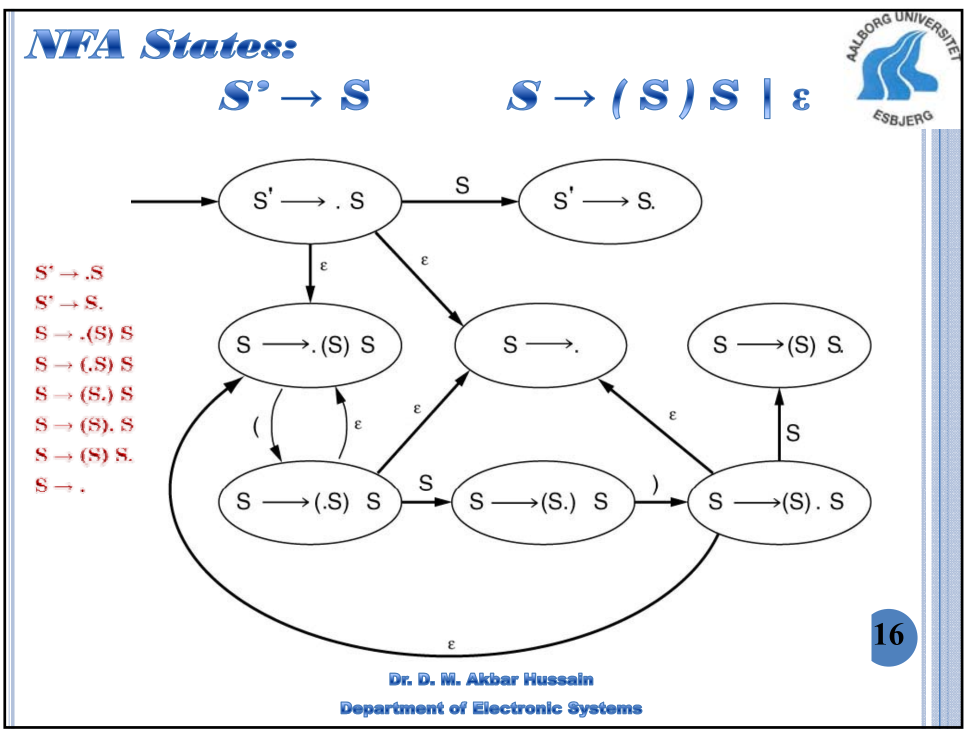
NFA States:
S' → S
S → (S) S | ε
S' ⟶ . S
S
S' ⟶ S.
ε
ε
S' → .S
S' → S.
S → .(S) S
S → (.S) S
S → (S.) S
S → (S). S
S → (S) S.
S → .
S ⟶ . (S) S
S ⟶ .
S ⟶ (S) S.
(
ε
ε
ε
S
S ⟶ (.S) S
S
S ⟶ (S.) S
)
S ⟶ (S) . S
ε
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Computing the DFA of sets of LR(0) items

- Add the augmentation item $S' \rightarrow .\ S$ to the start state of the DFA. Then add all the *initial items* for S to the state: $S \rightarrow .\ \alpha$. Continue by adding all the initial items for those non-terminals which appear right after the dot in any previous item (this is called the *closure* of the set of items).

- Every symbol that comes immediately after the dot gives rise to a transition to a state generated by adding closure items to the item with the dot moved past that symbol.

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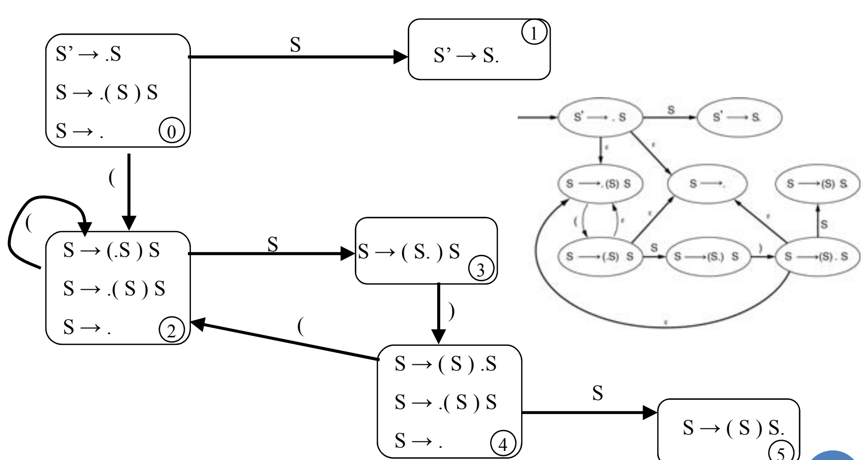
DFA Set of States:

$S \rightarrow (\ S\ )\ S \mid \varepsilon$



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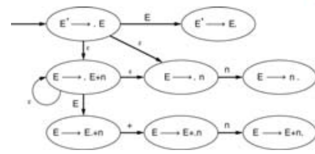
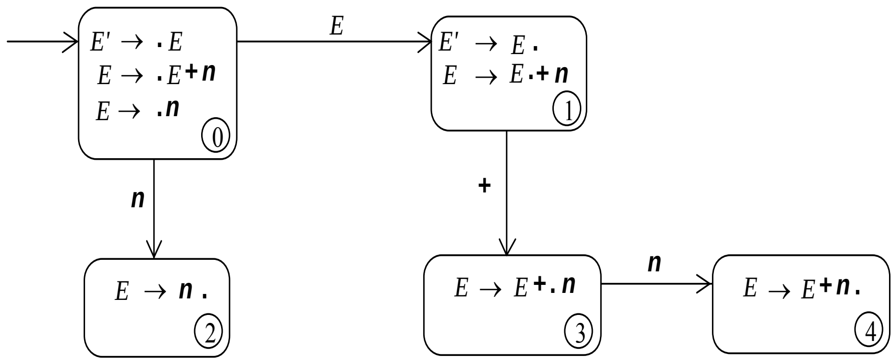
DFA Set of States:

$E \rightarrow E + n \mid n$





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LR(0) Grammar

Every state is distinguishable as either it contains shift actions or reduce actions, any other possibility will make either **shift-reduce conflict** or **reduce-reduce conflict.** Which will indicate that it is not an LR(0) grammar, so these rules are exclusive.

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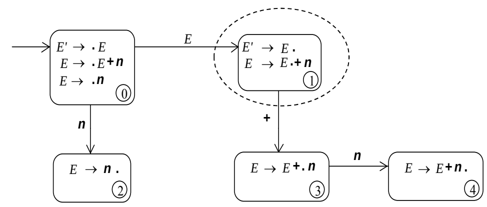
Sample Parse "n + n + n" string



	Parsing stack	Input	Action
1	$ 0	*n* + *n* + *n* $	shift 2
2	$ 0 *n* 2	+ *n* + *n* $	reduce $E \to$ ***n***
3	$ 0 *E* 1	+ *n* + *n* $	shift 3
4	$ 0 *E* 1 + 3	*n* + *n* $	shift 4
5	$ 0 *E* 1 + 3 *n* 4	+ *n* $	reduce $E \to E$ + ***n***
6	$ 0 *E* 1	+ *n* $	shift 3
7	$ 0 *E* 1 + 3	*n* $	shift 4
8	$ 0 *E* 1 + 3 *n* 4	$	reduce $E \to E$ + ***n***
9	$ 0 *E* 1	$	accept

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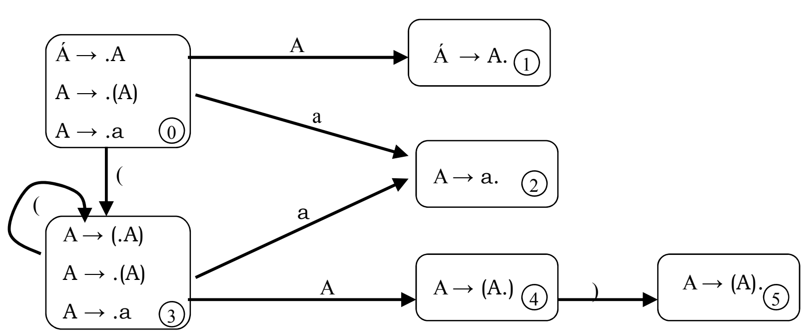
Example: A → (A) | a

The DFA of sets of items:



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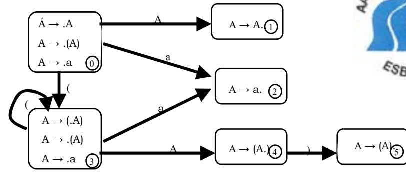
Parsing Actions: ((a))

Grammar: A → (A) | a



	Parsing Stack	Input	Action
1	$0	((a))$	Shift
2	$0 (3	(a))$	Shift
3	$0 (3 (3	a))$	Shift
4	$0 (3 (3 a 2	))$	Reduce A → a
5	$0 (3 (3 A 4	))$	Shift
6	$0 (3 (3 A 4) 5	)$	Reduce A → (A)
7	$0 (3 A 4	)$	Shift
8	$0 (3 A 4) 5	$	Reduce A → (A)
9	$0 A 1	$	Accept

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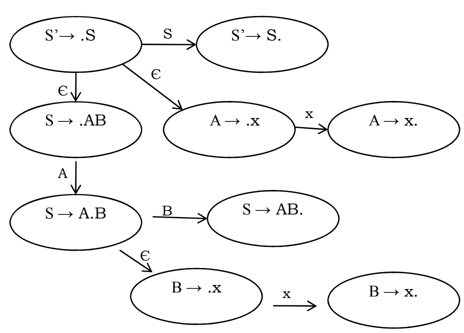
NFA LR(0) Items

Grammar: *S* → *A B*
A → x
B → x

S'→ .S
S → S.
S → .AB
S → A.B
S → AB.
A → .x
A → x.
B → .x
B → x.



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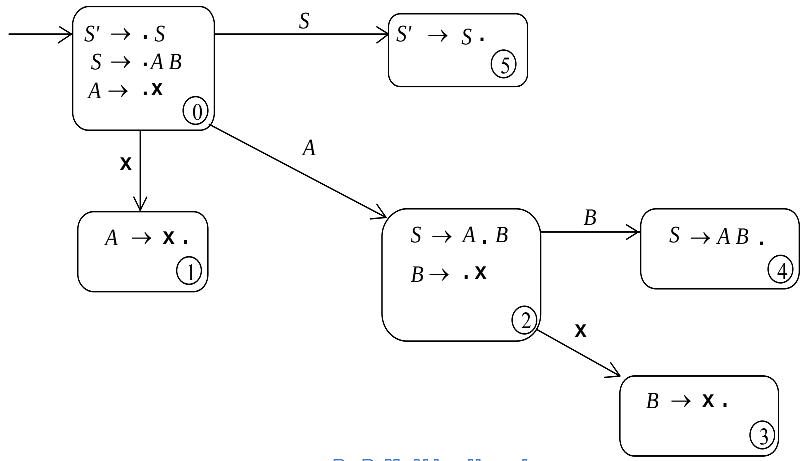
DFA States For:

$S \rightarrow A\ B$
$A \rightarrow \mathbf{x}$
$B \rightarrow \mathbf{x}$



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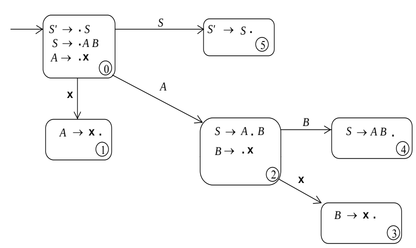
Sample Parse for "x x" string

$S \rightarrow A\ B$
$A \rightarrow \mathbf{x}$
$B \rightarrow \mathbf{x}$



	Parsing stack	Input	Action
1	$0	x x $	s1
2	$ 0 x 1	x $	r2 ($A \rightarrow$ x)
3	$ 0 *A* 2	x $	s3
4	$ 0 *A* 2 x 3	$	r3 ($B \rightarrow$ x)
5	$ 0 *A* 2 *B* 4	$	r1 ($S \rightarrow A\ B$)
6	$ 0 *S* 5	$	accept

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Yacc/Bison Parsing Tables

With the -v option ("verbose") Yacc generates a file y.output (Bison: <filename>.output) describing its parsing actions. For the same grammar

$$S \rightarrow A\ B$$
$$A \rightarrow \mathrm{x}$$
$$B \rightarrow \mathrm{x}$$

the output file looks as (next slide).

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Bison Table is Essentially the Same:

```
state 0
 'x'        shift, and go to state 1
 S go to state 5
 A go to state 2
state 1
 A  ->  'x' .(rule 2)
 $default reduce using rule 2 (A)
state 2
 S  ->  A . B (rule 1)
 'x'        shift, and go to state 3
 B go to state 4
```

```
state 3
 B  ->  'x'. (rule 3)
 $default            reduce using
    rule 3 (B)
state 4
 S  ->  A B . (rule 1)
 $default reduce using rule 1 (S)
state 5
   $ go to state 6
state 6
   $ go to state 7
state 7
   $default            accept
```

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Table Driven Bottom Up Parsing

- *ACTION | GOTO Table*

The set of DFA items & actions specified by the LR(0) algorithm can be combined into a parsing table, therefore it becomes a table driven parsing method.

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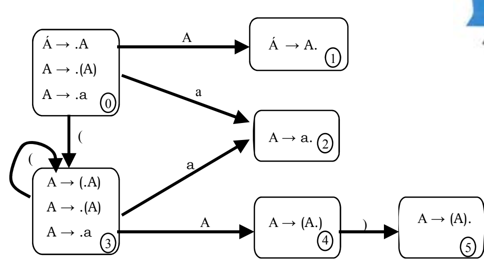
Table for Bottom Up Parsing

Grammar: A' → A
A → (A) | a



State	Action	Rule	Input			Goto
			(	a	)	A
0	Shift		3	2		1
1	Reduce	Reduce A' → A				
2	Reduce	Reduce A → a				
3	Shift		3	2		4
4	Shift				5	
5	Reduce	Reduce A → (A)				

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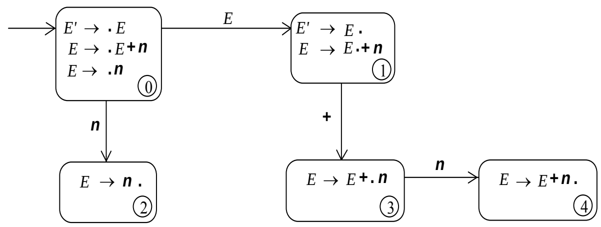
LR(0) Parsing Table



State	Action	Rule	Input		Goto
			n	+	E
0	shift		2		1
1	shift/reduce	E '→ E		3	
2	reduce	E → n			
3	shift		4		
4	reduce	E → E + n			

Is this a LR(0) Grammar ?

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Decision Problems in BU Parsing

- **Shift-reduce conflicts:** Come from ambiguities and almost always the right disambiguating rule is to shift (dangling-else).
- **Reduce-reduce conflicts:** More difficult, bottom-up parsers try to resolve them using Follow set contexts.
- There are no **shift-shift conflicts.**

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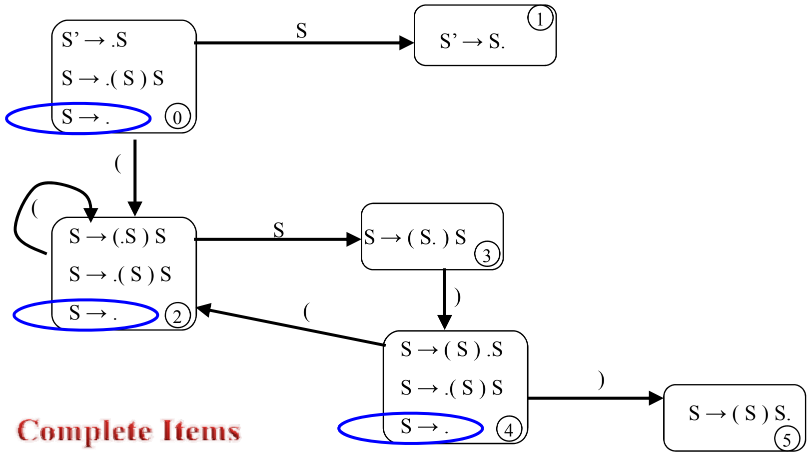
DFA Set of States: S → (S) S | ε



LR (0) Grammar ?

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SLR (1) Parsing

The power is increased by two actions:

1. Consulting the input token before a shift is executed.
2. Follow set is used for reduction execution.

   For example if there is complete item A → x. and the next token in the input is in the Follow (A) use the A → x. rule for reduction.

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SLR (1) Grammar

1. For any item A → x.Yc in a state s with a terminal Y, there is no complete item B → x. in s with Y in the Follow (B).
2. For any two complete items A → x., B → y. in s Follow (A) ∩ Follow (B) is empty.

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Reduce-Reduce Example

Grammar: $S \to A\ B$
$A \to \mathrm{x}$
$B \to \mathrm{x}$

Input: x x

Parse: (Follow(A) = {x}, Follow(B) = {$})

	Parsing stack	Input	Action
1	$	**x x** $	shift
2	$ **x**	**x** $	reduce $A \to$ **x**, reduce $B \to$ **x**
3	$ A	**x** $	shift
4	$ A **x**	$	reduce $B \to$ **x**
5	$ $A\ B$	$	reduce $S \to A\ B$
6	$ S	$	accept

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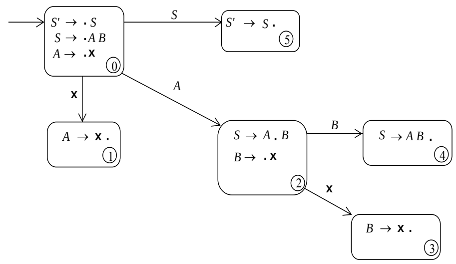
Parsing Table from the DFA

(1) $S \rightarrow A\ B$

(2) $A \rightarrow x$

(3) $B \rightarrow x$



State	Input		Goto		
	x	$	S	A	B
0	s1		5	2	
1	r2	r2			
2	s3				4
3	r3	r3			
4	r1	r1			
5		accept			

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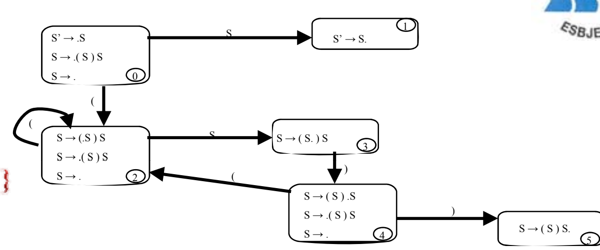
SLR (1) Parsing Table

S' → S

S → (S) S | ε

Follow (S') = { $ }

Follow (S) = { $,) }



State	Input			Goto
	(	)	$	S
0	S2	R (S → ε)	R (S → ε)	1
1			Accept	
2	S2	R (S → ε)	R (S → ε)	3
3		S4		
4	s2	R (S → ε)	R (S → ε)	5
5		R (S → (S) S)	R (S → (S) S)	

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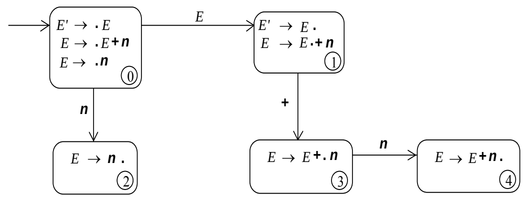
Parsing Table (*not LR(0)!*):



State	Input			Goto
	n	**+**	$	*E*
0	s2			1
1		s3	accept	
2		r ($E \rightarrow$ ***n***)	r ($E \rightarrow$ ***n***)	
3	s4			
4		r ($E \rightarrow E$ **+** ***n***)	r ($E \rightarrow E$ **+** ***n***)	

This table uses the SLR(1) rule: consult the Follow sets to decide between a shift and a reduce, or between two reduces:
Follow (E) = { + $ },
Follow (E') = {$}

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DANGLING-ELSE:

Grammar:

(1) $S \rightarrow I$
(2) $S \rightarrow$ other
(3) $I \rightarrow$ if S
(4) $I \rightarrow$ if S else S

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Dangling-else Example:

Grammar: $S \rightarrow I \mid \text{o}$

$I \rightarrow \text{i}\, S \mid \text{i}\, S\, \text{e}\, S$

Input: i i o e o

	Parsing stack	Input	Action
1	$	i i o e o $	shift
2	$ i	i o e o $	shift
3	$ i i	o e o $	shift
4	$ i i o	e o $	reduce $S \rightarrow \text{o}$
5	$ i i *S*	e o $	shift/reduce (shift)
6	$ i i *S* e	o $	shift
7	$ i i *S* e o	$	reduce $S \rightarrow \text{o}$
8	$ i i *S* e *S*	$	reduce $I \rightarrow \text{i}\, S\, \text{e}\, S$
9	$ i *I*	$	reduce $S \rightarrow I$
10	$ i *S*	$	reduce $I \rightarrow \text{i}\, S$
11	$ *I*	$	reduce $S \rightarrow I$
12	$ *S*	$	accept

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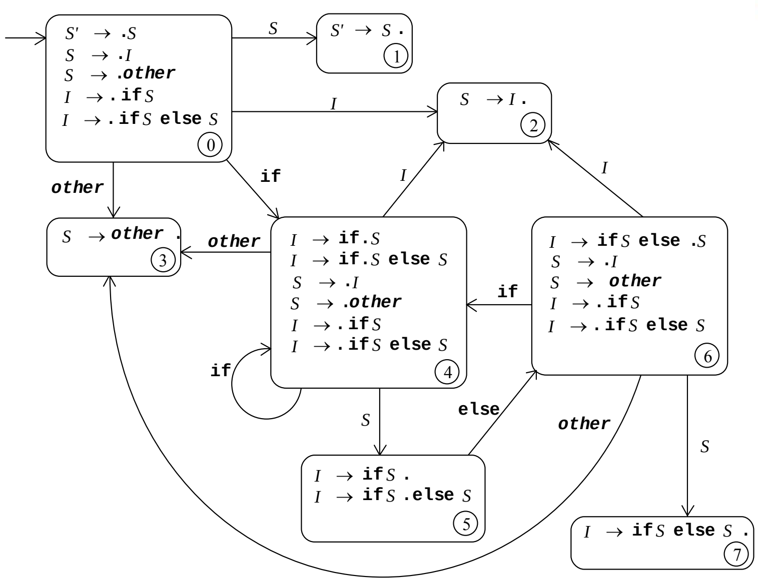
DFA - Dangling-else:



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First and Follow sets for Dangling-else

statement → *if-stmt* | *other*
if-stmt → if (*exp*) *statement else-part*
else-part → else *statement* | ε
exp → 0
First(*statement*) = {if, *other*}
First(*if-stmt*) = {if}
First(*else-part*) = {else, ε}
First(*exp*) = {0, 1}
Follow(*statement*) = {$,else}
Follow(*if-stmt*) = {$,else}
Follow(*else-part*) = {$,else}
Follow(*exp*) = {)}

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Parsing table (with conflict):

State	Input				Goto	
	if	else	other	$	S	I
0	s4		s3		1	2
1				accept		
2		r1		r1		
3		r2		r2		
4	s4		s3		5	2
5		s6/r3		r3		
6	s4		s3		7	2
7		r4		r4		

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When Follow Set Fails

Stmt → call_stmt | assign_stmt
call_stmt → identifier
assign_stmt → Var := exp
Var → Var [exp] | identifier
exp → Var | num

Simplified Version
S → id | V := E
V → id
E → V | n

Follow (S) = { $ }
Follow (V) = { $, := }

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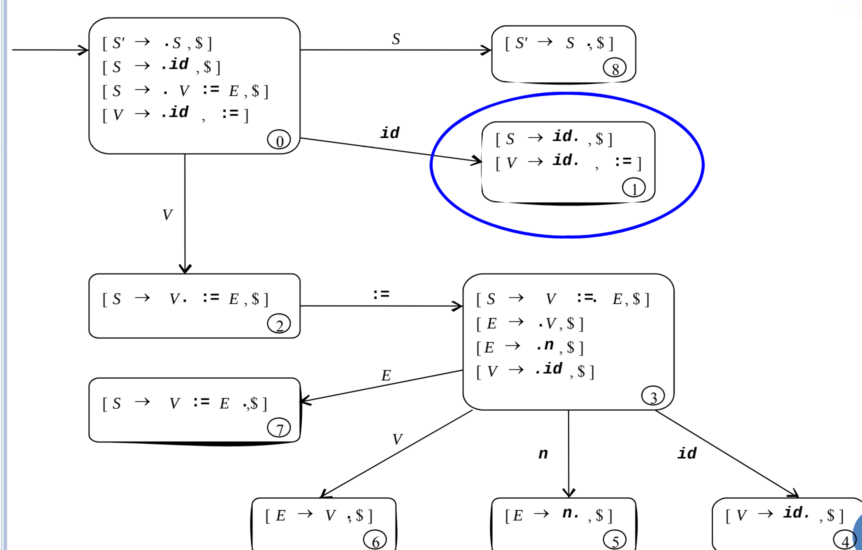
When Follow Set Fails



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LR (1) & LALR (1) Parsing

LR (1) resolve problems of SLR(1) parser

Price : Complexity

Modification of LR (1) to LALR (1):

Retaining most functionality of LR (1).

Keeping the efficiency of SLR (1).

In SLR (1) look-ahead is applied after construction of DFA's of LR (0) items.

For LR (1) look-ahead are built into the DFA's.

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LR (1) & LALR (1) Parsing

LR (1) item is: (LR (0) item + look-ahead token)

$[A \rightarrow \alpha.X\gamma, a]$

Given above LR (1) item if X is terminal then there will be transition to an item $[A \rightarrow \alpha X.\gamma, a]$.

If X is non-terminal then, there will be ε transition to $[X \rightarrow .\beta, b]$ for every $X \rightarrow .\beta$ and for every b in First (γa).

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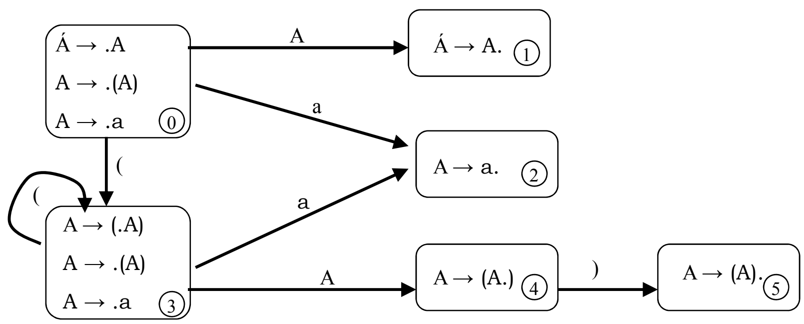
Example: A → (A) | a

LR (0) Items



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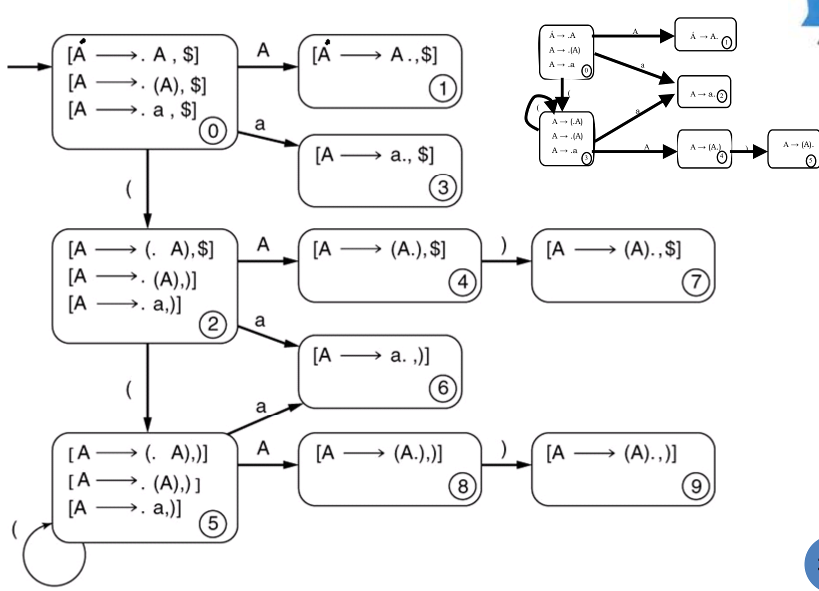
LR (1) DFA



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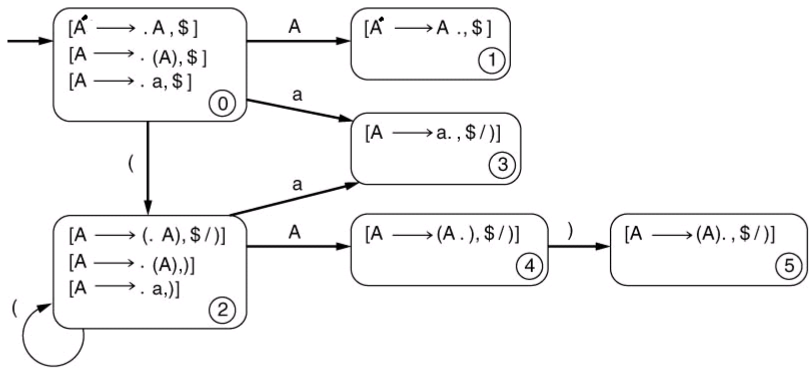
LALR (1) DFA



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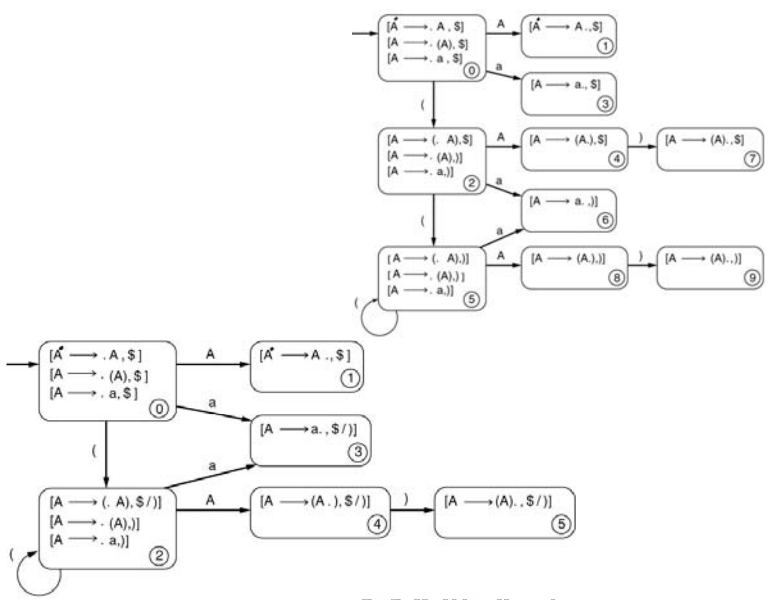
Comparison between LR(1) & LALR(1) States



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Shift Reduce Parsers differ in the use of Follow Set Information

- **LR(0)** parsers never consult the look-ahead at all.
- **SLR(1)** parsers use the Follow sets as previously constructed.
- **LR(1)** parsers use context to split the Follow sets into subsets for different parsing paths (huge, inefficient parsers).
- **LALR(1)** parsers: like LR(1) but coarser subsets are used (achieves most of the benefit, but much smaller and faster).

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Stack Problem in BU Parsers

Grammar: S → (S) S | ε   Grammar: S → S (S) | ε

Input String: ( ) ( )

Parsing Stack (BU)	Input	Parsing Stack (TD)
$0	( ) ( ) $	$0
$0 (2	) ( ) $	$0 S 1
$0 (2 S 3	) ( ) $	$0 S 1 (2
$0 (2 S 3) 4	( ) $	$0 S 1 (2 S 3
$0 (2 S 3) 4 (2	) $	$0 S 1 (2 S 3) 4
$0 (2 S 3) 4 (2 S 3	) $	$0 S 1
$0 (2 S 3) 4 (2 S 3) 4	$	$0 S 1 (2
$0 (2 S 3) 4 (2 S 3) 4 S 5	$	$0 S 1 (2 S 3
$ 0 (2 S 3) 4 5	$	$0 S 1 (2 S 3) 4
$ 0 S 1	$	$0 S 1

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