Chapter 12

Controller Design Based on Transient Response Criteria

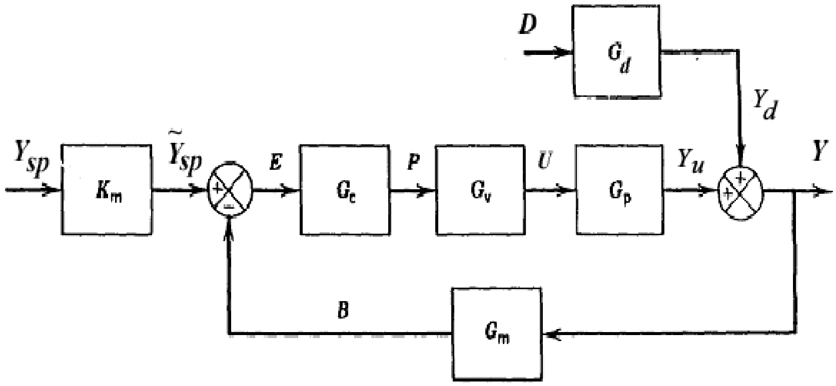


Figure 12.1. Block diagram for a standard feedback control system.

Chapter 12

Desirable Controller Features

0. Stable
1. Quick responding
2. Adequate disturbance rejection
3. Insensitive to model, measurement errors
4. Avoids excessive controller action
5. Suitable over a wide range of operating conditions

Impossible to satisfy all 5 unless self-tuning. Use "optimum sloppiness"

Chapter 12

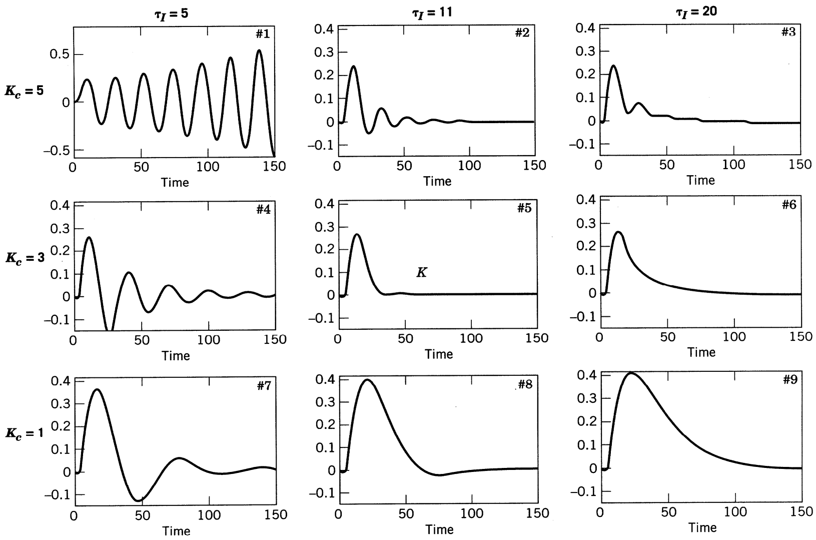


Figure 12.1 Unit-step disturbance responses for the candidate controllers (FOPTD Model: $K = 1$, $\theta = 4$, $\tau = 20$).

Chapter 12

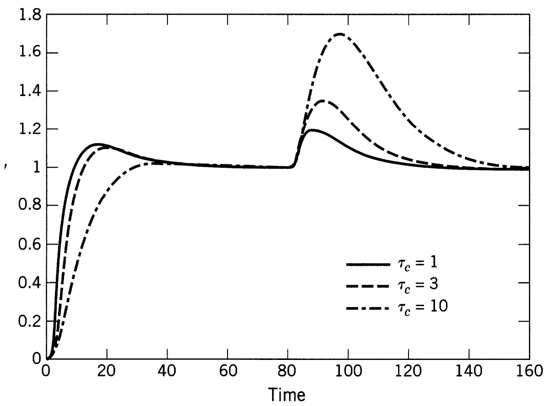


Figure 12.3 Simulation results for Example 12.1 (a): correct model gain.

Chapter 12

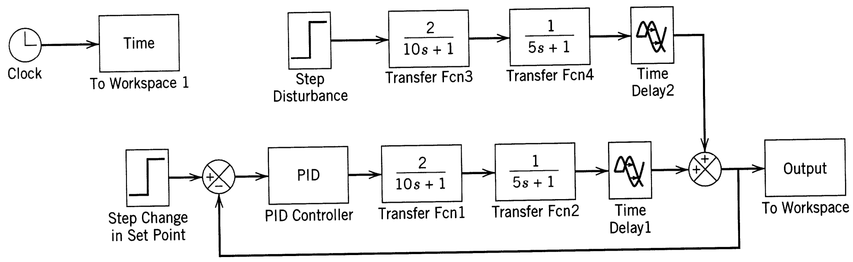


Figure 12.5 Simulink diagram for Example 12.1.

Chapter 12

Alternatives for Controller Design

1.Tuning correlations – most limited to 1st order plus dead time

2.Closed-loop transfer function - analysis of stability or response characteristics.

3.Repetitive simulation (requires computer software like MATLAB and Simulink)

4.Frequency response - stability and performance (requires computer simulation and graphics)

5.On-line controller cycling (field tuning)

Chapter 12

Controller Synthesis - Time Domain

Time-domain techniques can be classified into two groups:
(a) Criteria based on a few points in the response
(b) Criteria based on the entire response, or integral criteria

Approach (a): settling time, % overshoot, rise time, decay ratio (Fig. 5.10 can be viewed as closed-loop response)

Process model $G(s) = \dfrac{Ke^{-\theta s}}{\tau s + 1}$ (1st order)

Several methods based on 1/4 decay ratio have been proposed: Cohen-Coon, Ziegler-Nichols

Chapter 12

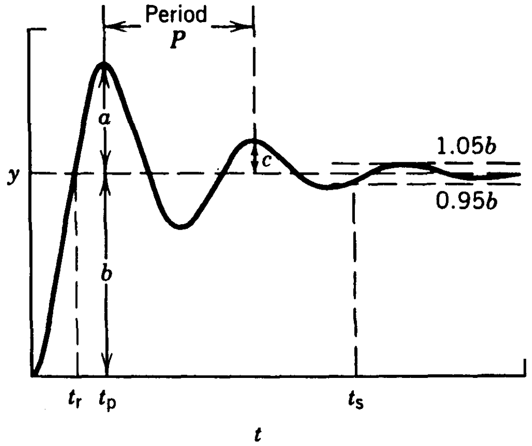


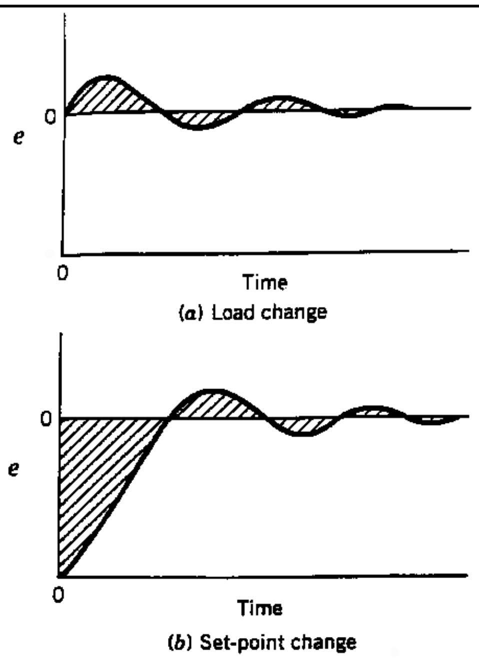
Figure 5.10. Performance characteristics for the step response of an underdamped process.

Chapter 12

Comparison of Ziegler-Nichols and Cohen-Coon Equations for Controller Tuning (1940's, 50's)

Controller	Ziegler-Nichols	Cohen-Coon
Proportional	$KK_C = (\tau/\theta)$	$KK_C = (\tau/\theta) + 1/3$
Proportional + Integral	$KK_C = 0.9(\tau/\theta)$ $\frac{\tau_I}{\tau} = 3.33(\theta/\tau)$	$KK_C = 0.9(\tau/\theta) + 0.083$ $\frac{\tau_I}{\tau} = \frac{\theta[3.33 + 0.33(\theta/\tau)]}{1.0 + 2.2(\theta/\tau)}$
Proportional + Integral + Derivative	$KK_C = 1.2(\tau/\theta)$ $\frac{\tau_I}{\tau} = 2.0(\theta/\tau)$ $\frac{\tau_D}{\tau} = 0.5(\theta/\tau)$	$KK_C = 1.35(\tau/\theta) + 0.270$ $\frac{\tau_I}{\tau} = \frac{\theta[32 + 6(\theta/\tau)]}{13 + 8(\theta/\tau)}$ $\frac{\tau_D}{\tau} = \frac{0.37(\theta/\tau)}{1.0 + 0.2(\theta/\tau)}$

Chapter 12



Graphical interpretation of IAE. The shaded area is the IAE value.

Chapter 12

Approach (b)

1. Integral of square error (ISE)

$$ISE = \int_0^\infty [e(t)]^2 dt$$

2. Integral of absolute value of error (IAE)

$$IAE = \int_0^\infty |e(t)| dt$$

3. Time-weighted IAE

$$ITAE = \int_0^\infty t|e(t)| dt$$

Pick controller parameters to minimize integral.

IAE allows larger deviation than ISE (smaller overshoots)
ISE longer settling time
ITAE weights errors occurring later more heavily

Approximate optimum tuning parameters are correlated with K, θ, τ (Table 12.3).

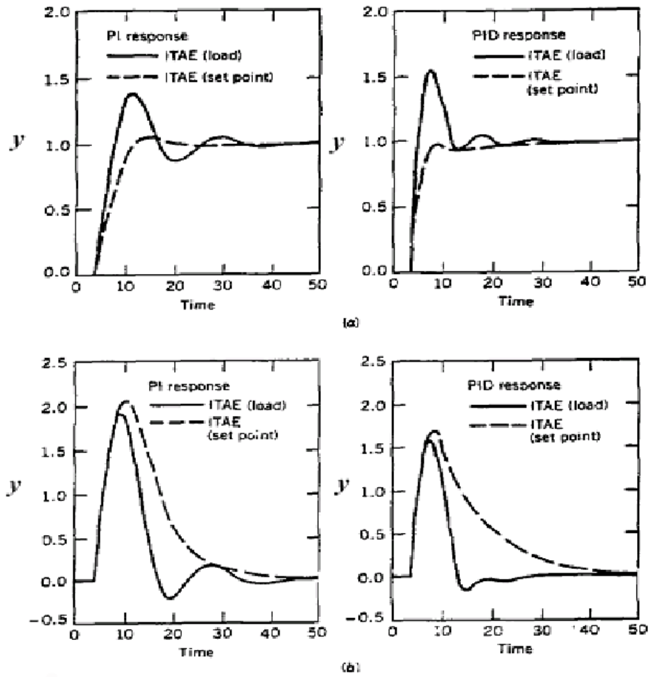
Chapter 12

Table 12.3 Controller Design Relations Based on the ITAE Performance Index and a First-Order plus Time-Delay Model

Type of Input	*Type of Controller*	*Mode*	*A*	*B*
Load	PI	P	0.859	−0.977
		I	0.674	−0.680
Load	PID	P	1.357	−0.947
		I	0.842	−0.738
		D	0.381	0.995
Set point	PI	P	0.586	−0.916
		I	1.03[b]	−0.165[b]
Set point	PID	P	0.965	−0.85
		I	0.796[b]	−0.1465[b]
		D	0.308	0.929

[a]Design relation: $Y = A(\theta/\tau)^B$ where $Y = KK_c$ for the proportional mode, τ/τ_I for the integral mode, and τ_D/τ for the derivative mode.
[b]For set-point changes, the design relation for the integral mode is $\tau/\tau_I = A + B(\theta/\tau)$. [8]

Chapter 12



Comparison of controllers designed using ITAE criteria for (a) set-point and (b) load changes.

Chapter 12

Summary of Tuning Relationships

1. K_C is inversely proportional to $K_P K_V K_M$.

2. K_C decreases as θ/τ increases.

3. τ_I and τ_D increase as θ/τ increases (typically $\tau_D = 0.25\ \tau_I$).

4. Reduce K_c, when adding more integral action; increase K_c, when adding derivative action

5. To reduce oscillation, decrease K_C and increase τ_I .

Chapter 12

Disadvantages of Tuning Correlations

1. Stability margin is not quantified.
2. Control laws can be vendor - specific.
3. First order + time delay model can be inaccurate.
4. K_p, τ, and θ can vary.
5. Resolution, measurement errors decrease stability margins.
6. ¼ decay ratio not conservative standard (too oscillatory).

Chapter 12

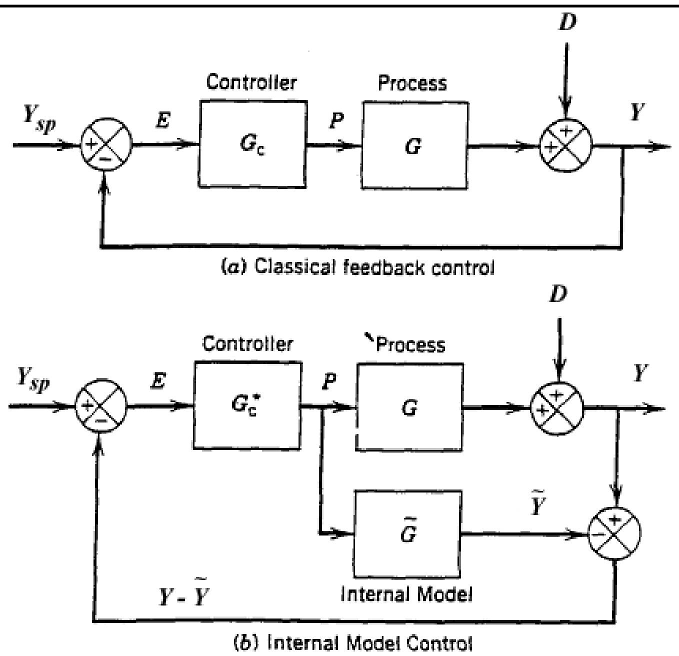


Figure 12.2. Feedback control strategies.

Chapter 12

Direct Synthesis

$$\frac{Y}{Y_{sp}} = \frac{G_C G}{1 + G_C G}$$ (G includes G_m, G_v)

1. Specify closed-loop response (transfer function)

$$\left(\frac{Y}{Y_{sp}}\right)_d$$

2. Need process model, G (= $G_P G_M G_V$)

3. Solve for G_c,

$$G_C = \frac{1}{G}\left(\frac{\left(\frac{Y}{Y_{sp}}\right)_d}{1-\left(\frac{Y}{Y_{sp}}\right)_d}\right)$$

Chapter 12

Example: Second Order Process with PI Controller Can Yield Second Order Closed-loop Response

$$G(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

PI: $G_C(s) = \frac{K_C(\tau_I s + 1)}{\tau_I s}$ or $G_C(s) = K_C\left(1 + \frac{1}{\tau_I s}\right)$

Let $\tau_I = \tau_1$, where $\tau_1 > \tau_2$

Canceling terms,

$$\frac{Y}{Y_{sp}} = \frac{KK_C}{\tau_1 s(\tau_2 s + 1) + KK_C}$$ Check gain (s = 0)

Chapter 12

2nd order response with...

$$\tau = \sqrt{\frac{\tau_1\tau_2}{K_C K}} \quad \text{and} \quad \zeta = \frac{1}{2}\sqrt{\frac{\tau_1}{\tau_2 K_C K}}$$

Select K_c to give $\zeta = 0.4 - 0.5$ (overshoot)

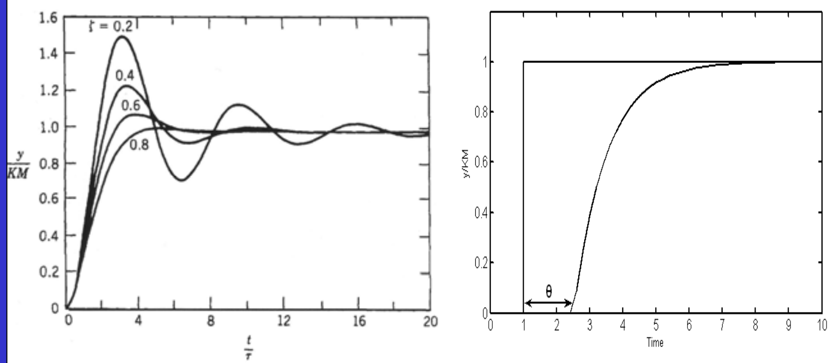


Figure. Step response of underdamped second-order processes and first-order process.

Chapter 12

Direct Synthesis

$$\frac{Y}{Y_{sp}} = \frac{G_C G}{1 + G_C G}$$ (G includes G_m, G_v)

1. Specify closed-loop response (transfer function)

$$\left(\frac{Y}{Y_{sp}}\right)_d$$

2. Need process model, $\tilde{G}$ (= $G_P G_M G_V$)

3. Solve for G_c,

$$G_C = \frac{1}{\tilde{G}}\left(\frac{\left(\frac{Y}{Y_{sp}}\right)_d}{1 - \left(\frac{Y}{Y_{sp}}\right)_d}\right) \quad (12\text{-}3b)$$

Specify Closed – Loop Transfer Function

$$\left(\frac{Y}{Y_{sp}}\right)_d = \frac{e^{-\theta s}}{\tau_c s+1} \qquad (12-6)$$

(first – order response, no offset)

$$\left(\tau_c = speed\ of\ response, \theta = process\ time\ delay\ in\ G\right)$$

But other variations of (12-6) can be used (e.g., replace time delay with polynomial approximation)

If $\theta = 0$, then (12 - 3b) yields $G_c = \frac{1}{\tilde{G}} \cdot \frac{1}{\tau_c s})$ (12 - 5)

For $\tilde{G} = \frac{K}{\tau s+1}, G_c = \frac{\tau s+1}{K\tau_c s} = \frac{\tau}{K\tau_c} + \frac{1}{K\tau_c s}$ (PI)

Chapter 12

Deviation of PI Controller for FOPTD Process

Consider the standard first-order-plus-time-delay model,

$$\tilde{G}(s) = \frac{Ke^{-\theta s}}{\tau s+1} \qquad \text{(12-10)}$$

Specify closed-loop response as FOPTD (12-6), but approximate $e^{-\theta s} \approx 1 - \theta s$.

Substituting and rearranging gives a PI controller,

$G_c = K_c\left(1 + 1/\tau_I s\right)$, with the following controller settings:

$$K_c = \frac{1}{K}\frac{\tau}{\theta + \tau_c}, \qquad \tau_I = \tau \qquad \text{(12-11)}$$

Chapter 12

Derivation of PID Controller for FOPTD Process:

let $\left(\frac{Y}{Y_{sp}}\right)_d = \frac{1-\theta/2\, s}{\tau_c s+1}$ $\quad \tilde{G}(s) = \frac{Ke^{-\theta s}}{\tau s+1} \approx \frac{K(1-\theta/2\, s)}{(\tau s+1)(1+\theta/2\, s)}$

$$G_c = \frac{1}{\tilde{G}}\left(\frac{\left(\frac{Y}{Y_{sp}}\right)_d}{1-\left(\frac{Y}{Y_{sp}}\right)_d}\right) \qquad (12\text{-}3b)$$

$$G_c = \frac{(\tau s+1)\left(1+\theta/2\, s\right)}{K\left(1-\theta/2\, s\right)} \bullet \frac{\frac{1-\theta/2\, s}{\tau_c s+1}}{1-\frac{1-\theta/2\, s}{\tau_c s+1}} = \frac{(\tau s+1)\left(1+\theta/2\, s\right)}{K\left(\theta/2+\tau_c\right)s} \qquad (12\text{-}2a)$$

$$K_c = \frac{1}{K}\frac{2\left(\frac{\tau}{\theta}\right)+1}{2\left(\frac{\tau_c}{\theta}\right)+1} \qquad \tau_I = \frac{\theta}{2}+\tau \qquad \tau_D = \frac{\tau}{2(\tau/\theta)+1} \qquad (12\text{-}30)$$

Chapter 12

Second-Order-plus-Time-Delay (SOPTD) Model

Consider a second-order-plus-time-delay model,

$$\tilde{G}(s) = \frac{Ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)} \qquad (12\text{-}12)$$

Use of FOPTD closed-loop response (12-6) and time delay approximation gives a PID controller in parallel form,

$$G_c = K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right) \qquad (12\text{-}13)$$

where

$$K_c = \frac{1}{K}\frac{\tau_1+\tau_2}{\tau_c+\theta}, \qquad \tau_I = \tau_1+\tau_2, \qquad \tau_D = \frac{\tau_1\tau_2}{\tau_1+\tau_2} \qquad (12\text{-}14)$$

Chapter 12

Table 12.1 IMC-Based PID Controller Settings for $G_c(s)$ (Chien and Fruehauf, 1990)

Case	Model	K_cK	τ_I	τ_D
A	$\frac{K}{\tau s+1}$	$\frac{\tau}{\tau_c}$	τ	—
B	$\frac{K}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{\tau_1+\tau_2}{\tau_c}$	$\tau_1+\tau_2$	$\frac{\tau_1\tau_2}{\tau_1+\tau_2}$
C	$\frac{K}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{2\zeta\tau}{\tau_c}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
D	$\frac{K(-\beta s+1)}{\tau^2 s^2+2\zeta\tau s+1},\ \beta>0$	$\frac{2\zeta\tau}{\tau_c+\beta}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
E	$\frac{K}{s}$	$\frac{2}{\tau_c}$	$2\tau_c$	—
F	$\frac{K}{s(\tau s+1)}$	$\frac{2\tau_c+\tau}{\tau_c^2}$	$2\tau_c+\tau$	$\frac{2\tau_c\tau}{2\tau_c+\tau}$
G	$\frac{Ke^{-\theta s}}{\tau s+1}$	$\frac{\tau}{\tau_c+\theta}$	τ	—
H	$\frac{Ke^{-\theta s}}{\tau s+1}$	$\frac{\tau+\frac{\theta}{2}}{\tau_c+\frac{\theta}{2}}$	$\tau+\frac{\theta}{2}$	$\frac{\tau\theta}{2\tau+\theta}$
I	$\frac{K(\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{\tau_1+\tau_2-\tau_3}{\tau_c+\theta}$	$\tau_1+\tau_2-\tau_3$	$\frac{\tau_1\tau_2-(\tau_1+\tau_2-\tau_3)\tau_3}{\tau_1+\tau_2-\tau_3}$
J	$\frac{K(\tau_3 s+1)e^{-\theta s}}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{2\zeta\tau-\tau_3}{\tau_c+\theta}$	$2\zeta\tau-\tau_3$	$\frac{\tau^2-(2\zeta\tau-\tau_3)\tau_3}{2\zeta\tau-\tau_3}$
K	$\frac{K(-\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}{\tau_c+\tau_3+\theta}$	$\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}$	$\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}+\frac{\tau_1\tau_2}{\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}$
L	$\frac{K(-\tau_3 s+1)e^{-\theta s}}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}{\tau_c+\tau_e+\theta}$	$2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}$	$\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}+\frac{\tau^2}{2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}$
M	$\frac{Ke^{-\theta s}}{s}$	$\frac{2\tau_c+\theta}{(\tau_c+\theta)^2}$	$2\tau_c+\theta$	—
N	$\frac{Ke^{-\theta s}}{s}$	$\frac{2\tau_c+\theta}{\left(\tau_c+\frac{\theta}{2}\right)^2}$	$2\tau_c+\theta$	$\frac{\tau_c\theta+\frac{\theta^2}{4}}{2\tau_c+\theta}$
O	$\frac{Ke^{-\theta s}}{s(\tau s+1)}$	$\frac{2\tau_c+\tau+\theta}{(\tau_c+\theta)^2}$	$2\tau_c+\tau+\theta$	$\frac{(2\tau_c+\theta)\tau}{2\tau_c+\tau+\theta}$

Chapter 12

Example 12.1

Use the DS design method to calculate PID controller settings for the process:

$$G=\frac{2e^{-s}}{(10s+1)(5s+1)}$$

Consider three values of the desired closed-loop time constant: τ = 1, 3, and 10. Evaluate the controllers for unit step changes in both the set point and the disturbance, assuming that $G_d = G$. Perform the evaluation for two cases:

a. The process model is perfect ($\tilde{G} = G$).

b. The model gain is $\tilde{K} = 0.9$, instead of the actual value, $K = 2$. This model error could cause a robustness problem in the controller for $K = 2$.

$$\tilde{G}=\frac{0.9e^{-s}}{(10s+1)(5s+1)}$$

The IMC controller settings for this example are:

	$\tau_c = 1$	$\tau_c = 3$	$\tau_c = 10$
$K_c\left(\tilde{K} = 2\right)$	**3.75**	**1.88**	**0.682**
$K_c\left(\tilde{K} = 0.9\right)$	**8.33**	**4.17**	**1.51**
τ_I	**15**	**15**	**15**
τ_D	**3.33**	**3.33**	**3.33**

Note only K_c is affected by the change in process gain.

Chapter 12

The values of K_c decrease as τ_c increases, but the values of τ_I and τ_D do not change, as indicated by Eq. 12-14.

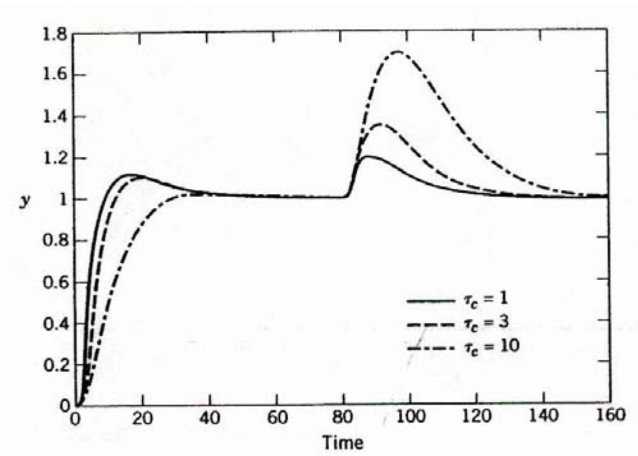


Figure 12.3 Simulation results for Example 12.1 (a): correct model gain.

Chapter 12

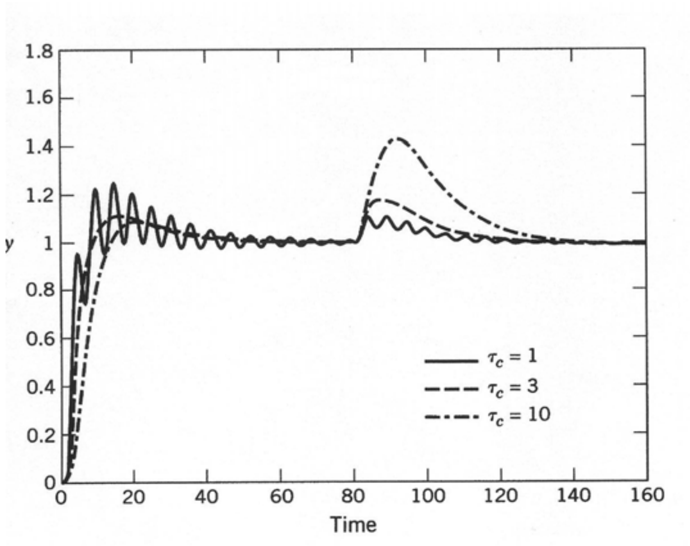


Figure 12.4 Simulation results for Example 12.1 (b): incorrect model gain.

Chapter 12

Controller Tuning Relations

Model-based design methods such as DS and IMC produce PI or PID controllers for certain classes of process models, with one tuning parameter τ_c (see Table 12.1)

How to Select τ_c?

- Several IMC guidelines for τ_c have been published for the model in Eq. 12-10:

1. $\tau_c / \theta > 0.8$ and $\tau_c > 0.1\tau$ (Rivera et al., 1986)
2. $\tau > \tau_c > \theta$ (Chien and Fruehauf, 1990)
3. $\tau_c = \theta$ (Skogestad, 2003)

Chapter 12

Tuning for Lag-Dominant Models

- First- or second-order models with relatively small time delays $(\theta / \tau << 1)$ are referred to as *lag-dominant models*.
- The IMC and DS methods provide satisfactory set-point responses, but very slow disturbance responses, because the value of τ_I is very large. $(\tau_I = \tau)$
- Fortunately, this problem can be solved in three different ways.

Method 1: Integrator Approximation

Approximate $\tilde{G}(s) = \dfrac{Ke^{-\theta s}}{\tau s + 1}$ by $\tilde{G}(s) = \dfrac{K^* e^{-\theta s}}{s}$

where $K^* = K / \tau$.

- Then can use the IMC tuning rules (Rule M or N) to specify the controller settings.

Chapter 12

Method 2. Limit the Value of τ_I

- Skogestad (2003) has proposed limiting the value of τ_I:

$$\tau_I = \min\left\{\tau_1, 4\left(\tau_c + \theta\right)\right\} \qquad (12\text{-}34)$$

where τ_1 is the largest time constant (if there are two).

Method 3. Design the Controller for Disturbances, Rather Set-point Changes

- The desired CLTF is expressed in terms of $(Y/D)_d$, rather than $(Y/Y_{sp})_d$
- *Reference*: Chen & Seborg (2002)

Chapter 12

Example 12.4

Consider a lag-dominant model with $\theta/\tau = 0.01$:

$$\tilde{G}(s) = \frac{100}{100s+1}e^{-s}$$

Design three PI controllers:

a) IMC $(\tau_c = 1)$

b) IMC $(\tau_c = 2)$ based on the integrator approximation in Eq. 12-33

c) IMC $(\tau_c = 1)$ with Skogestad's modification (Eq. 12-34)

Chapter 12

Evaluate the three controllers by comparing their performance for unit step changes in both set point and disturbance. Assume that the model is perfect and that $G_d(s) = G(s)$.

Solution

The PI controller settings are:

Controller	K_c	τ_I
(a) IMC	**0.5**	**100**
(b) Integrator approximation	**0.556**	**5**
(c) Skogestad	**0.5**	**8**

Chapter 12

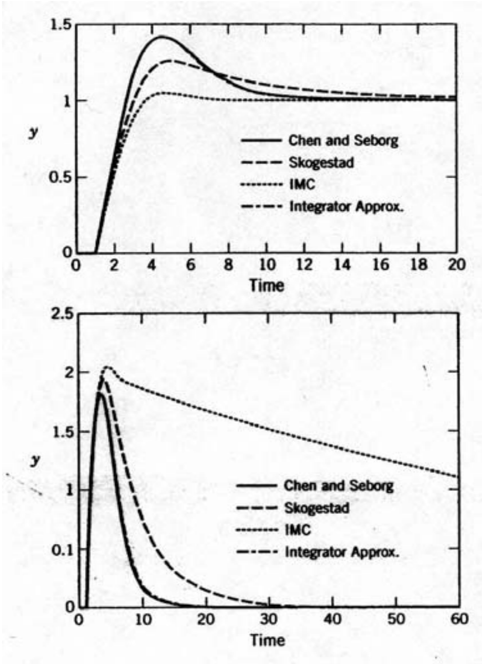


Figure 12.8. Comparison of set-point responses (top) and disturbance responses (bottom) for Example 12.4. The responses for the integrator approximation and Chen and Seborg (discussed in textbook) methods are essentially identical.

Chapter 12

On-Line Controller Tuning

1. ***Controller tuning inevitably involves a tradeoff between performance and robustness.***
2. ***Controller settings do not have to be precisely determined.*** In general, a small change in a controller setting from its best value (for example, ±10%) has little effect on closed-loop responses.
3. ***For most plants, it is not feasible to manually tune each controller.*** Tuning is usually done by a control specialist (engineer or technician) or by a plant operator. Because each person is typically responsible for 300 to 1000 control loops, it is not feasible to tune every controller.
4. ***Diagnostic techniques for monitoring control system performance are available.***

Chapter 12

Controller Tuning and Troubleshooting Control Loops

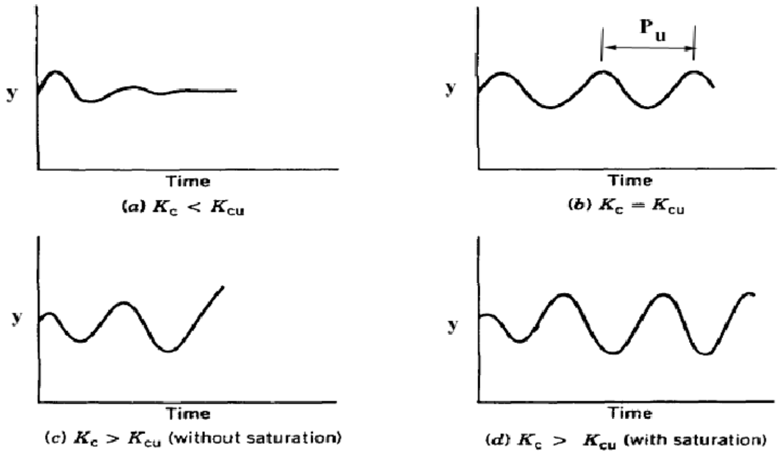


Figure 12.12. Experimental determination of the ultimate gain K_{cu}.

Chapter 12

Ziegler-Nichols Rules:

These well-known tuning rules were published by Z-N in 1942:

controller	K_c	τ_I	τ_D
P	0.5 K_{CU}	-	-
PI	0.45 K_{CU}	$P_U/1.2$	-
PID	0.6 K_{CU}	$P_U/2$	$P_U/8$

Z-N controller settings are widely considered to be an "industry standard".

Z-N settings were developed to provide 1/4 decay ratio -- too oscillatory?

Chapter 12

Modified Z-N settings for PID control

controller	K_c	τ_I	τ_D
original	$0.6\ K_{CU}$	$P_U/2$	$P_U/8$
Some overshoot	$0.33\ K_{CU}$	$P_U/2$	$P_U/3$
No overshoot	$0.2\ K_{CU}$	$P_U/3$	$P_U/2$

Chapter 12

Table 12.6 Controller Settings based on the Continuous Cycling Method

Ziegler-Nichols	K_c	τ_I	τ_D
P	$0.5K_{cu}$	—	—
PI	$0.45K_{cu}$	$P_u/1.2$	—
PID	$0.6K_{cu}$	$P_u/2$	$P_u/8$
Tyreus-Luyben†	K_c	τ_I	τ_D
PI	$0.31K_{cu}$	$2.2P_u$	—
PID	$0.45K_{cu}$	$2.2P_u$	$P_u/6.3$

† Luyben and Luyben (1997).

Chapter 12

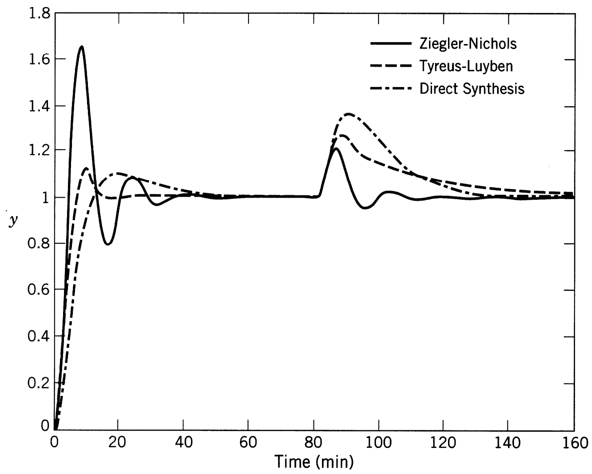
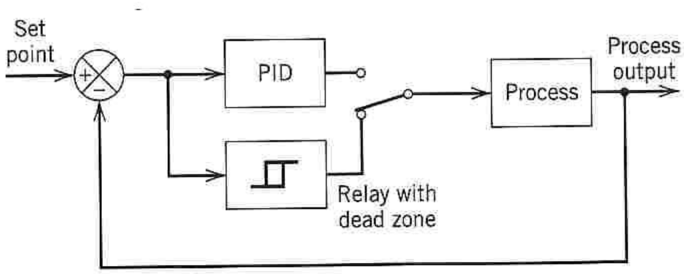
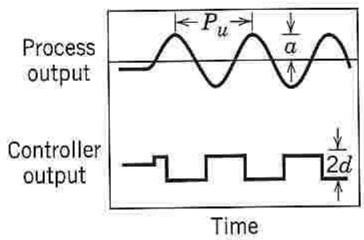


Figure 12.13 Comparison of PID controllers for Example 12.7.

Chapter 12





$$K_{CU} = \frac{4d}{\pi a}$$

Use Z-N settings

Figure 12.14 Auto-tuning using a relay controller.

Chapter 12

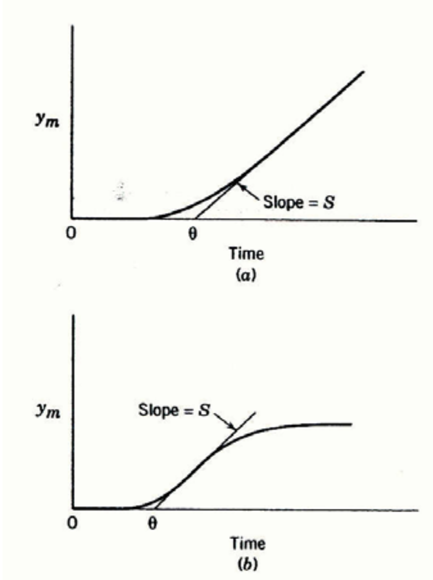


Figure 12.15 Typical process reaction curves: (a) non-self-regulating process, (b) self-regulating process.

Chapter 12

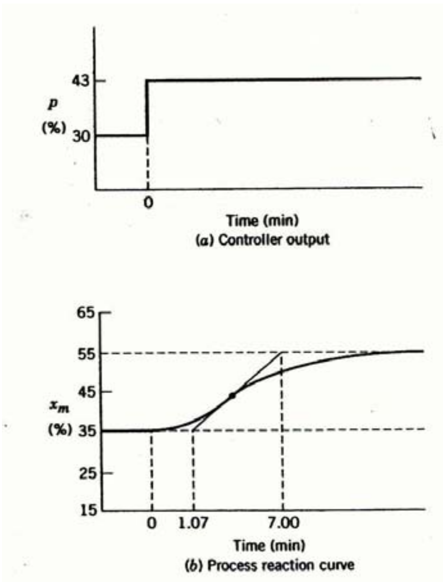


Figure 12.16 Process reaction curve for Example 12.8.

Chapter 12

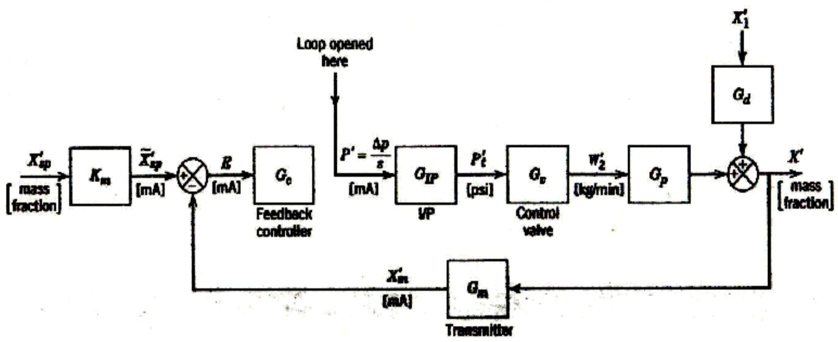


Figure 12.17 Block diagram for Example 12.8.

Chapter 12