

# EN4 Course: Fundamental Control Theory and Modeling

Zhenyu Yang, H.332, Tel: 99407608, email: yang@et.aau.dk

## MM2: Essentials for Feedback Control

### 1 Reading

- Section 2.1 (models of mechanical systems, p.20-33);
- Section 3.1 (Laplace transform, p.86-110)
- Section 3.2 (Block diagram, p.111-118)

### 2 Content

- Essentials in using ODE model
- Refresh of Laplace transform
- Block diagram transformation
- Matlab/Simulink issues

### 3 Exercise

1. Consider a pendulum system which up-end of the pendulum is pivoted on a fixed surface. Through the first-principle modeling, a mathematical model can be obtained as

$$T_c - mgl\sin\theta = I\ddot{\theta}, \quad (1)$$

where  $T_c$  is the applied external torque,  $\theta$  is the pendulum's deviated angle from the vertical position,  $I$  is the moment of inertia of the pendulum, and it can be estimated as  $I = ml^2$ .

- Is this ODE model (1) linear or nonlinear? and why?
  - Suppose the motion is small enough that we can let  $\sin\theta \approx \theta$ , get a linear ODE model from (1) based on this approximation.
  - Suppose there is no any external force acting on the system, i.e.,  $T_c = 0$ , but the pendulum has an initial angle position  $\theta(0) = 0.5rad$  and initial angular velocity  $\dot{\theta}(0) = 0$ , can you image the dynamic behavior of the pendulum?
  - Based on above assumption, get a quantitative description of the pendulum performance. (Hint: derive the solution of an ODE)
  - Write a piece of M-file using the ode23 or ode45 solver in Matlab to simulate the considered system behavior. Compare the simulated result with the theoretical solution you obtained from last question.
  - Regard  $T_c$  as the system input and  $\theta$  as the system output, derive the transfer function of the considered linear system.
  - What're the system poles and zeros? what kind of information we can gain by this pole-zero analysis?
  - Can you manage to get a Simulink model of this considered system? and simulate it under the condition that  $\theta(0) = 0.5rad$ ,  $\dot{\theta}(0) = 0$  and  $T_c = 0$  as well.
  - Suppose the system is exposed to some wind disturbance, which can be regarded as an external torque, denoted as  $T_d$ , acting on the pendulum at the opposite direction of  $T_c$ . Can you extend this consideration into model (1)? so that what kind of system (SISO, SIMO, MISO, MIMO) you obtain?
  - Suppose the wind disturbance can be modeled as  $T_f = K_f\dot{\theta}$ , where  $K_f$  is some constant value, can you update the pendulum model and simulate the new system? What a kind of different result do you get, comparing with the situation where  $T_f$  is neglected?
2. Check the modeling of a crane system (or inverted pendulum) studied in Section 2.1 from the CC textbook. A linearized model can be obtained as

$$\begin{aligned} (I + m_p l^2)\ddot{\theta} + m_p g l \theta &= -m_p l \ddot{x} \\ (m_t + m_p)\ddot{x} + b\dot{x} + m_p l \dot{\theta} &= u \end{aligned} \quad (2)$$

- By regrading  $u$  as system input and  $\theta$  as system output, derive the transfer function of above system.
- Try to implement a simulink model of the original nonlinear system, i.e.,

$$\begin{aligned}(I + m_p l^2)\ddot{\theta} + m_p g l \sin\theta &= -m_p l \ddot{x} \cos\theta \\ (m_t + m_p)\ddot{x} + b\dot{x} + m_p l \ddot{\theta} \cos\theta - m_p l \dot{\theta}^2 \sin\theta &= u.\end{aligned}\tag{3}$$