MM10 Frequency Response Design

Readings:

- FC: p389-407: lead compensation

What Have We Talked about in MM9?

- Control design based on Bode plot
  - Stability margins (Gain margin and phase margin)
  - Transient performance
  - Steady-state performance
- Nyquist Diagram
  - What's Nyquist diagram?
  - What we can gain from Nyquist diagram
- Matlab functions: bode(), margin(), nyquist()

Nyquist Criterion for Stability (MM9)

The Nyquist criterion states that:

- P = the number of open-loop (unstable) poles of G(s)H(s)
- N = the number of times the Nyquist diagram encircles –1
  - clockwise encirclements of -1 count as positive encirclements
  - counter-clockwise (or anti-clockwise) encirclements of -1 count as negative encirclements
- Z = the number of right half-plane (positive, real) poles of the closed-loop system
- The important equation:

$$Z = P + N$$

Goals for this lecture (MM10)

- **An illustrative example**
  - **Frequency response analysis**
  - **Frequency response design**
- Lead and lag compensators
  - What's a lead/lag compensator?
  - Their frequency features
- A systematical procedure for lead compensator design
- A practical design example – Beam and Ball Control

An Illustrative Example: **Antenna Position Control**

Control system design for a satellite tracking antenna (one-dimentional)

- Design specifications:
  - Overshoot to a step input less than 16%;
  - Settling time to 2% to be less than 10 sec.;
  - Tracking error to a ramp input of slope 0.01rad/sec to be less than 0.01rad;
  - Sampling time to give at at least 10 samples in a rise time.

Example: **Mathematical Modeling**

- System model:

$$J\ddot{\theta} + B\dot{\theta} = T_c + T_d$$

- Transfer function:

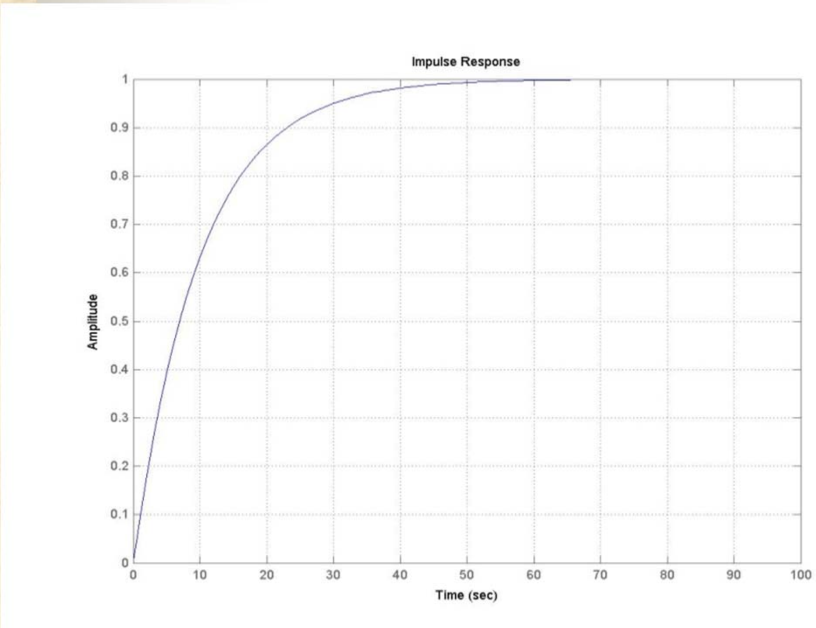
$$\frac{\theta(s)}{U(s)} = \frac{1}{s(\frac{s}{a}+1)}, \quad a = \frac{B}{J} = 0.1, \quad u(t) = \frac{T_c(t)}{B}$$

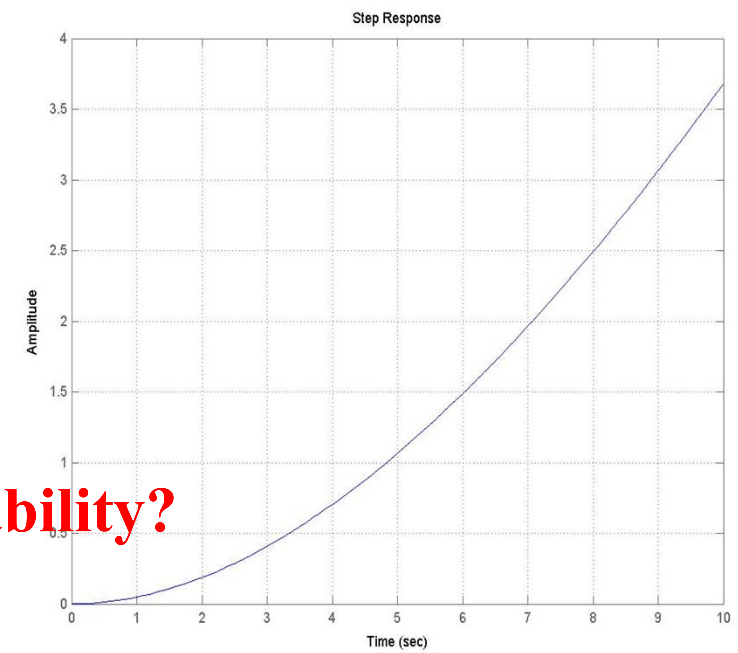
Example: **Open-Loop Analysis**

- Transfer function: $\frac{\theta(s)}{U(s)} = \frac{1}{s(\frac{s}{a}+1)}, \quad a = \frac{B}{J} = 0.1, \quad u(t) = \frac{T_c(t)}{B}$

- Open-loop properties
  - Step response
  - Impulse response

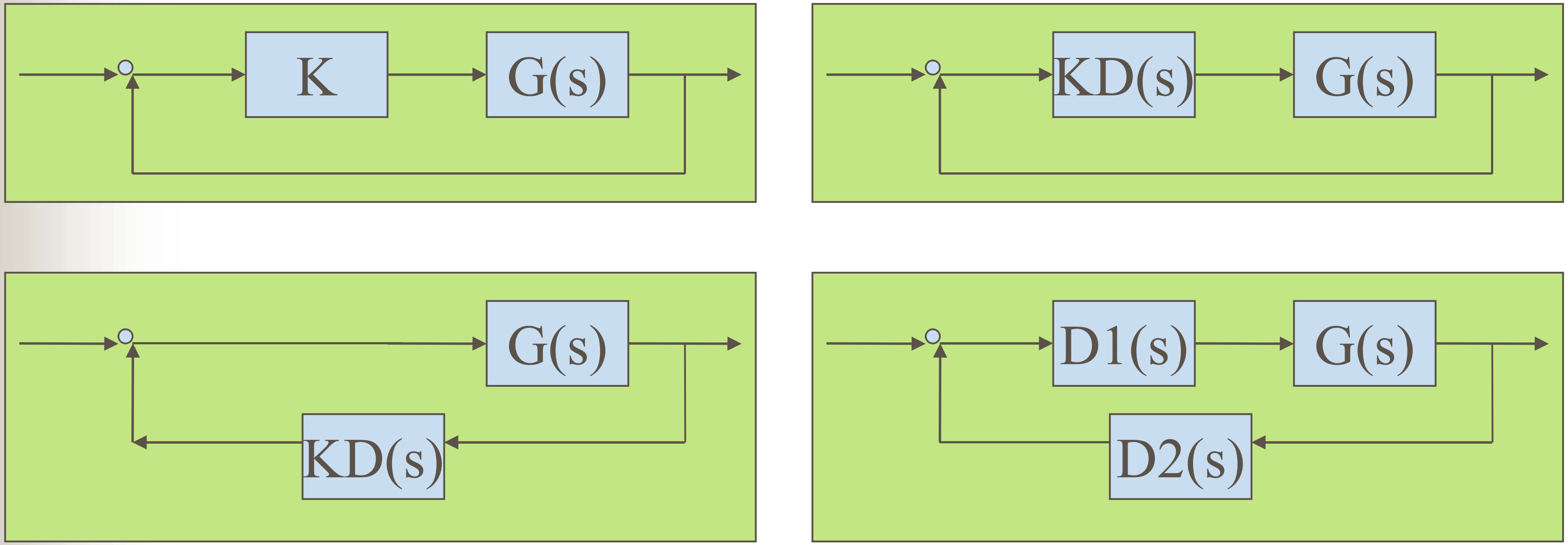
```
impulse(tf(1,[10 1 0])); figure;
step(tf(1,[10 1 0]),10)
```





Stability?

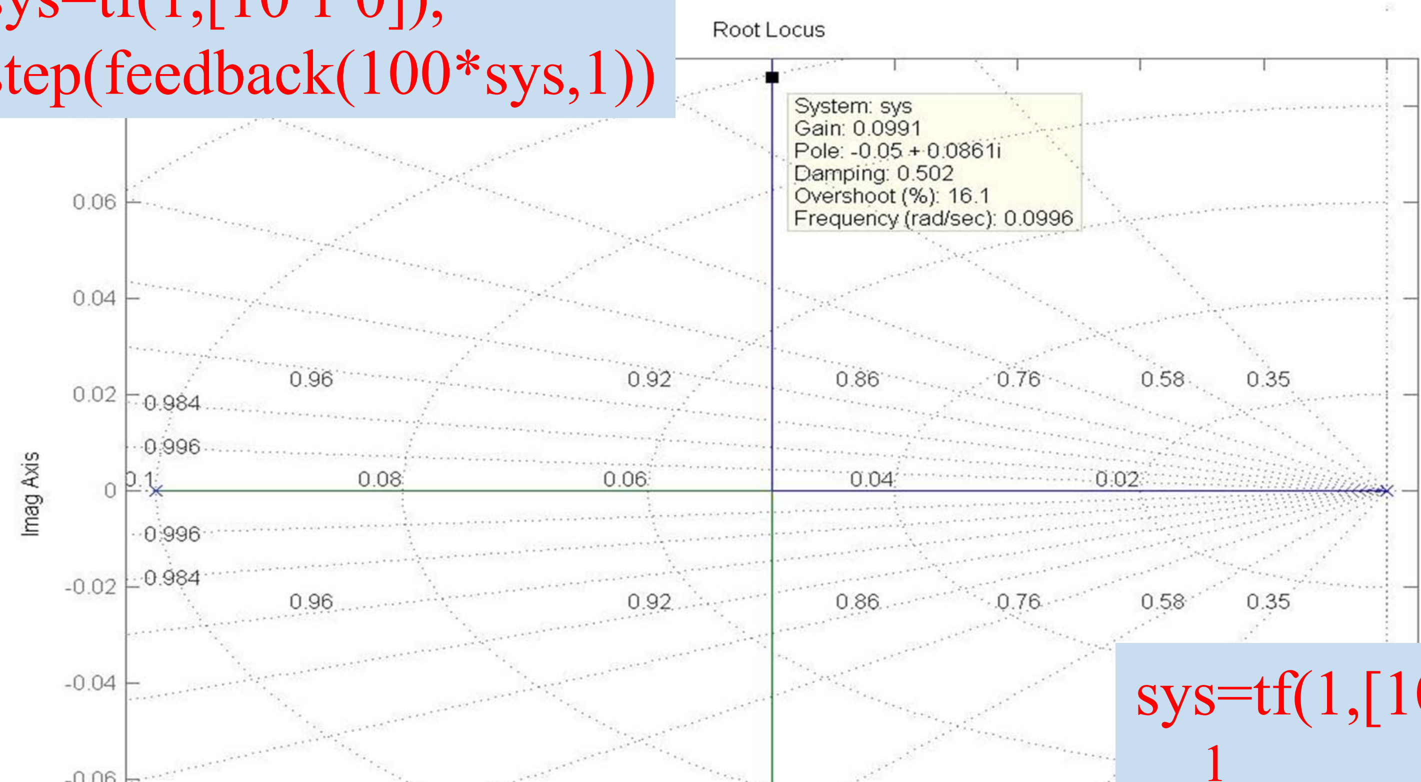
Example: **Possible Control Structure**

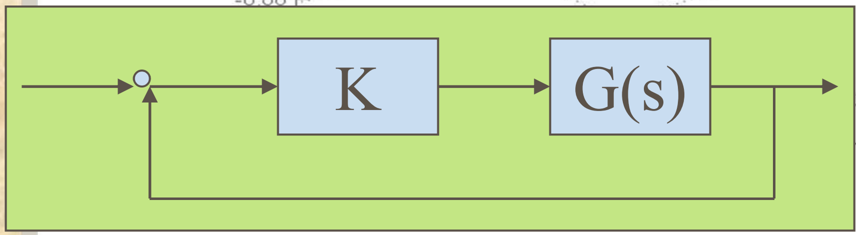


- Different control structures
  - Cascade controller: Gain, dynamic compensator?
  - Feedback controller: Gain, dynamic compensator?
  - Single or multiple loops?
  - .....

Example: **Just Gain Controller?**

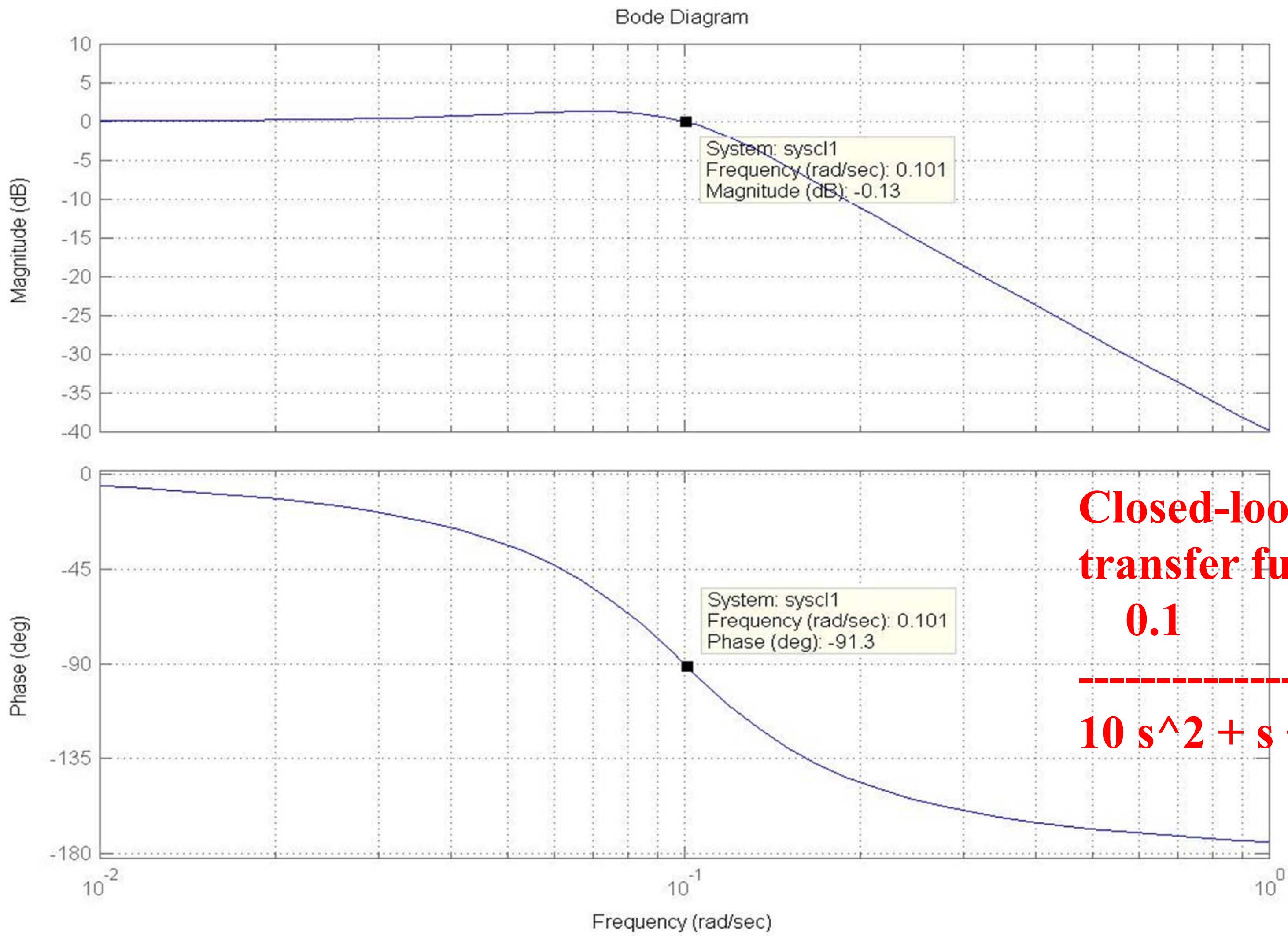
sys=tf(1,[10 1 0]),
step(feedback(100*sys,1))





sys=tf(1,[10 1 0]),
1
----
10 s^2 + s
rlocus(sys)

syscl1=feedback(0.1*sys,1); bode(syscl1) ; figure, margin(0.1*sys)



Closed-loop transfer function

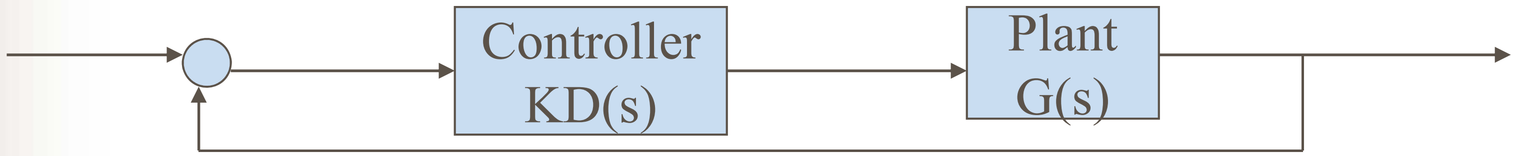
$$\frac{0.1}{10 s^2 + s + 0.1}$$

Dynamic Compensation

- **Objective:**

  If a satisfactory process dynamics can not be obtained by a gain adjustment alone, some modification or compensation of the process dynamics is needed

$$1 + KD(s)G(s) = 0$$



Lead and Lag Compensators

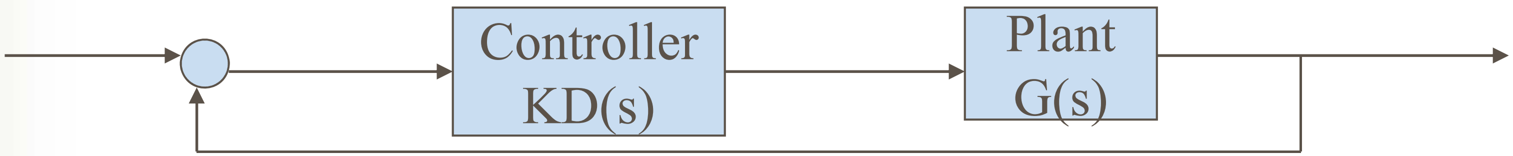
- **Lead compensation:** acts mainly to lower rise time and decrease the transient overshoot:

  D(s)=(s+z)/(s+p) with $z < p$

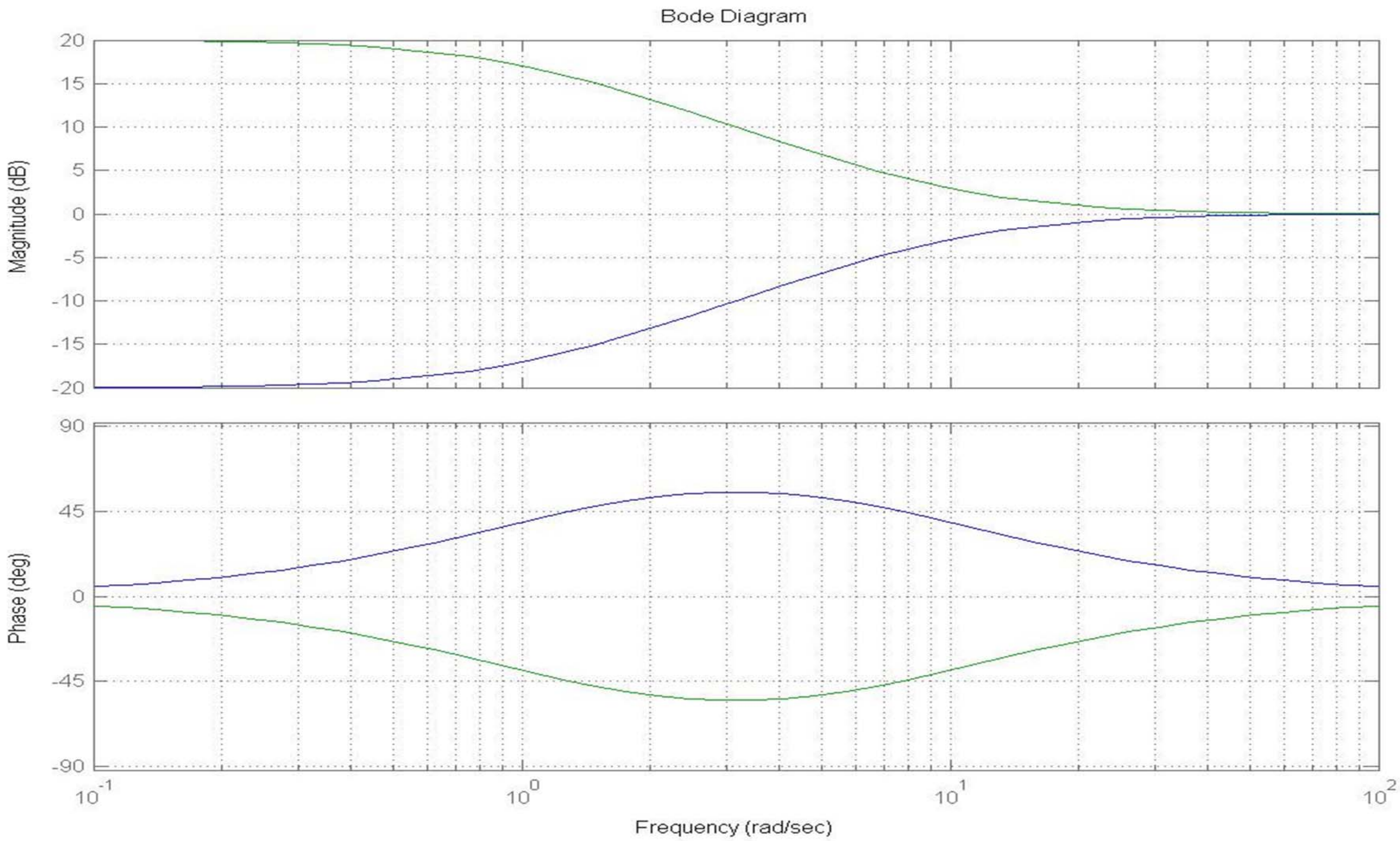
- **Lag compensation:** acts mainly to improve the steady-state accuracy:

  D(s)=(s+z)/(s+p) with $z > p$

- Compensationis typically placed in series with the plant in the feedforward path

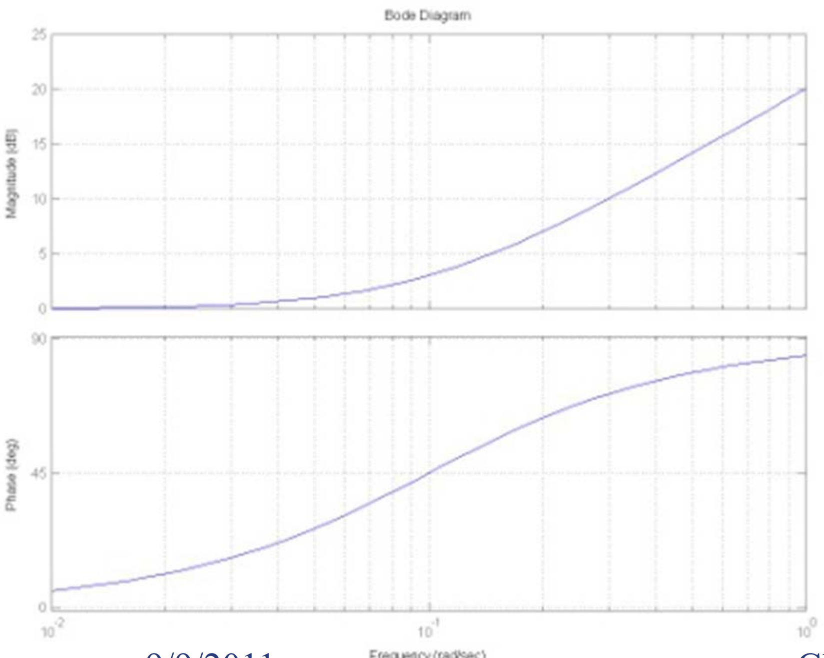


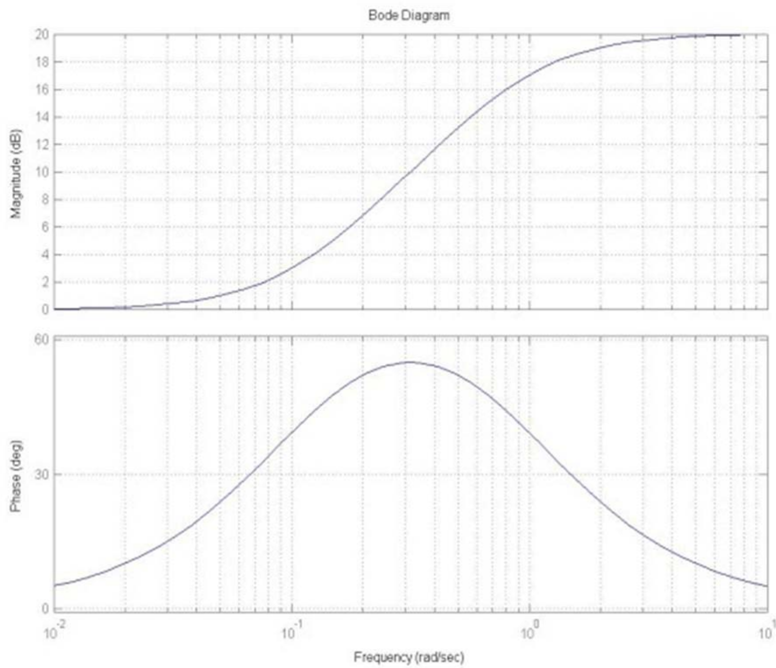
Frequency Properties of Lead and Lag



Lead and PD Controllers (I)

- **PD compensation**: **$D(s)=K(T_D s+1)$**
  - Increasing the phase margin
  - Amplify the high frequency noise
- **Lead compensation:** **$D(s)=K(Ts+1)/(\alpha Ts+1)$, $\alpha<1$**





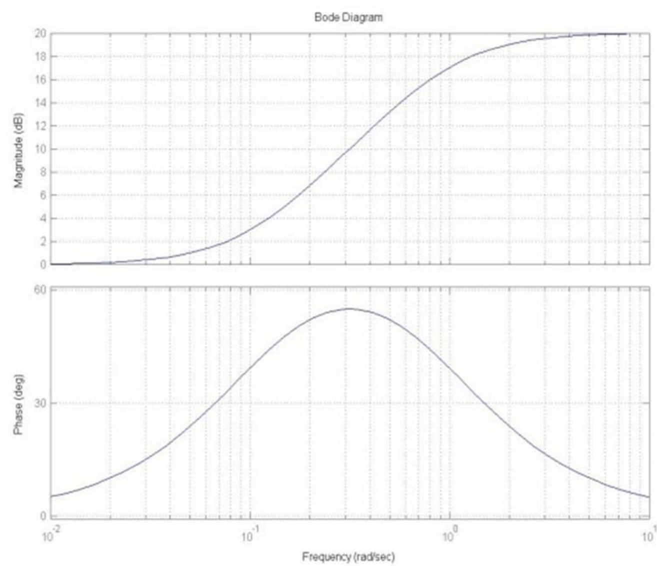
Lead and PD Controllers (II)

- **Lead compensation:** **D(s)=K(Ts+1)/(αTs+1), α <1**
  - Lead compensator is a high-pass filter (app.PD control)
  - It is used whenever substantial improvement in damping is required
  - The maximum phase contribution is

Example:
sysD=tf([10 1],[1 1])
bode(sysD)

$$\omega_{max} = \frac{1}{T\sqrt{\alpha}}$$

$$\alpha = \frac{1 - \sin \beta_{max}}{1 + \sin \beta_{max}}$$



Example: **PD Controller for Antenna Control (I)**

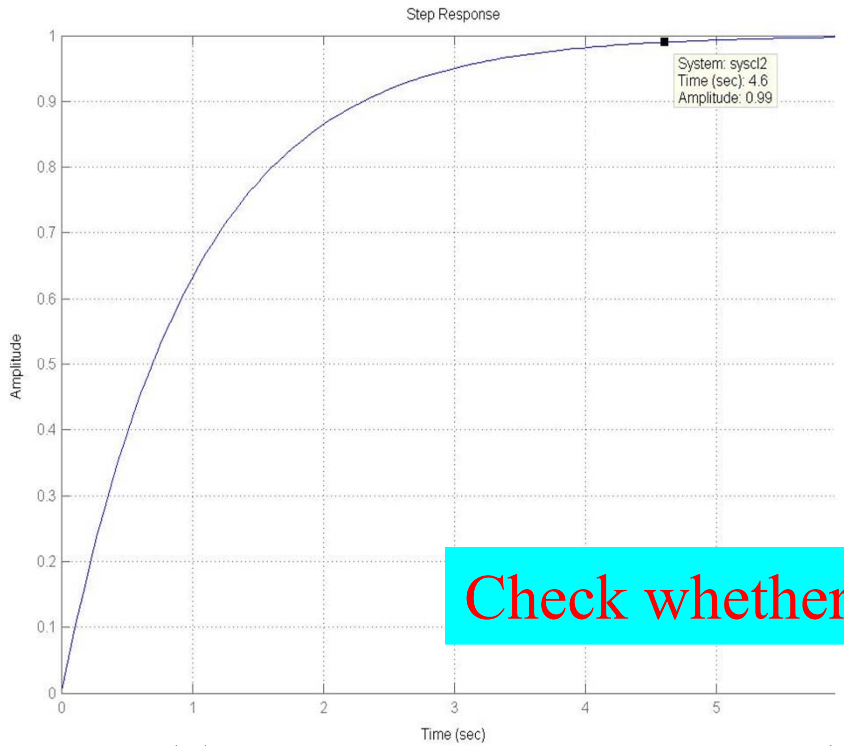
- Design the low frequency gain **K** with respect to the steady-state error specification: Tracking error to a ramp input of slope 0.01rad/sec to be less than 0.01rad
- Ramp Input (R(s) = 1/s^2): $e(\infty) = \dfrac{1}{\lim_{s\to 0} sG(s)} = \dfrac{1}{K_r} \Rightarrow K_r = \lim_{s\to 0} sG(s)$ (MM5)
- Antenna system case..... **K=1**

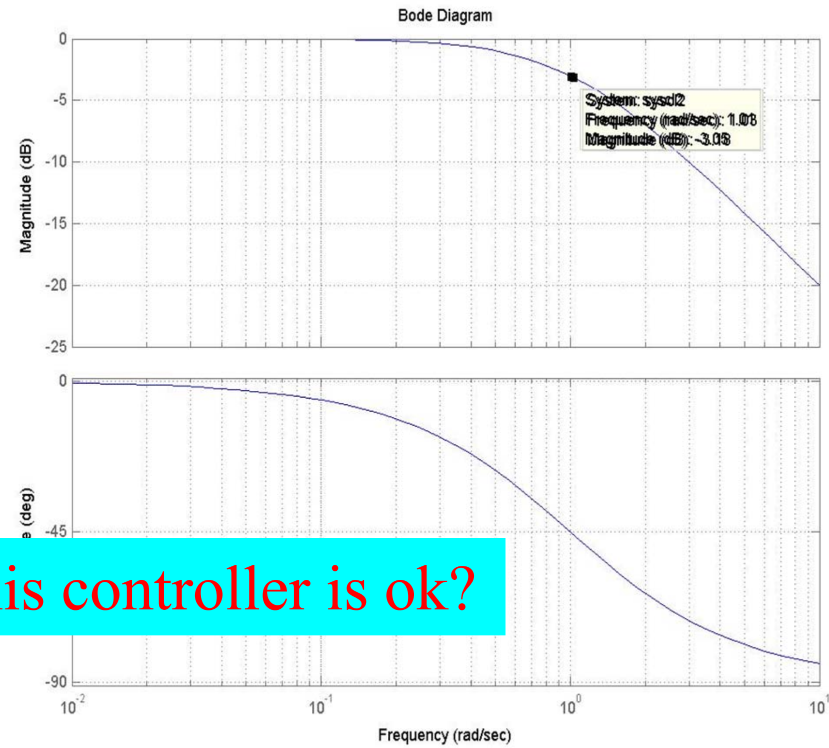
Example: **PD Controller for Antenna Control (II)**

- **PD controller: D(s)=10s+1**

sysd=tf(1,[10.0 1.0 0])*tf([10.0 1.0],1);

syscl2=feedback(sysd,1); step(syscl2)





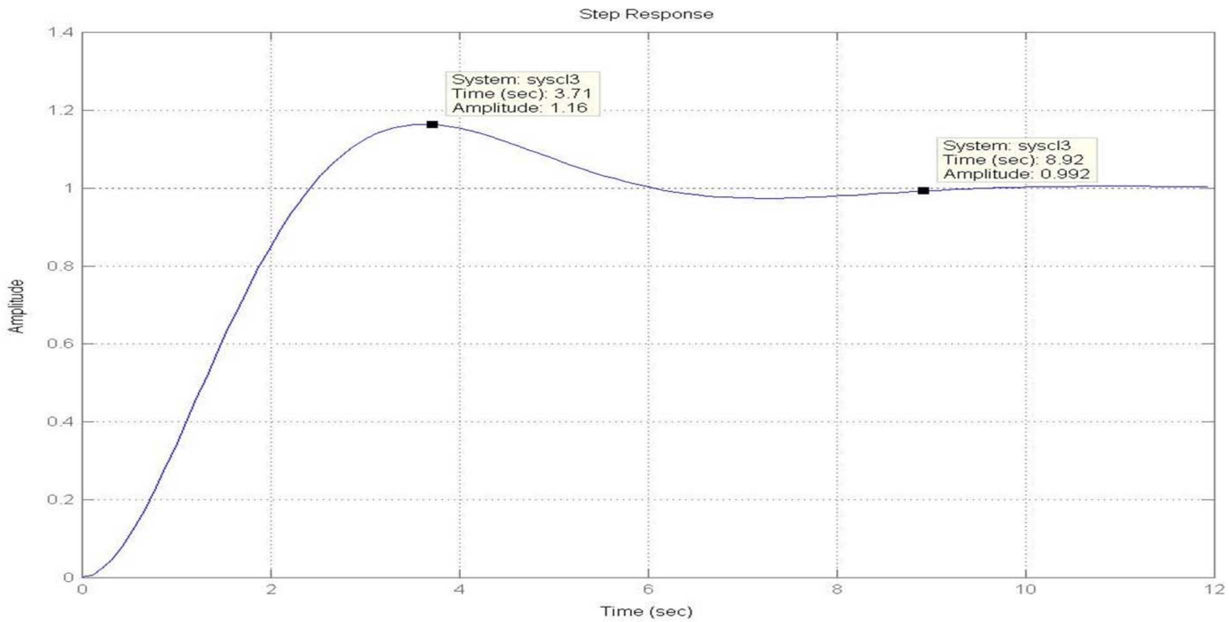
Check whether this controller is ok?

Example: **Intuitive Lead Design**

- Design the low frequency gain **K** with respect to the steady-state error specification Antenna system case..... **K=1**
- **Lead controller: D(s)=(10s+1)/(s+1)**

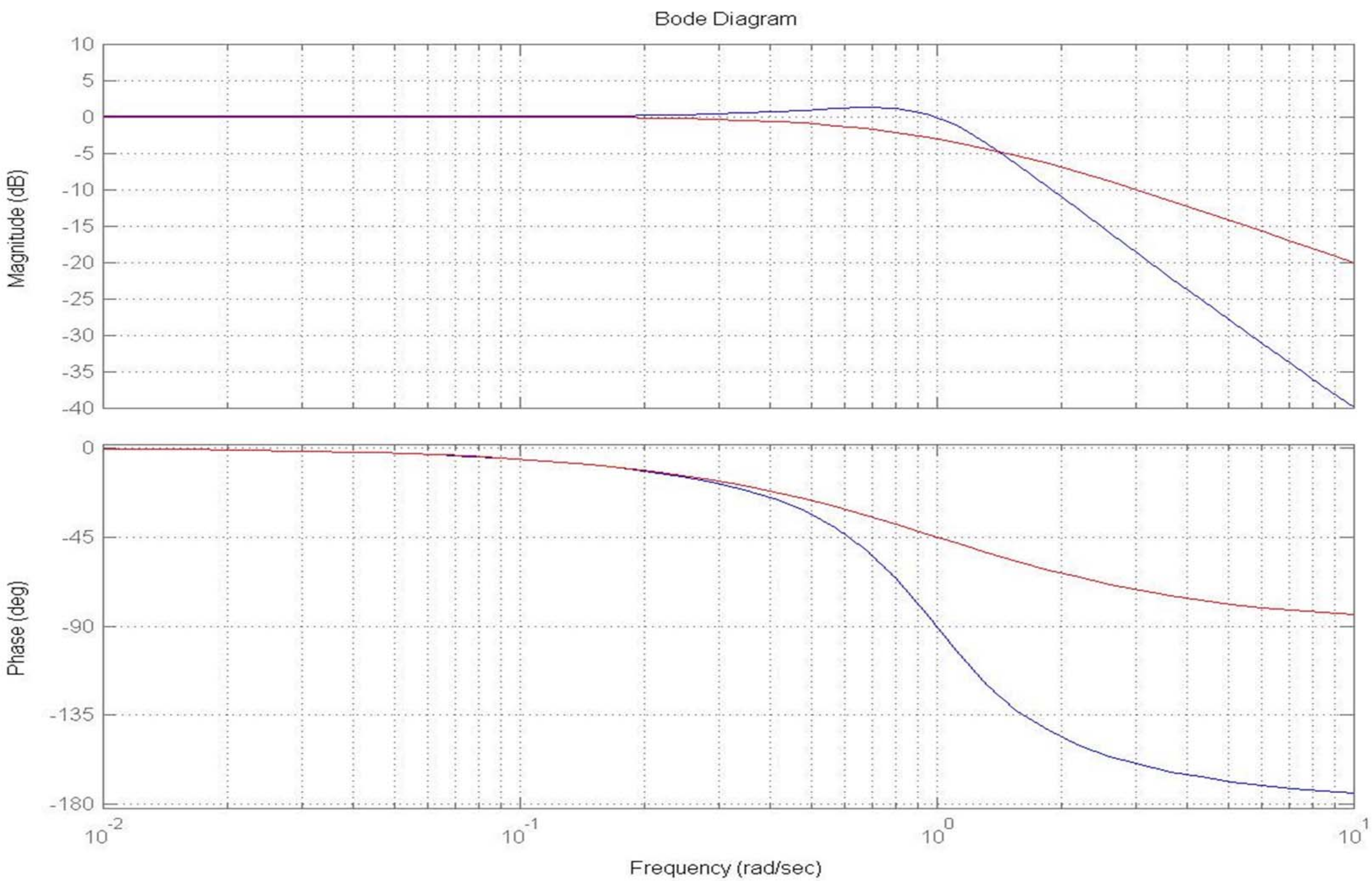
sysd=tf(1,[10.0 1.0 0])*tf([10.0 1.0],[1 1]);

syscl3=feedback(sysd,1); step(syscl3)



Example: **Comparison of PD and Lead Design**

- bode(syscl2,syscl3); grid



Goals for this lecture (MM10)

- An illustrative example
  - Frequency response analysis
  - Frequency response design

- Lead and lag compensators
  - What's a lead/lag compensator?
  - Their frequency features

- **A systematical procedure for lead compensator design**

Lead Compensator Design Procedure (I)

- **Step 1:** Design the low frequency gain **K** with respect to the staedy-state error specification

Antenna system case..... **K=1**

  - Step Input (R(s) = 1/s): $e(\infty) = \dfrac{1}{1+\lim_{s\to 0} G(s)} = \dfrac{1}{1+K_p} \Rightarrow K_p = \lim_{s\to 0} G(s)$
  - Ramp Input (R(s) = 1/s^2): $e(\infty) = \dfrac{1}{\lim_{s\to 0} sG(s)} = \dfrac{1}{K_v} \Rightarrow K_v = \lim_{s\to 0} sG(s)$
  - Parabolic Input (R(s) = 1/s^3): $e(\infty) = \dfrac{1}{\lim_{s\to 0} s^2 G(s)} = \dfrac{1}{K_a} \Rightarrow K_a = \lim_{s\to 0} s^2 G(s)$

Lead Compensator Design Procedure (II)

- **Step 2**: Determine the needed phase lead
  - Original system PM:

    sys=tf(1,[10 1 0]), margin(sys)... PM=18 at 0.308
  - Expected PM:

    Expected overshoot limit (16%)

    ⇩

    Dampling ratio $\xi \geq 0.5$

    ⇩

    Expected PM ≈ 100*0.5=50 degree
  - Directly needed phase lead: **50-18=32 degree**
  - Expected phase lead: **32+ (7~10) degree**

Lead Compensator Design Procedure (III)

Lead compensation: **D(s)=K(Ts+1)/(αTs+1), α <1**

- **Step 3**: Determine coefficient **α**

$$\alpha = \frac{1-\sin\beta_{\max}}{1+\sin\beta_{\max}} = \frac{1-\sin 40}{1+\sin 40} = 0.2174$$

- **Step 4**: Determine coefficient **T**

$$T = \frac{1}{\omega_{\max}\sqrt{\alpha}} = \frac{1}{(\omega_n/2)\sqrt{\alpha}} = \frac{2}{0.92\sqrt{\alpha}} = 4.622$$

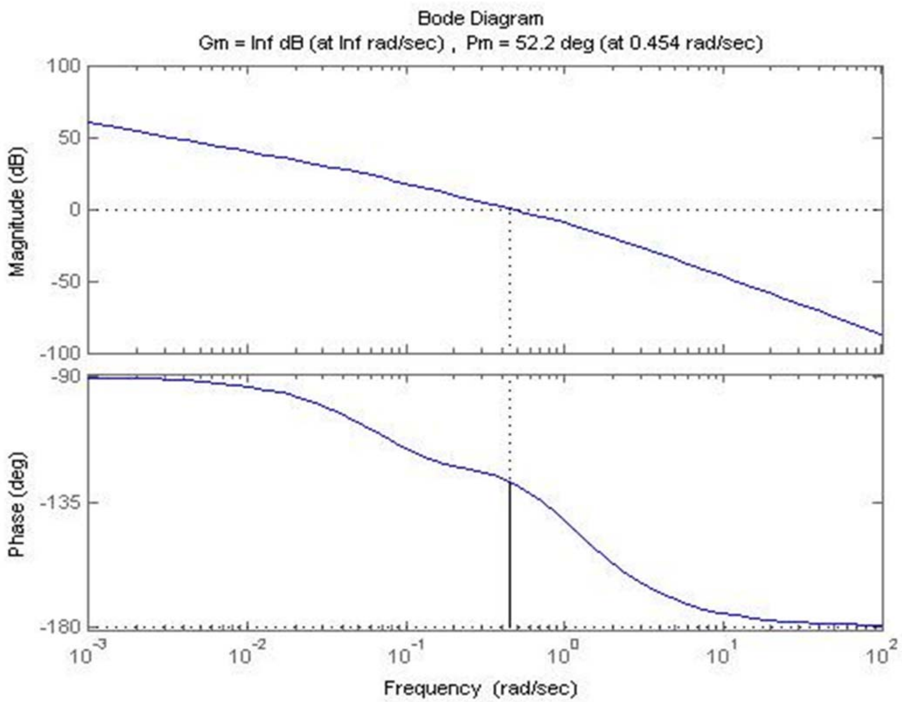
$$\omega_{\max} = \frac{1}{T\sqrt{\alpha}}$$

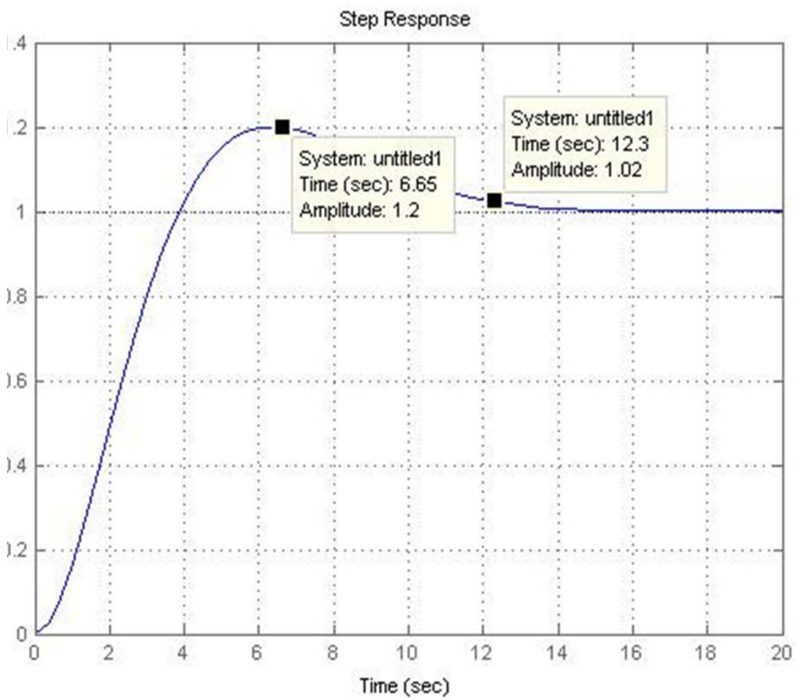
$$\alpha = \frac{1-\sin\beta_{\max}}{1+\sin\beta_{\max}}$$

Lead Compensator Design Procedure (IV)

- **Step 5**: Draw the compensated frequency response, check PM

sysD=tf([4.622 1],[1.0137 1]); sysC=sys*sysD; margin(sysC); step(feedback(sysC,1))





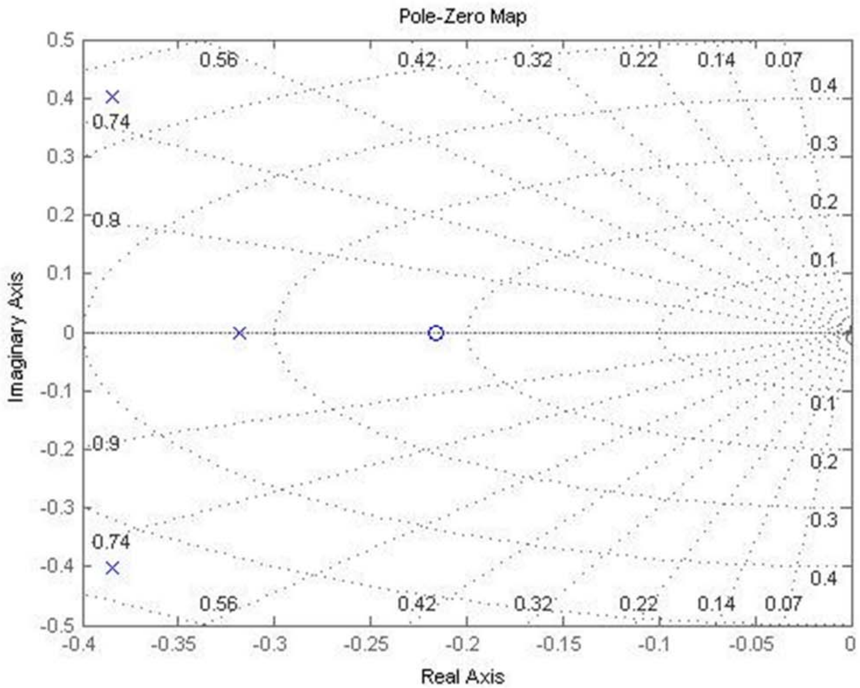
Why this Lead Compensator doesn't work? (I)

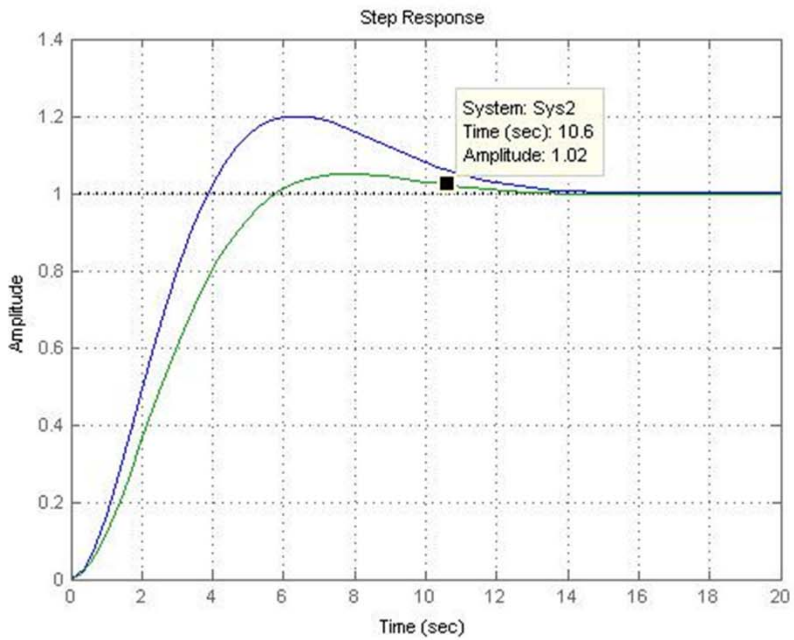
- Check the poles & zeros of the closed-loop...

syscl=feedback(sysC,1); pzmap(syscl);

- Compare with a standard 2nd-order system...

Sys2=tf(0.3099, conv([1 0.384-0.403i], [1 0.384+0.403i])) ;





Lead Compensator Design Procedure (V)

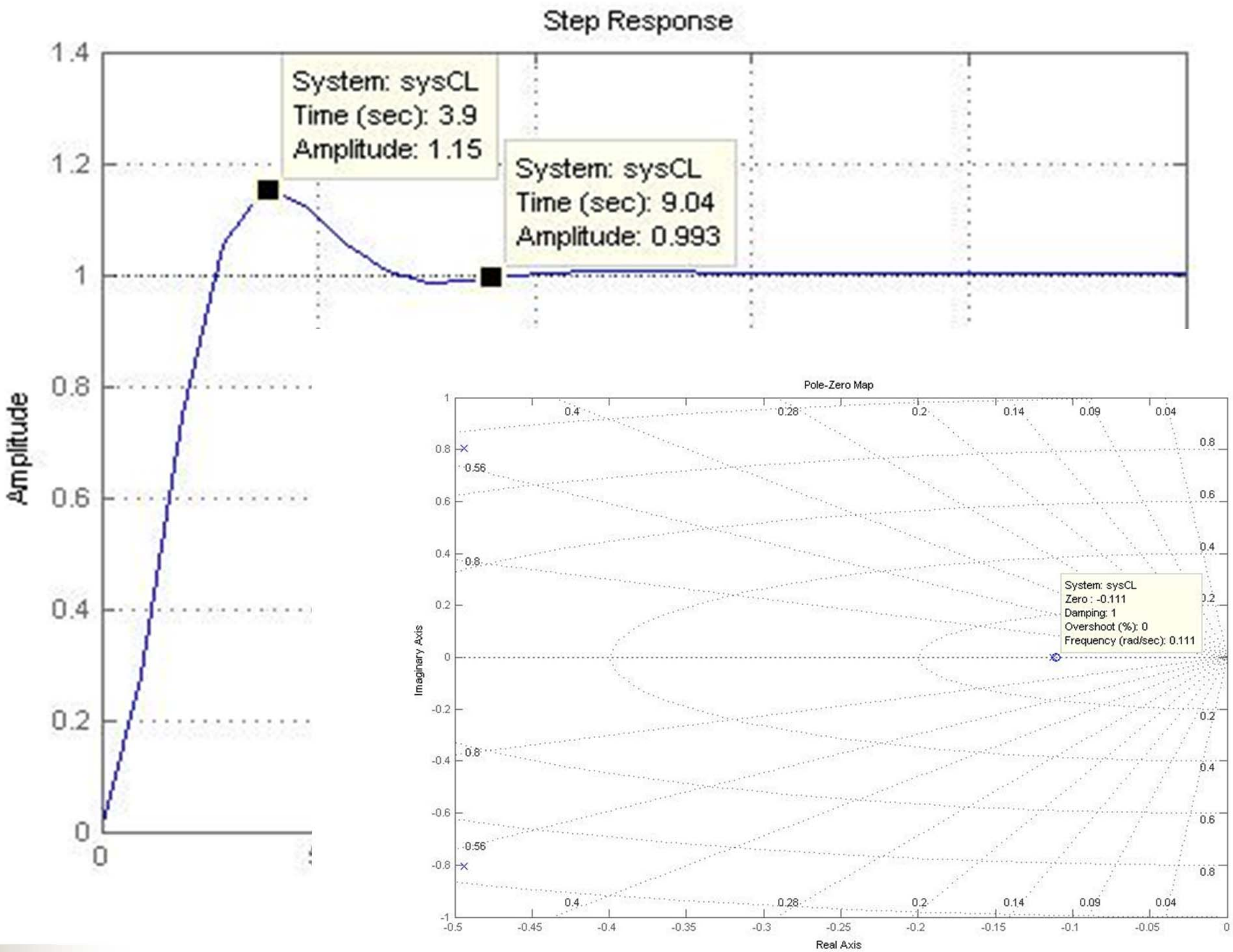
- **Step 6**: Iterate on the design until all specifications are met

```
sysD=tf([9.7457 1],[1.0911 1]) ; sysC=sys*sysD; margin(sysC);
sysCL=feedback(sysC,1); step(sysCL)
sysD=tf([9 1],[1 1]);  sysCL=feedback(sys*sysD,1); step(sysCL)
```

$$\alpha = \frac{1 - \sin \beta_{max}}{1 + \sin \beta_{max}} = \frac{1 - \sin 53}{1 + \sin 53} = 0.112$$

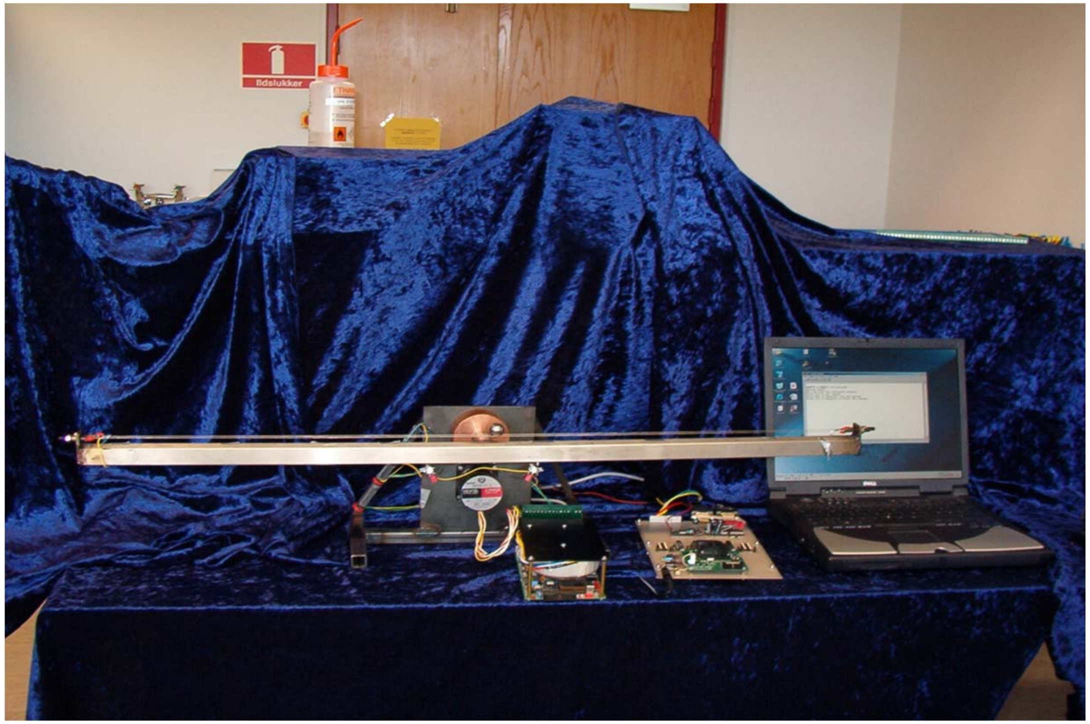
$$T = \frac{1}{\omega_{max} \sqrt{\alpha}} = \frac{1}{(\omega_n / 1.5)\sqrt{\alpha}} = \frac{3}{0.92 \sqrt{\alpha}} = 9.7457$$

$$D(s) = \frac{9.7457 \, s + 1}{1.0911 \, s + 1}$$


Step Response
System: sysCL
Time (sec): 3.9
Amplitude: 1.15
System: sysCL
Time (sec): 9.04
Amplitude: 0.993
Amplitude
Pole-Zero Map
System: sysCL
Zero : -0.111
Damping: 1
Overshoot (%): 0
Frequency (rad/sec): 0.111
Imaginary Axis
Real Axis

4.1 What's B&B System?

- **System:** A ball rolls along the track of a beam that is pivoted at some position.
- **Objective:** To steadily place the ball at any given position along the track
- **Strategy:** To control the track angle through the control of a servo motor



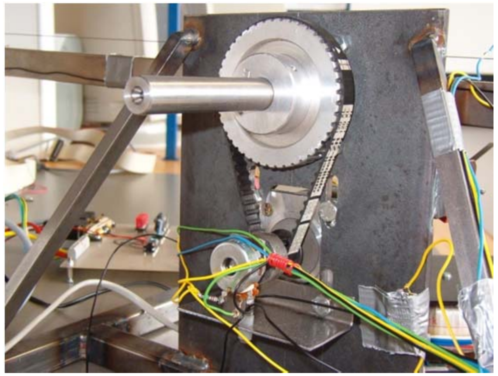
4.2 Why focus on B&B System?

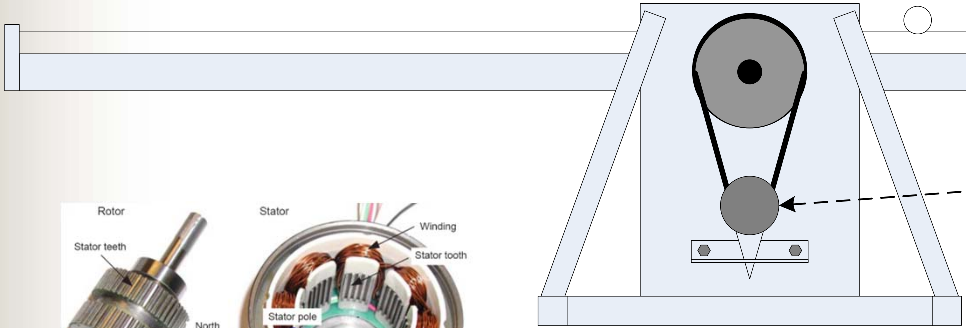
- The ball and beam apparatus demonstrates the control problems associated with **unstable systems**.
- An example of such a system is a missile during launch; active control is required to prevent the missile going unstable and toppling over.

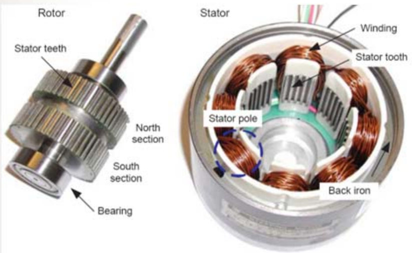
4.4 AUE Beam and Ball System

Implement at least one control method 5-10% overshoot and 3-6 second settling time

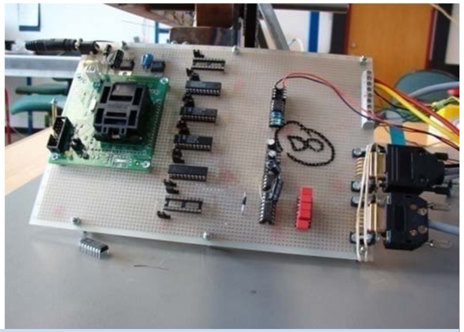
The potentiometer and axle





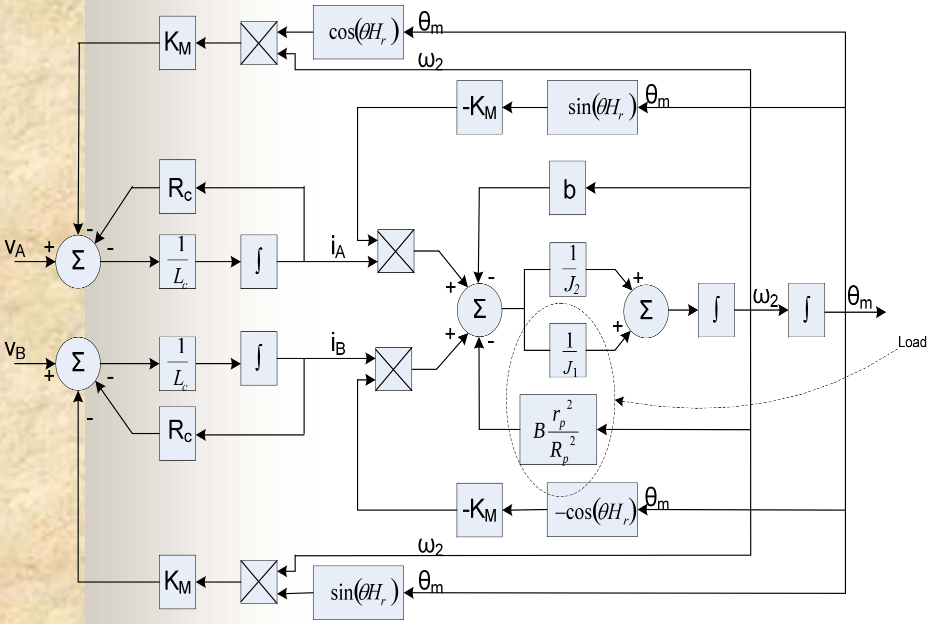


a hybrid stepping motor

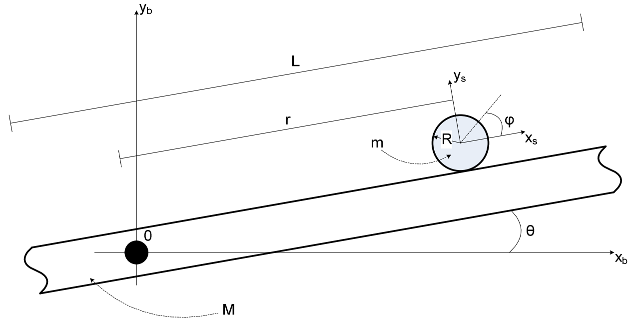


TI MSP430 fix-point

4.4.1 Modelling the AUE B&B System



Block diagram of stepping motor and load

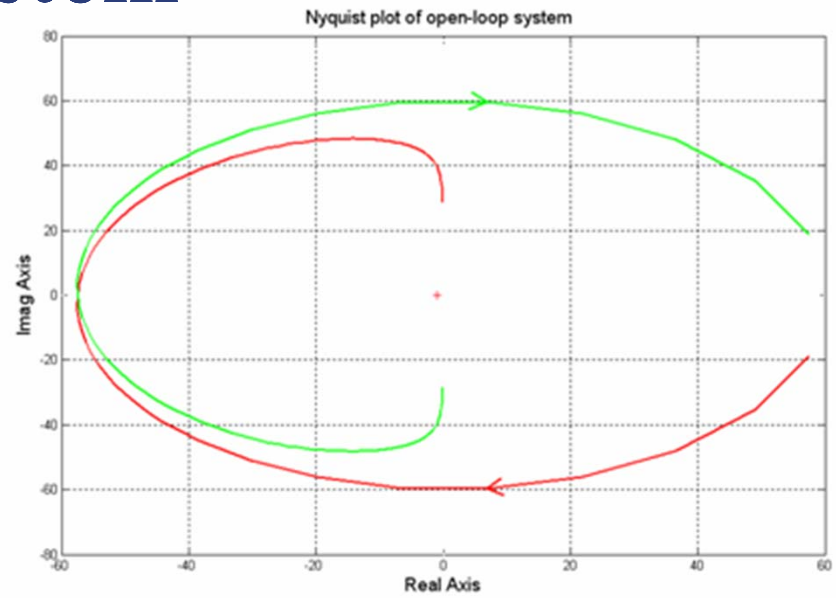


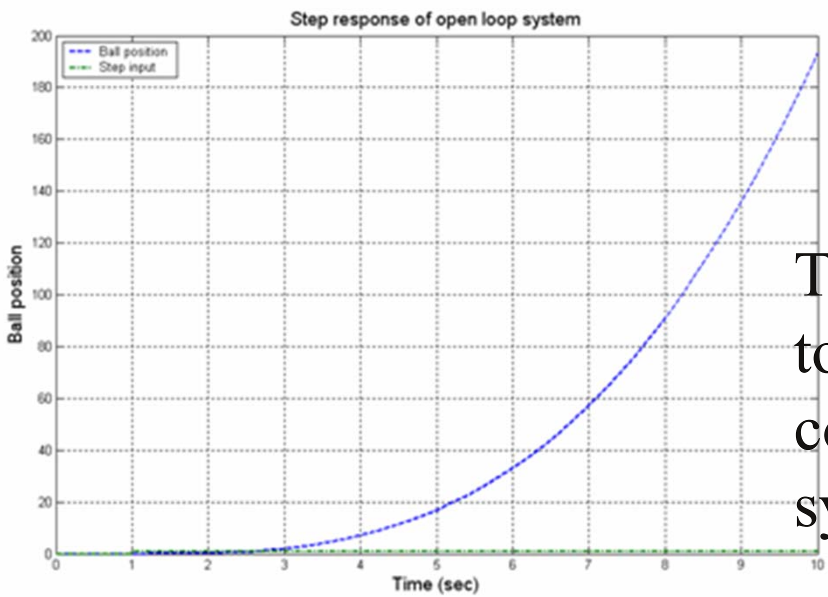
Lagrangian modelling technique

$$\frac{r(s)}{\theta(s)} = \frac{m \cdot g}{\left(\frac{J_{ball}}{R^2} + m\right) \cdot s^2}$$

4.4.2 Analysis of the AUE B&B System

From the nyquist plot, it can be Observed that the system is unstable since -1 is encircled clockwise by the nyquist plot

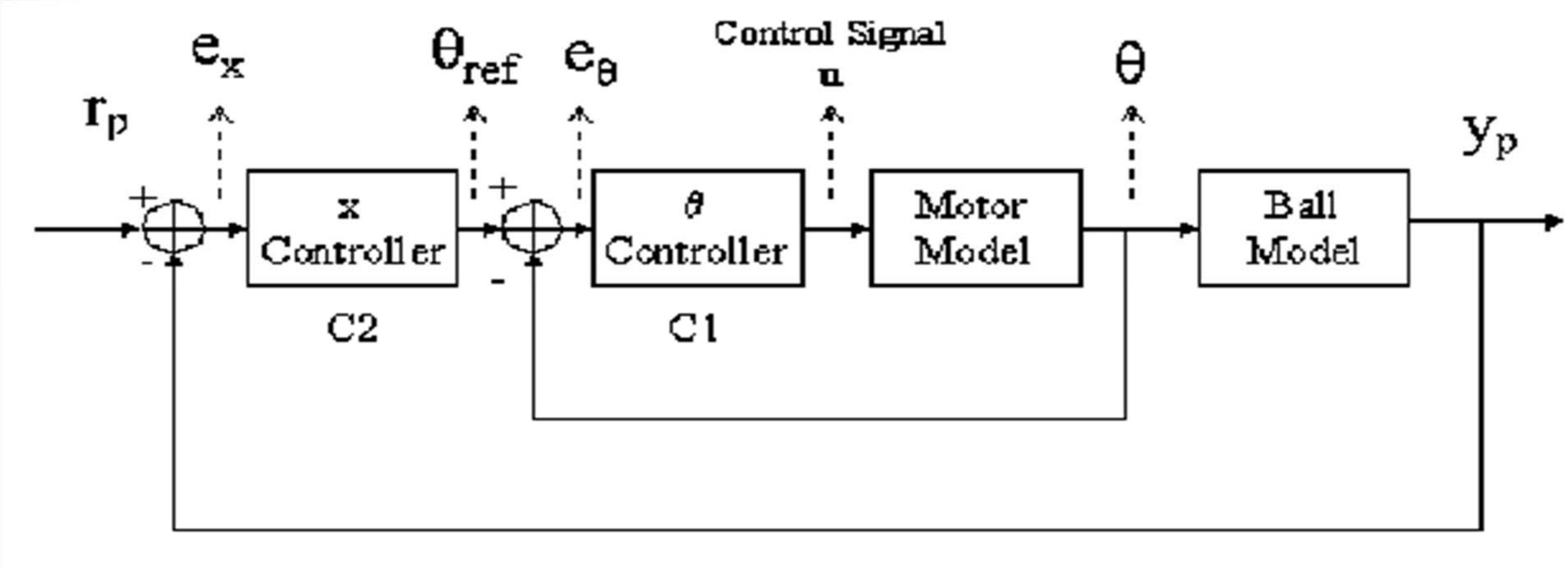




The system is unstable if it is exposed to a step input. From this can it be concluded that the
system needs some kind of controller.

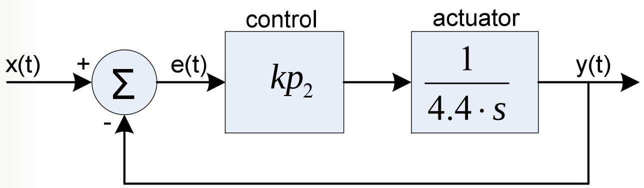
4.4.3 Control Strategy for the B&B System

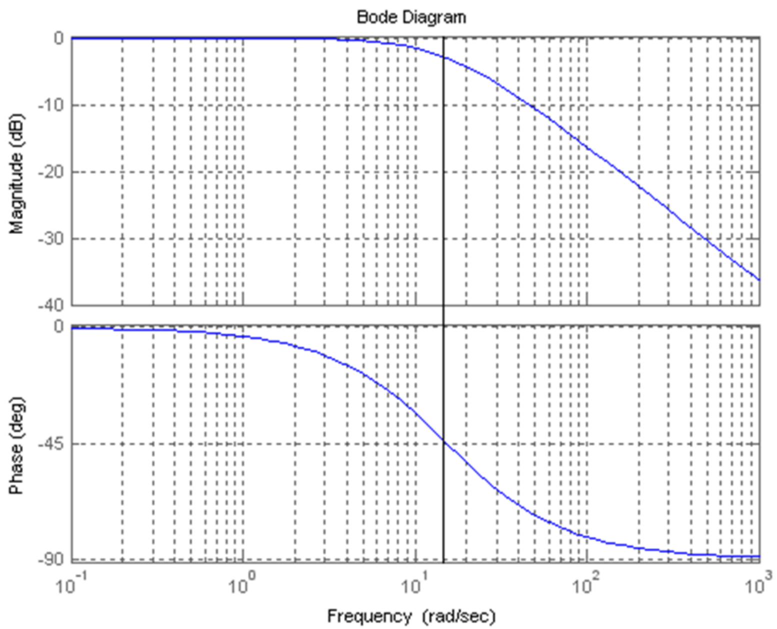
- Cascade control
  - Master loop (outer loop)
  - Slave loop (inner loop)



4.4.4 Control Design for Slave Loop

- The block "control" contains the *P-controller*, *Kp2*.
- The slave loop must be faster enough (e.g., 10 times faster) comparing with the master loop

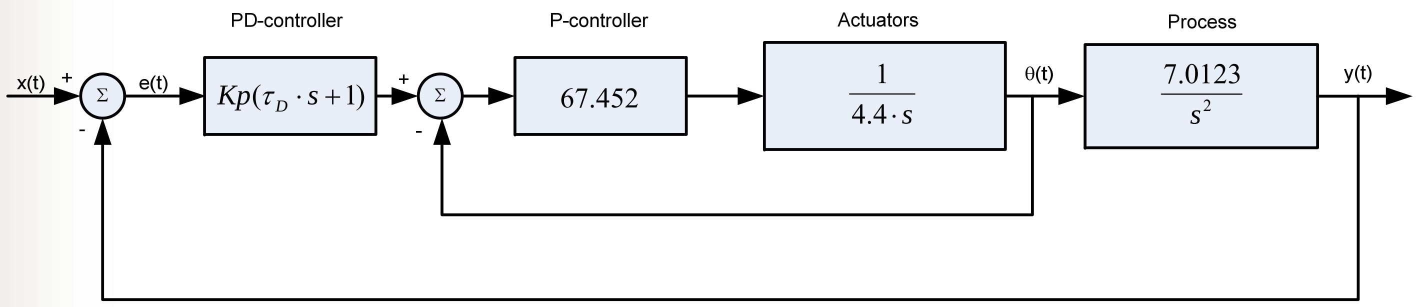


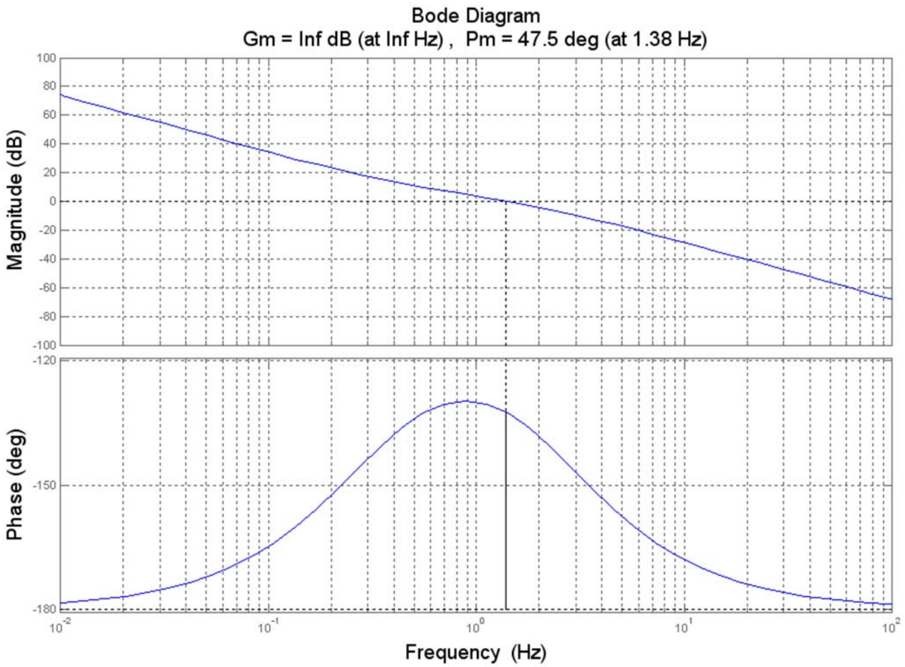


Through bode plot it can be seen that the system has a cutoff frequency at appr. 15 rad/sec

The system is settled in approximately 0.3 sec

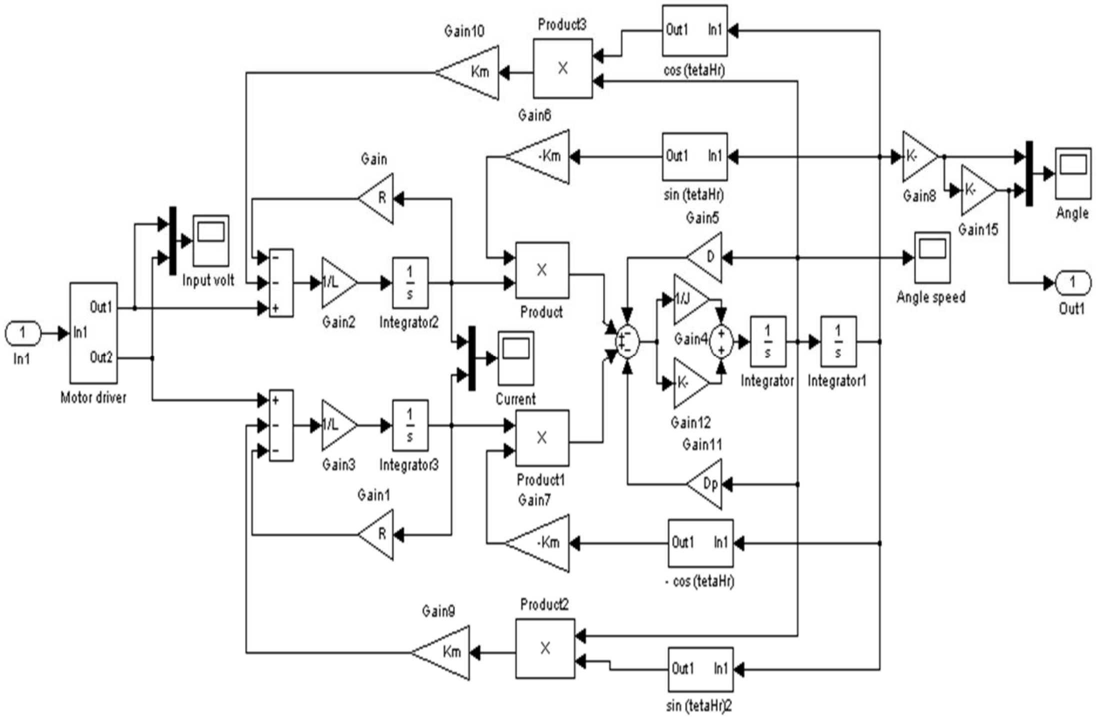
4.4.5 Control Design for Master Loop

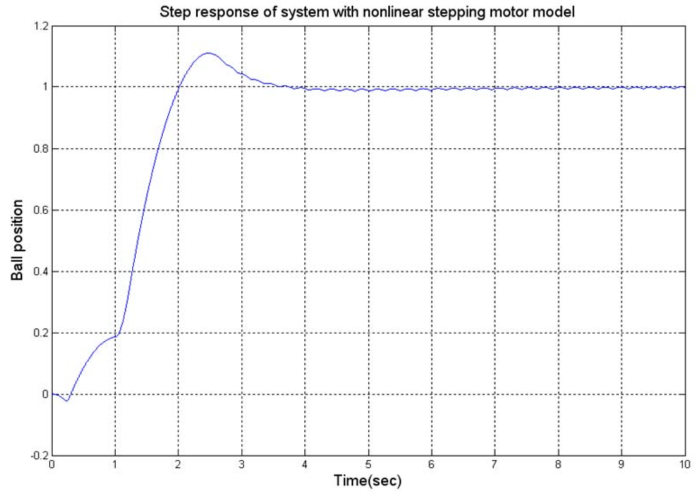


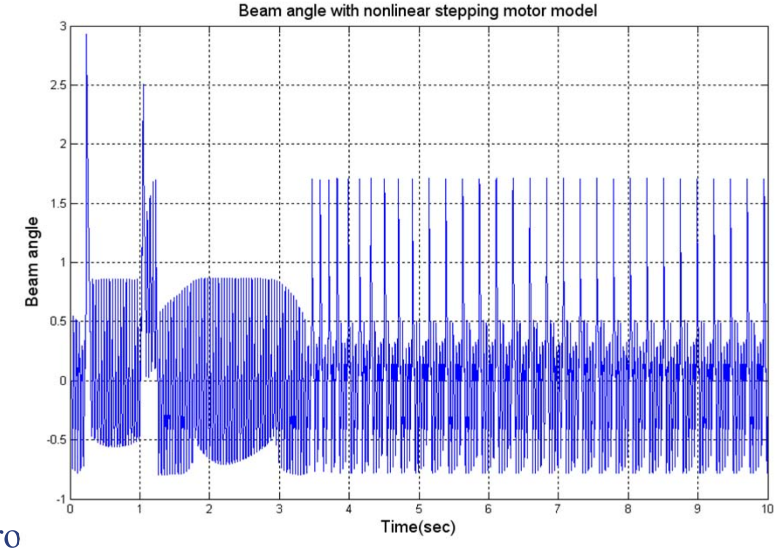


the system has a phase margin of 47.5 deg at a frequency of 1.38 Hz

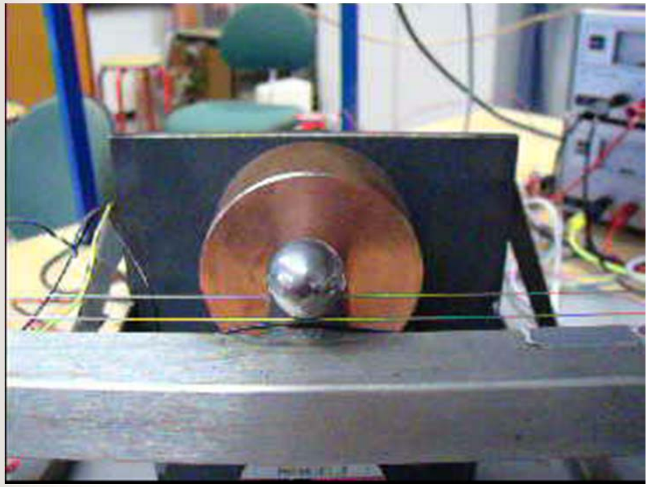
4.4.6 Simulation Tests

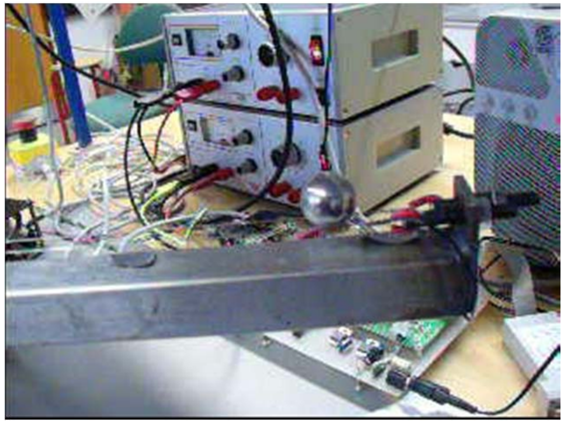






4.4.7 Real Test Videos





Exercise

Could you repeat the antenna design using

1. Continuous lead compensation;
2. Emulation method for digital control;

Such that the design specifications:

- Overshoot to a step input less than 5%;
- Settling time to 1% to be less than 14 sec.;
- Tracking error to a ramp input of slope 0.01rad/sec to be less than 0.01rad;
- Sampling time to give at at least 10 samples in a rise time.

(Write your analysis and program on a paper!)