Controller Design Based on Transient Response Criteria

Desirable Controller Features

0. Stable
1. Quick responding
2. Adequate disturbance rejection
3. Insensitive to model, measurement errors
4. Avoids excessive controller action
5. Suitable over a wide range of operating conditions

Impossible to satisfy all 5 unless self-tuning. Use “optimum sloppiness"
Chapter 12

Figure 12.1 Unit-step disturbance responses for the candidate controllers (FOPTD Model: $K = 1$, $\theta = 4$, $\tau = 20$).

Figure 12.3 Simulation results for Example 12.1 (a): correct model gain.
Alternatives for Controller Design

1. Tuning correlations – most limited to 1st order plus dead time
2. Closed-loop transfer function - analysis of stability or response characteristics.
3. Repetitive simulation (requires computer software like MATLAB and Simulink)
4. Frequency response - stability and performance (requires computer simulation and graphics)
5. On-line controller cycling (field tuning)
Controller Synthesis - Time Domain

Time-domain techniques can be classified into two groups:
(a) Criteria based on a few points in the response
(b) Criteria based on the entire response, or integral criteria

Approach (a): settling time, % overshoot, rise time, decay ratio (Fig. 5.10 can be viewed as closed-loop response)

Process model \[ G(s) = \frac{Ke^{-\theta_s}}{\tau s + 1} \] (1st order)

Several methods based on 1/4 decay ratio have been proposed: Cohen-Coon, Ziegler-Nichols

![Figure 5.10. Performance characteristics for the step response of an underdamped process.](image)
### Comparison of Ziegler-Nichols and Cohen-Coon Equations for Controller Tuning (1940’s, 50’s)

<table>
<thead>
<tr>
<th>Controller</th>
<th>Ziegler-Nichols</th>
<th>Cohen-Coon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional</td>
<td>[ KK_c = \left( \frac{\theta}{\tau} \right) ]</td>
<td>[ KK_c = \left( \frac{\theta}{\tau} \right) + \frac{1}{3} ]</td>
</tr>
<tr>
<td>Proportional + Integral</td>
<td>[ KK_c = 0.9 \left( \frac{\theta}{\tau} \right) ] [ \frac{\tau_i}{\tau} = 3.33 \left( \frac{\theta}{\tau} \right) ]</td>
<td>[ KK_c = 0.9 \left( \frac{\theta}{\tau} \right) + 0.083 ] [ \frac{\tau_i}{\tau} = \frac{0.33 + 0.33 \left( \frac{\theta}{\tau} \right)}{1.0 + 2.2 \left( \frac{\theta}{\tau} \right)} ]</td>
</tr>
<tr>
<td>Proportional + Integral + Derivative</td>
<td>[ KK_c = 1.2 \left( \frac{\theta}{\tau} \right) ] [ \frac{\tau_i}{\tau} = 2.0 \left( \frac{\theta}{\tau} \right) ] [ \frac{\tau_d}{\tau} = 0.5 \left( \frac{\theta}{\tau} \right) ]</td>
<td>[ KK_c = 1.35 \left( \frac{\theta}{\tau} \right) + 0.270 ] [ \frac{\tau_i}{\tau} = \frac{0.32 + 0.32 \left( \frac{\theta}{\tau} \right)}{1.3 + 8 \left( \frac{\theta}{\tau} \right)} ] [ \frac{\tau_d}{\tau} = \frac{0.37 \left( \frac{\theta}{\tau} \right)}{1.0 + 0.2 \left( \frac{\theta}{\tau} \right)} ]</td>
</tr>
</tbody>
</table>

---

### Graphical Interpretation of IAE

The shaded area is the IAE value.
Approach (b)

1. Integral of square error (ISE)
   \[ \text{ISE} = \int_{0}^{\infty} e(t)^2 \, dt \]

2. Integral of absolute value of error (IAE)
   \[ \text{IAE} = \int_{0}^{\infty} |e(t)| \, dt \]

3. Time-weighted IAE
   \[ \text{ITAE} = \int_{0}^{\infty} t |e(t)| \, dt \]

Pick controller parameters to minimize integral.

- IAE allows larger deviation than ISE (smaller overshoots)
- ISE longer settling time
- ITAE weights errors occurring later more heavily

Approximate optimum tuning parameters are correlated with K, \( \theta \), \( \tau \) (Table 12.3).

<table>
<thead>
<tr>
<th>Type of Input</th>
<th>Type of Controller</th>
<th>Mode</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>PI</td>
<td>P</td>
<td>0.859</td>
<td>-0.977</td>
</tr>
<tr>
<td>Load</td>
<td></td>
<td>I</td>
<td>0.674</td>
<td>-0.680</td>
</tr>
<tr>
<td></td>
<td></td>
<td>P</td>
<td>1.357</td>
<td>-0.947</td>
</tr>
<tr>
<td></td>
<td>PID</td>
<td>I</td>
<td>0.842</td>
<td>-0.738</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D</td>
<td>0.381</td>
<td>0.995</td>
</tr>
<tr>
<td>Set point</td>
<td>PI</td>
<td>P</td>
<td>0.586</td>
<td>-0.916</td>
</tr>
<tr>
<td>Set point</td>
<td></td>
<td>I</td>
<td>1.03^b</td>
<td>-0.165^b</td>
</tr>
<tr>
<td></td>
<td>PID</td>
<td>I</td>
<td>0.965</td>
<td>-0.85</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D</td>
<td>0.308</td>
<td>0.929</td>
</tr>
</tbody>
</table>

\(^{a}\text{Design relation: } Y = A(t/\tau)\text{ where } Y = KK_i \text{ for the proportional mode, } \tau/\tau_i \text{ for the integral mode, and } \tau/\tau_d \text{ for the derivative mode.}\)

\(^{b}\text{For set-point changes, the design relation for the integral mode is } \tau/\tau_i = A + B(0/\tau_i).\)
Summary of Tuning Relationships

1. $K_C$ is inversely proportional to $K_pK_vK_M$.

2. $K_C$ decreases as $\theta/\tau$ increases.

3. $\tau_I$ and $\tau_D$ increase as $\theta/\tau$ increases (typically $\tau_D = 0.25 \tau_I$).

4. Reduce $K_C$, when adding more integral action; increase $K_C$, when adding derivative action.

5. To reduce oscillation, decrease $K_C$ and increase $\tau_I$. 
Disadvantages of Tuning Correlations

1. Stability margin is not quantified.
2. Control laws can be vendor-specific.
3. First order + time delay model can be inaccurate.
4. $K_p$, $\tau$, and $\theta$ can vary.
5. Resolution, measurement errors decrease stability margins.
6. $1/4$ decay ratio not conservative standard (too oscillatory).

Figure 12.2. Feedback control strategies.
Direct Synthesis

\[ \frac{Y}{Y_{sp}} = \frac{G_cG}{1+G_cG} \quad (G \text{ includes } G_m, G_v) \]

1. Specify closed-loop response (transfer function)

\[ \left( \begin{array}{c}
Y \\
Y_{sp}
\end{array} \right)_{d} \]

2. Need process model, \( G (= G_PG MG_V) \)

3. Solve for \( G_c \),

\[ G_c = \frac{1}{G} \left( \begin{array}{c}
\frac{y}{y_{sp}} \\
1 - \frac{y}{y_{sp}}
\end{array} \right) \]

Example: Second Order Process with PI Controller Can Yield Second Order Closed-loop Response

\[ G(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)} \]

PI: \( G_c(s) = \frac{K_c(\tau_1 s + 1)}{\tau_1 s} \quad \text{or} \quad G_c(s) = K_c \left( 1 + \frac{1}{\tau_1 s} \right) \)

Let \( \tau_1 = \tau_1 \), where \( \tau_1 > \tau_2 \)

Canceling terms,

\[ \frac{Y}{Y_{sp}} = \frac{KK_c}{\tau_1 s (\tau_2 s + 1) + KK_c} \quad \text{Check gain } (s = 0) \]
2\textsuperscript{nd} order response with...
\[ \tau = \sqrt{\frac{\tau_1 \tau_2}{K_c K}} \quad \text{and} \quad \zeta = \frac{1}{2} \sqrt{\frac{\tau_1}{\tau_2 K_c K}} \]

Select $K_c$ to give $\zeta = 0.4 - 0.5$ (overshoot)

Figure. Step response of underdamped second-order processes and first-order process.

**Direct Synthesis**

\[ \frac{Y}{Y_{sp}} = \frac{G_c G}{1 + G_c G} \quad (G \text{ includes } G_{in}, G_v) \]

1. Specify closed-loop response (transfer function)
\[ \left( \frac{Y}{Y_{sp}} \right)_d \]

2. Need process model, $\bar{G}$ ($= G p_{G_v} G_v$)

3. Solve for $G_c$
\[ G_c = \frac{1}{G} \left( \frac{Y}{Y_{sp}} \right)_d \left( 1 - \frac{Y}{Y_{sp}} \right)_d \] (12-3b)
Specify Closed – Loop Transfer Function

\[
\frac{\bar{Y}}{\bar{Y}_p} = \frac{e^{-\theta s}}{\tau_c s + 1} \quad (12-6)
\]

(first – order response, no offset)

\[
(\tau_c = \text{speed of response}, \theta = \text{process time delay in } G)
\]

But other variations of (12-6) can be used (e.g., replace time delay with polynomial approximation)

If \( \theta = 0 \), then (12 - 3b) yields \( G_c = \frac{1}{G} \cdot \frac{1}{\tau_c s} \) \hspace{1cm} (12 - 5)

For \( \tilde{G} = \frac{K}{\tau s + 1} \), \( G_c = \frac{\tau s + 1}{K\tau_s} = \frac{\tau}{K\tau_c} + \frac{1}{K\tau_s} \) \hspace{1cm} (PI)

Deviation of PI Controller for FOPTD Process

Consider the standard first-order-plus-time-delay model,

\[
\tilde{G}(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \quad (12-10)
\]

Specify closed-loop response as FOPTD (12-6), but approximate \( e^{-\theta s} \approx 1 - \theta s \).

Substituting and rearranging gives a PI controller,

\[
G_c = K_c (1 + 1/\tau_f s), \text{ with the following controller settings:}
K_c = \frac{1}{K \theta + \tau_c}, \quad \tau_f = \tau \quad (12-11)
\]
Derivation of PID Controller for FOPTD Process:

Let \( \frac{Y}{\hat{Y}} = \frac{1-\frac{\theta}{2}s}{\tau_s s + 1} \)

\( \tilde{G}(s) = \frac{K e^{-\theta s}}{\tau_s + 1} \approx \frac{K(1-\frac{\theta}{2}s)}{(\tau_s + 1)(1+\frac{\theta}{2}s)} \)

\[ G_c = \frac{1}{\tilde{G}} \left( \frac{\frac{Y}{\hat{Y}}}{1-\frac{\theta}{2}s} \right) \] (12-3b)

\[ G_c = \frac{(\tau_s + 1)(1+\frac{\theta}{2}s)}{K(1-\frac{\theta}{2}s)} \cdot \frac{1-\frac{\theta}{2}s}{\tau_s s + 1} = \frac{K(\tau_s + 1)(1+\theta^2/2s)}{1-\frac{\theta}{2}s} \] (12-2a)

\[ K_c = K \frac{2\left(\frac{\tau_s}{\theta}+1\right)}{2\left(\frac{\tau_s}{\theta}+1\right)} \quad \tau_f = \frac{\theta}{2} + \tau \quad \tau_D = \frac{\tau}{2(\tau_s/\theta)+1} \] (12-30)

Second-Order-plus-Time-Delay (SOPTD) Model

Consider a second-order-plus-time-delay model,

\[ \tilde{G}(s) = \frac{K e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)} \] (12-12)

Use of FOPTD closed-loop response (12-6) and time delay approximation gives a PID controller in parallel form,

\[ G_c = K_c \left( 1 + \frac{1}{\tau_f s} + \tau_D s \right) \] (12-13)

where

\[ K_c = \frac{1}{K} \frac{\tau_1 + \tau_2}{\tau_c + \theta}, \quad \tau_f = \tau_1 + \tau_2, \quad \tau_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} \] (12-14)
Consider three values of the desired closed-loop time constant: \( \tau = 1, 3, \text{ and } 10 \). Evaluate the controllers for unit step changes in both the set point and the disturbance, assuming that \( G_d = G \).

Perform the evaluation for two cases:

a. The process model is perfect (\( \bar{G} = G \)).

b. The model gain is \( \bar{K} = 0.9 \), instead of the actual value, \( K = 2 \). This model error could cause a robustness problem in the controller for \( K = 2 \). \( \bar{G} = \frac{0.9e^{-s}}{10s + 1}(5s + 1) \)

Example 12.1

Use the DS design method to calculate PID controller settings for the process:

\[
G = \frac{2e^{-s}}{(10s + 1)(5s + 1)}
\]

Consider three values of the desired closed-loop time constant: \( \tau = 1, 3, \text{ and } 10 \). Evaluate the controllers for unit step changes in both the set point and the disturbance, assuming that \( G_d = G \).

Perform the evaluation for two cases:

a. The process model is perfect (\( \bar{G} = G \)).

b. The model gain is \( \bar{K} = 0.9 \), instead of the actual value, \( K = 2 \). This model error could cause a robustness problem in the controller for \( K = 2 \). \( \bar{G} = \frac{0.9e^{-s}}{10s + 1}(5s + 1) \)
The IMC controller settings for this example are:

<table>
<thead>
<tr>
<th></th>
<th>$\tau_c = 1$</th>
<th>$\tau_c = 3$</th>
<th>$\tau_c = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_c (\hat{K} = 2)$</td>
<td>3.75</td>
<td>1.88</td>
<td>0.682</td>
</tr>
<tr>
<td>$K_c (\hat{K} = 0.9)$</td>
<td>8.33</td>
<td>4.17</td>
<td>1.51</td>
</tr>
<tr>
<td>$\tau_f$</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>$\tau_D$</td>
<td>3.33</td>
<td>3.33</td>
<td>3.33</td>
</tr>
</tbody>
</table>

Note only $K_c$ is affected by the change in process gain.

The values of $K_c$ decrease as $\tau_c$ increases, but the values of $\tau_I$ and $\tau_D$ do not change, as indicated by Eq. 12-14.

Figure 12.3 Simulation results for Example 12.1 (a): correct model gain.
Controller Tuning Relations

Model-based design methods such as DS and IMC produce PI or PID controllers for certain classes of process models, with one tuning parameter $\tau_c$ (see Table 12.1)

**How to Select $\tau_c$?**

- Several IMC guidelines for $\tau_c$ have been published for the model in Eq. 12-10:

1. $\tau_c / \theta > 0.8$ and $\tau_c > 0.1\tau$  
   (Rivera et al., 1986)
2. $\tau > \tau_c > 0$  
   (Chien and Fruehauf, 1990)
3. $\tau_c = \theta$  
   (Skogestad, 2003)

Figure 12.4  Simulation results for Example 12.1 (b): incorrect model gain.
Tuning for Lag-Dominant Models

- First- or second-order models with relatively small time delays ($0/\tau<1$) are referred to as lag-dominant models.

- The IMC and DS methods provide satisfactory set-point responses, but very slow disturbance responses, because the value of $\tau_I$ is very large. ($\tau_I = \tau$)

- Fortunately, this problem can be solved in three different ways.

Method 1: Integrator Approximation

\[ \tilde{G}(s) = \frac{Ke^{-0s}}{\tau s + 1} \text{ by } \tilde{G}(s) = \frac{K* e^{-0s}}{s} \]

where $K* = K / \tau$.

- Then can use the IMC tuning rules (Rule M or N) to specify the controller settings.

Method 2. Limit the Value of $\tau_I$

- Skogestad (2003) has proposed limiting the value of $\tau_I$:

\[ \tau_I = \min\{\tau_I, 4(\tau_c + \theta)\} \quad (12-34) \]

where $\tau_I$ is the largest time constant (if there are two).

Method 3. Design the Controller for Disturbances, Rather than Set-point Changes

- The desired CLTF is expressed in terms of $(Y/D)_d$ rather than $(Y/Y_d)_d$

Reference: Chen & Seborg (2002)
Example 12.4

Consider a lag-dominant model with $\theta / \tau = 0.01$:

$$\tilde{G}(s) = \frac{100}{100s + 1} e^{-s}$$

Design three PI controllers:

a) IMC ($\tau_c = 1$)

b) IMC ($\tau_c = 2$) based on the integrator approximation in Eq. 12-33

c) IMC ($\tau_c = 1$) with Skogestad’s modification (Eq. 12-34)

Evaluate the three controllers by comparing their performance for unit step changes in both set point and disturbance. Assume that the model is perfect and that $G_d(s) = G(s)$.

Solution

The PI controller settings are:

<table>
<thead>
<tr>
<th>Controller</th>
<th>$K_c$</th>
<th>$\tau_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) IMC</td>
<td>0.5</td>
<td>100</td>
</tr>
<tr>
<td>(b) Integrator approximation</td>
<td>0.556</td>
<td>5</td>
</tr>
<tr>
<td>(c) Skogestad</td>
<td>0.5</td>
<td>8</td>
</tr>
</tbody>
</table>
Chapter 12

On-Line Controller Tuning

1. *Controller tuning inevitably involves a tradeoff between performance and robustness.*

2. *Controller settings do not have to be precisely determined.* In general, a small change in a controller setting from its best value (for example, ±10%) has little effect on closed-loop responses.

3. *For most plants, it is not feasible to manually tune each controller.* Tuning is usually done by a control specialist (engineer or technician) or by a plant operator. Because each person is typically responsible for 300 to 1000 control loops, it is not feasible to tune every controller.

4. *Diagnostic techniques for monitoring control system performance are available.*
Controller Tuning and Troubleshooting Control Loops

Ziegler-Nichols Rules:
These well-known tuning rules were published by Z-N in 1942:

<table>
<thead>
<tr>
<th>controller</th>
<th>$K_c$</th>
<th>$\tau_l$</th>
<th>$\tau_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>$0.5 , K_{CU}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>PI</td>
<td>$0.45 , K_{CU}$</td>
<td>$P_U/1.2$</td>
<td>-</td>
</tr>
<tr>
<td>PID</td>
<td>$0.6 , K_{CU}$</td>
<td>$P_U/2$</td>
<td>$P_U/8$</td>
</tr>
</tbody>
</table>

Z-N controller settings are widely considered to be an "industry standard".

Z-N settings were developed to provide 1/4 decay ratio -- too oscillatory?
Modified Z-N settings for PID control

<table>
<thead>
<tr>
<th>controller</th>
<th>$K_c$</th>
<th>$\tau_1$</th>
<th>$\tau_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>original</td>
<td>$0.6 , K_{CU}$</td>
<td>$P_u/2$</td>
<td>$P_u/8$</td>
</tr>
<tr>
<td>Some overshoot</td>
<td>$0.33 , K_{CU}$</td>
<td>$P_u/2$</td>
<td>$P_u/3$</td>
</tr>
<tr>
<td>No overshoot</td>
<td>$0.2 , K_{CU}$</td>
<td>$P_u/3$</td>
<td>$P_u/2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>controller</th>
<th>$K_c$</th>
<th>$\tau_1$</th>
<th>$\tau_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>$0.5K_{cu}$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>PI</td>
<td>$0.45K_{cu}$</td>
<td>$P_u/1.2$</td>
<td>—</td>
</tr>
<tr>
<td>PID</td>
<td>$0.6K_{cu}$</td>
<td>$P_u/2$</td>
<td>$P_u/8$</td>
</tr>
</tbody>
</table>

**Table 12.6 Controller Settings based on the Continuous Cycling Method**

<table>
<thead>
<tr>
<th>Ziegler-Nichols</th>
<th>$K_c$</th>
<th>$\tau_1$</th>
<th>$\tau_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>$0.5K_{cu}$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>PI</td>
<td>$0.45K_{cu}$</td>
<td>$P_u/1.2$</td>
<td>—</td>
</tr>
<tr>
<td>PID</td>
<td>$0.6K_{cu}$</td>
<td>$P_u/2$</td>
<td>$P_u/8$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tyreus-Luyben†</th>
<th>$K_c$</th>
<th>$\tau_1$</th>
<th>$\tau_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>$0.31K_{cu}$</td>
<td>$2.2P_u$</td>
<td>—</td>
</tr>
<tr>
<td>PID</td>
<td>$0.45K_{cu}$</td>
<td>$2.2P_u$</td>
<td>$P_u/6.3$</td>
</tr>
</tbody>
</table>

† Luyben and Luyben (1997).
Figure 12.13 Comparison of PID controllers for Example 12.7.

Figure 12.14 Auto-tuning using a relay controller.
Figure 12.15 Typical process reaction curves: (a) non-self-regulating process, (b) self-regulating process.

Figure 12.16 Process reaction curve for Example 12.8.
Figure 12.17 Block diagram for Example 12.8.