MM11 Root Locus Design Method

Reading material: FC pp.270-328

What have we talked in lecture (MM10)?

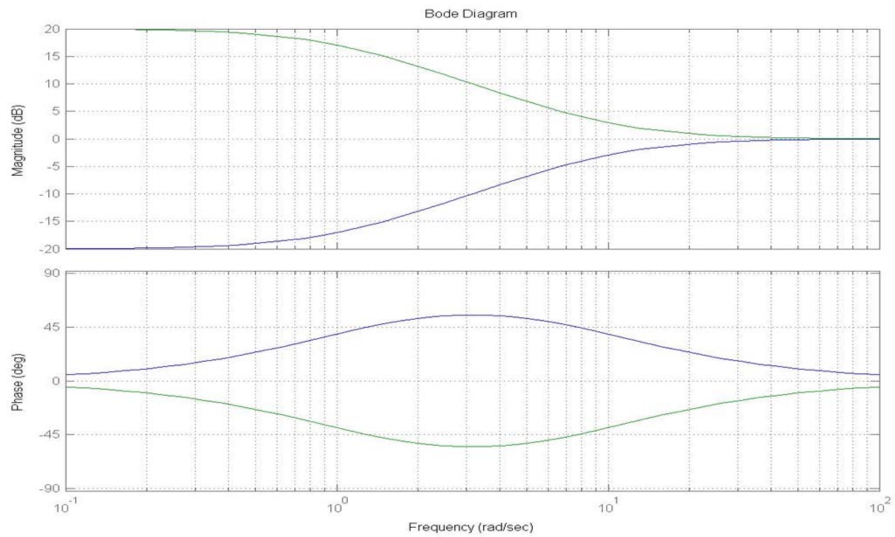
- Lead and lag compensators

  D(s)=(s+z)/(s+p)

  with z < p or z > p

  D(s)=K(Ts+1)/(αTs+1),

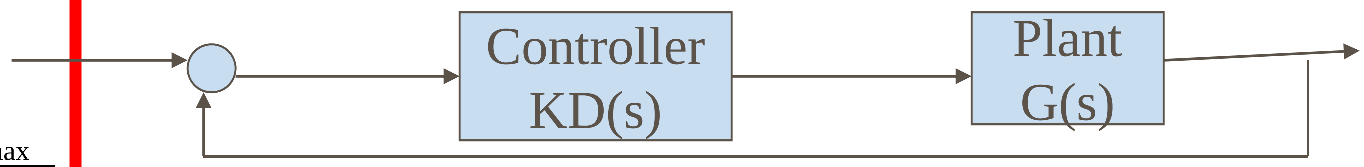
  with α <1 or α >1



- A systematical procedure for lead compensator design

$$\omega_{max} = \frac{1}{T\sqrt{\alpha}}$$

$$\alpha = \frac{1 - \sin\beta_{max}}{1 + \sin\beta_{max}}$$



Goals for this lecture (MM11)

- **What's root locus?**
  - **Definition**
  - **features**

- How to sketch a root locus?
  - Manually
  - Matlab - rlocus()

- How to use root locus for control design?

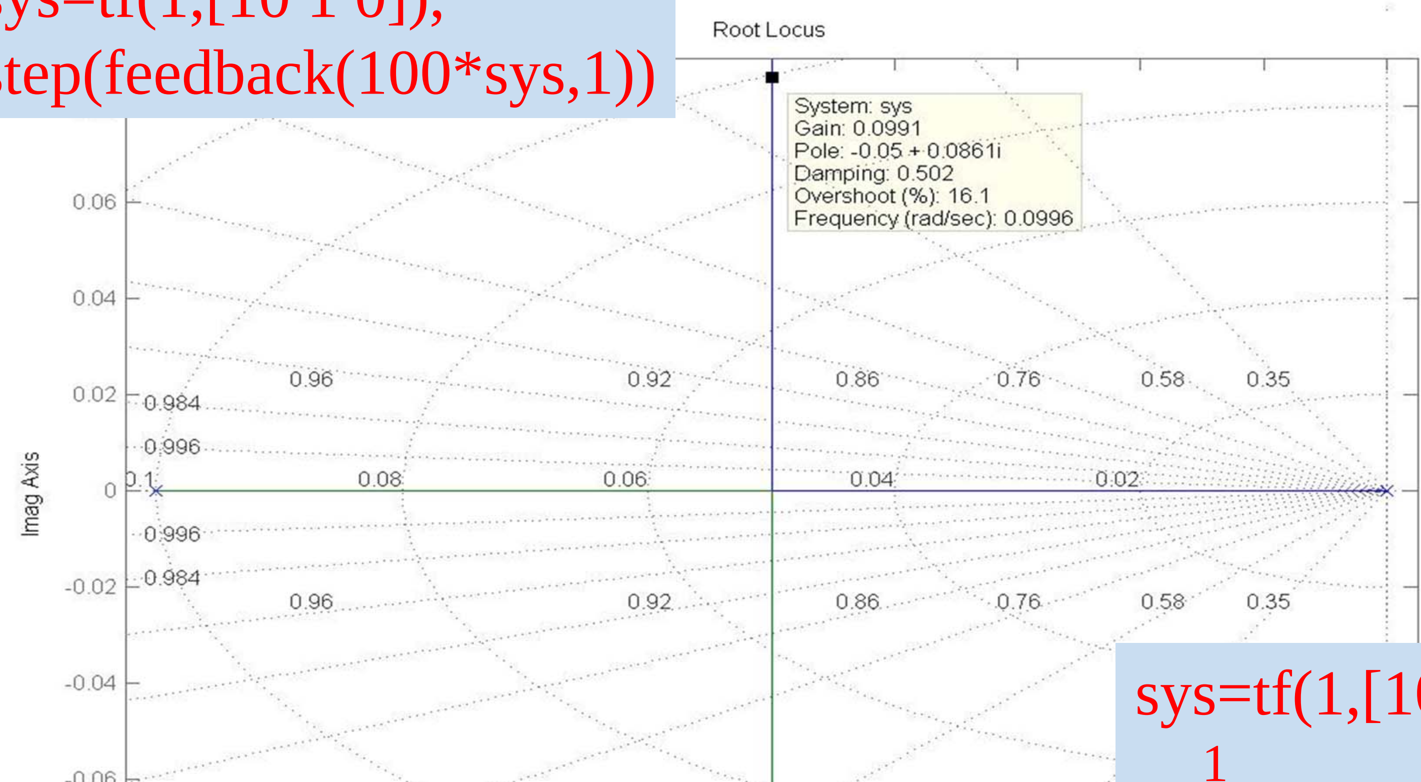
Illustrative Example: Antenna Control from MM10?

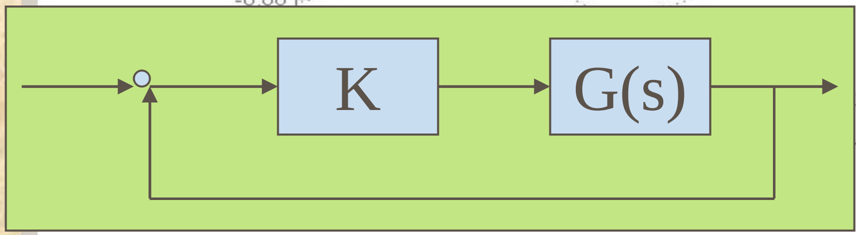
Control system design for a satellite tracking antenna

- Design specifications:
  - Overshoot to a step input less than 16%;
  - Settling time to 2% to be less than 10 sec.;
  - Tracking error to a ramp input of slope 0.01rad/sec to be less than 0.01rad;
  - Sampling time to give at at least 10 samples in a rise time.

Example: **Just Gain Controller?**

```
sys=tf(1,[10 1 0]),
step(feedback(100*sys,1))
```

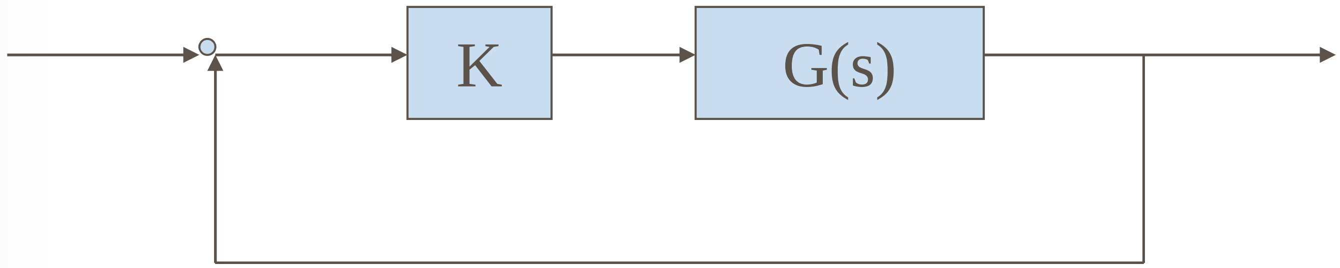





```
sys=tf(1,[10 1 0]),
    1
-------------
10 s^2 + s
```

rlocus(sys)

1. Introduction - Root Locus



Open-loop trans. Func.: **KG(s)**;
Closed-loop trans. Func.: **KG(s)/(1+KG(s))**
Sensitivity function: **1/(1+KG(s))**

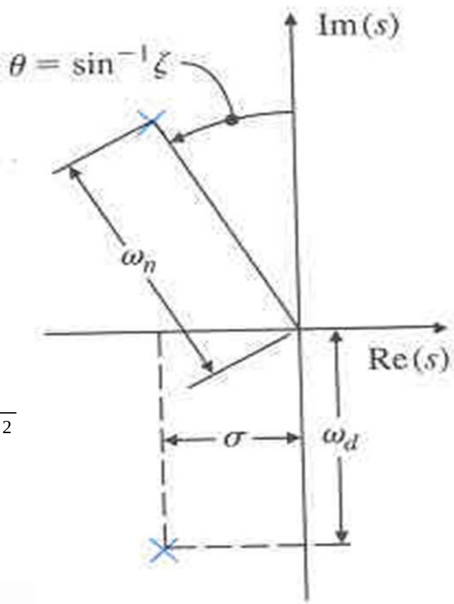
- The root locus of an (open-loop) transfer function **KG(s)** is a plot of the locations (locus) of all possible closed loop poles with proportional gain **K** and unity feedback
- From the root locus we can select a gain such that our closed-loop system will perform the way we want

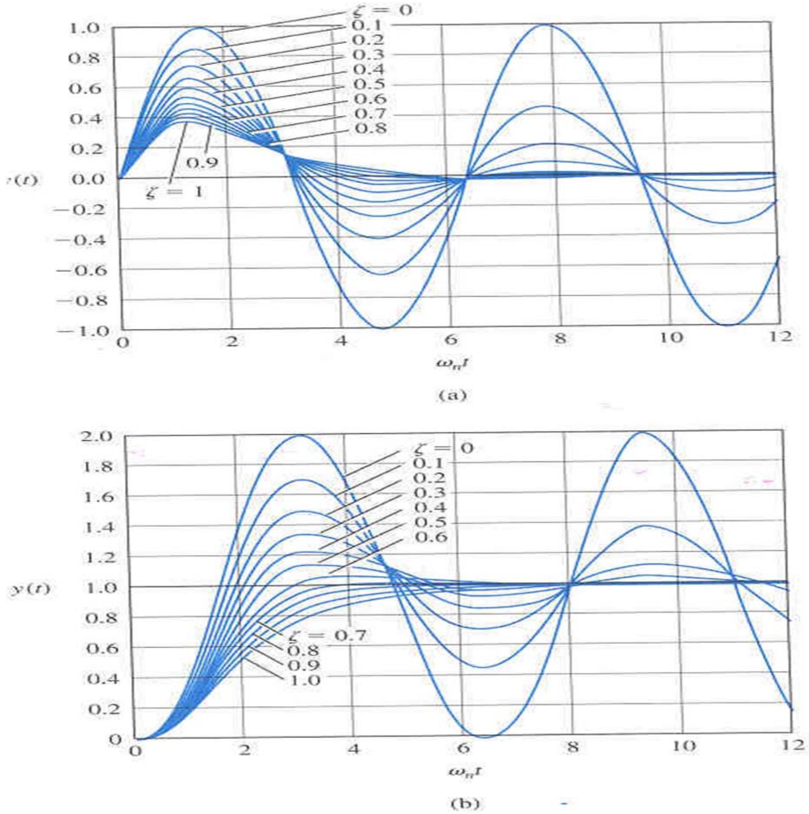
Root Locus: **Effects of Poles**

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad h(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\sigma t} \sin(\omega_d t) 1(t)$$

ζ : *damping ratio* $\quad \omega_n$: *naturalfre quency* $\quad \sigma = \zeta\omega_n, \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$

FIGURE 3.11
s-plane plot for a pair of complex poles





$$t_r \cong \frac{1.8}{\omega_n}$$

$$t_s \cong \frac{4.6}{\zeta\omega_n} \cong \frac{4.6}{\sigma}$$

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}, \quad 0 \le \zeta \le 1$$

$$t_p = \frac{\pi}{\omega_d}, \qquad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

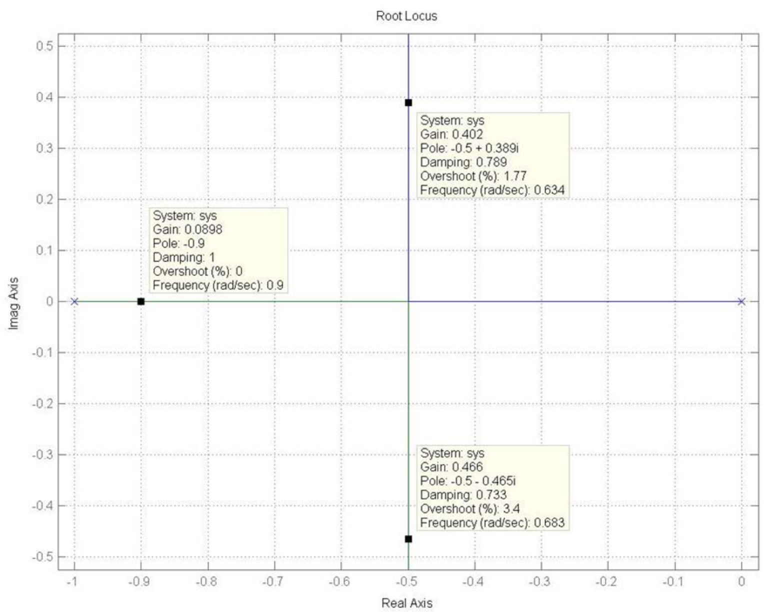
Root Locus: **Definition**

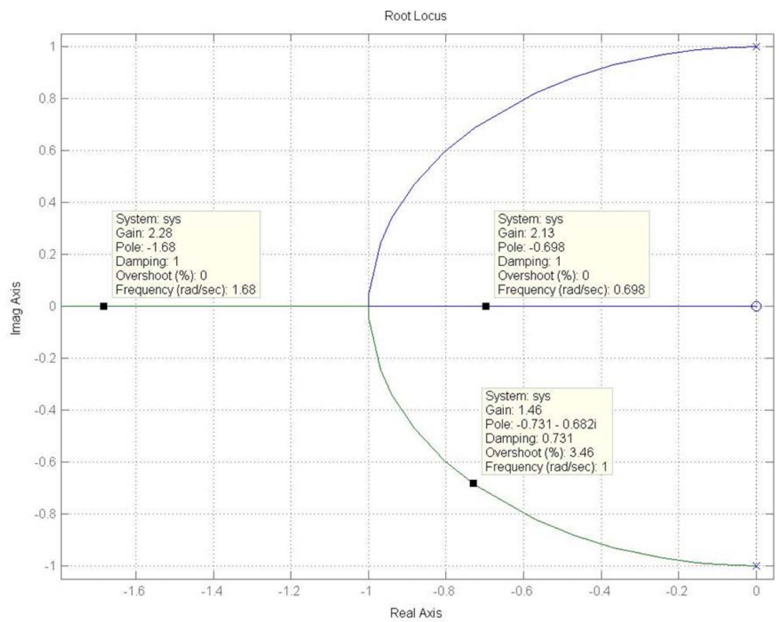
- **Root locus** is the set of vaues of **s** for which the follwing equation holds for some positive real vaue of **K**

$$1+KG(s)=0$$

Example1:
G(s)=1/s(s+1)

Example2:
G(s)=1/s(s+c)





How to Sketch a Root Locus?

Principles:

- The root locus must have n branches, where n is the number of poles of G(s)
- Each branch starts at a pole of G(s) and goes to a zero of G(s)
- If G(s) has more poles than zeros: $m < n$, we say that G(s) has zeros at infinity, i.e., there are n-m number of branches of the root locus that go to infinity (asymptotes)

Rules for manually sketching a Root Locus

- The phase **G(s)** of is always **180 (180 locus)**

Summary: Guidelines for Plotting a 180° Root Locus

1. Mark poles with an × and zeros with a ○.
2. Draw the locus on the real axis to the left of an odd number of real poles plus zeros.
3. Draw the asymptotes, centered at α and leaving at angles ϕ_l, where

$$n - m = \text{number of asymptotes}$$

$$\alpha = \frac{\sum p_i - \sum z_i}{n - m} = \frac{-a_1 + b_1}{n - m},$$

$$\phi_l = \frac{180^\circ + 360^\circ(l-1)}{n - m}, \qquad l = 1, 2, \ldots, n - m.$$

4. Compute locus departure angles from the poles and arrival angles at the zeros:

$$q\phi_{\text{dep}} = \sum \psi_i - \sum \phi_i - 180^\circ - 360^\circ l,$$

$$q\psi_{\text{arr}} = \sum \phi_i - \sum \psi_i + 180^\circ + 360^\circ l,$$

where q is the order of the pole or zero and l takes on q integer values so that the angles are between $\pm 180^\circ$.
5. If further refinement is required at the stability boundary: Assume $s_0 = j\omega_0$, and compute the point(s) where the locus crosses the imaginary axis for positive values of K, and/or use Routh's stability criterion. (This step may not be necessary.)
6. Use the results from the study of multiple roots to help in sketching how locus segments come together and break away: Two segments come together at 180° and break away at ±90°. Three locus segments approach each other at relative angles of 120° and depart at angles rotated by 60°.
7. Complete the locus, using the facts developed in the previous steps and making reference to the illustrative loci for guidance. The locus branches start at poles and end at zeros or infinity.

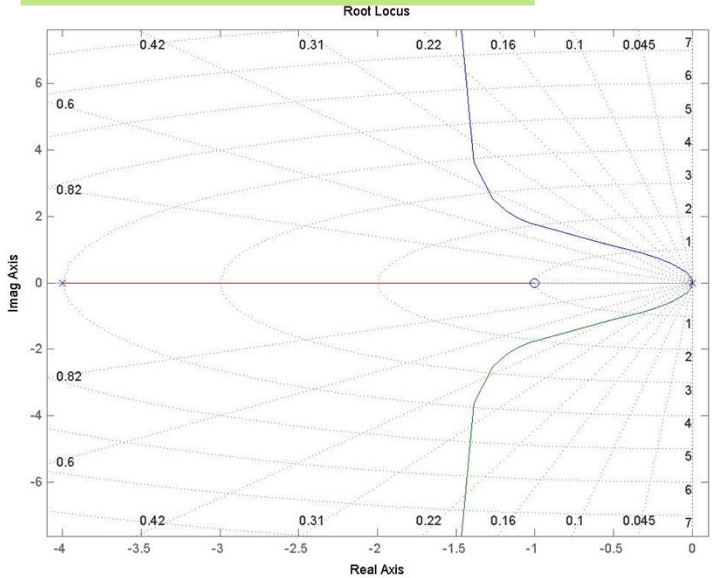
Plotting a Root Locus Using Matlab (I)

rlocus(sys)

Example3:

G(s)=(s+1)/s^2(s+4)

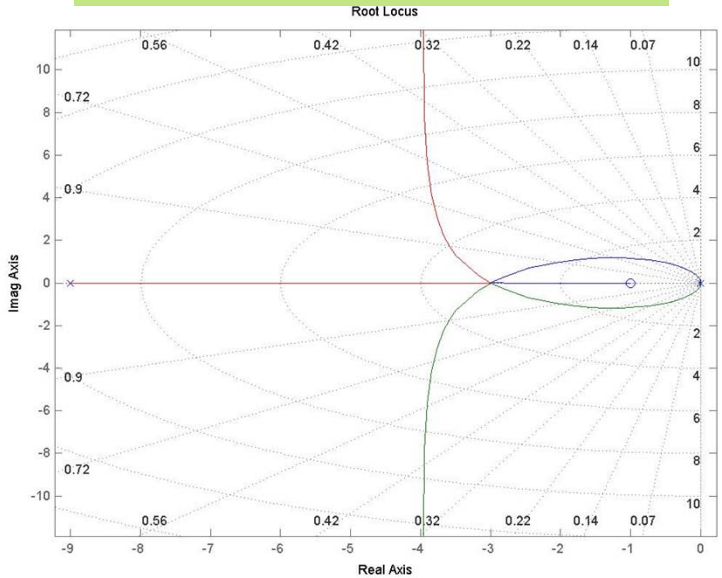
```
num=[1 1];
den=[1 4 0 0];
sys=tf(num,den);
rlocus(sys); sgrid
```



Example4:

G(s)=(s+1)/s^2(s+9)

```
num=[1 1];
den=[1 9 0 0];
sys=tf(num,den);
rlocus(sys); sgrid
```



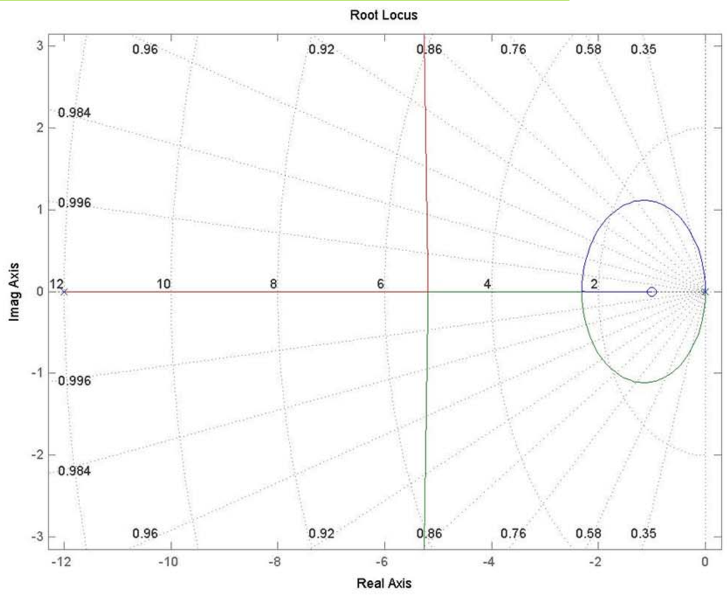
Plotting a Root Locus Using Matlab (II)

rlocus(sys)

Example5:

G(s)=(s+1)/s²(s+12)

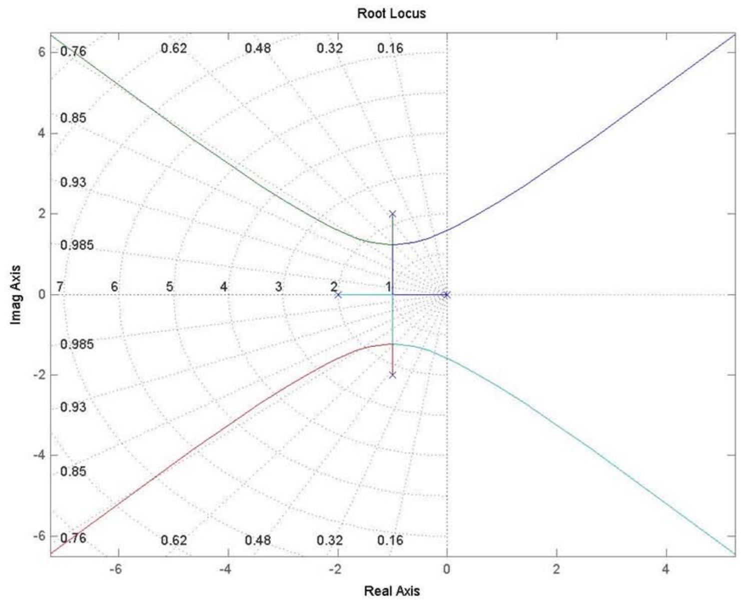
```
num=[1 1];
den=[1 12 0 0];
sys=tf(num,den);
rlocus(sys); sgrid
```



Example6:

G(s)=1/[s(s+2)[(s+1)²+4]]

```
num=[1];
den=conv([1 2 0],[1 2 5]);
sys=tf(num,den);
rlocus(sys); sgrid
```



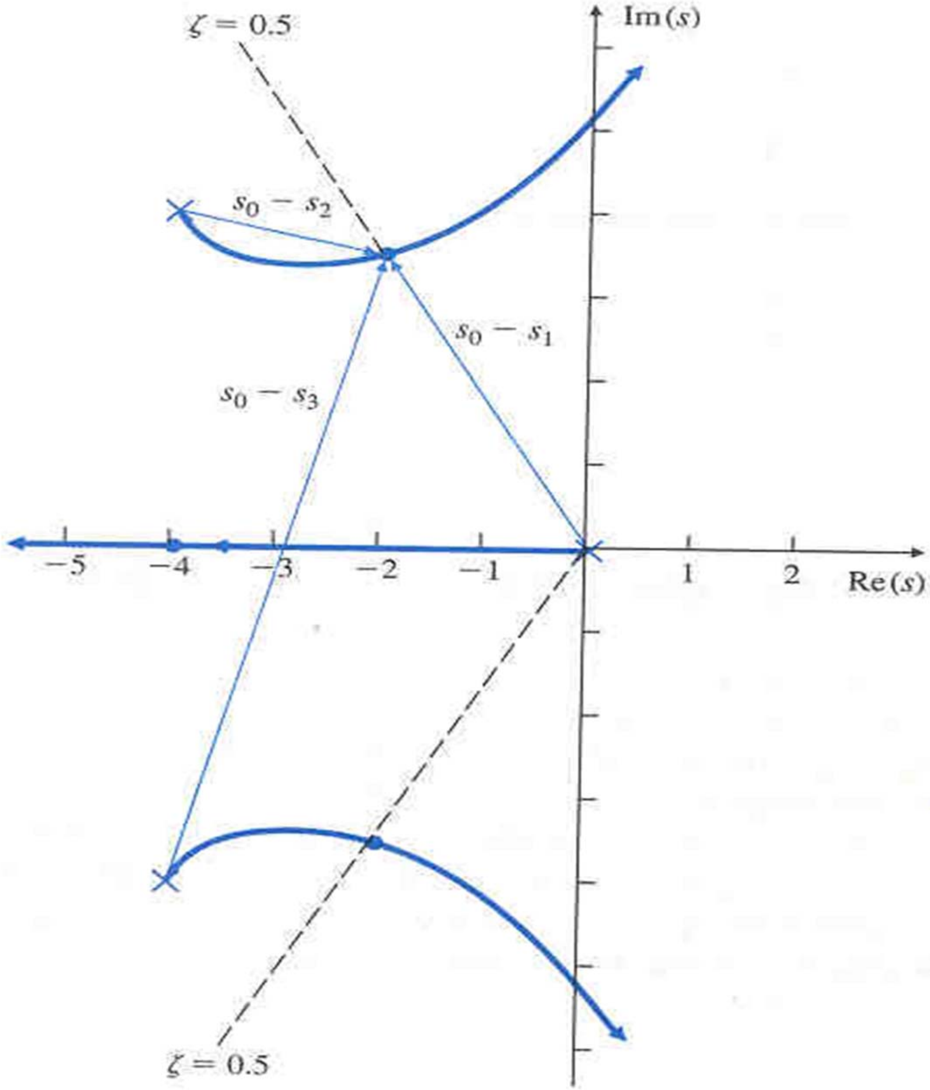
Control Design Using Root Locus (I)

- **Objective:** select a particular value of K that will meet the specifications for static and dynamic

  $$1+KG(s)=0$$

- **Magnitude condition:**

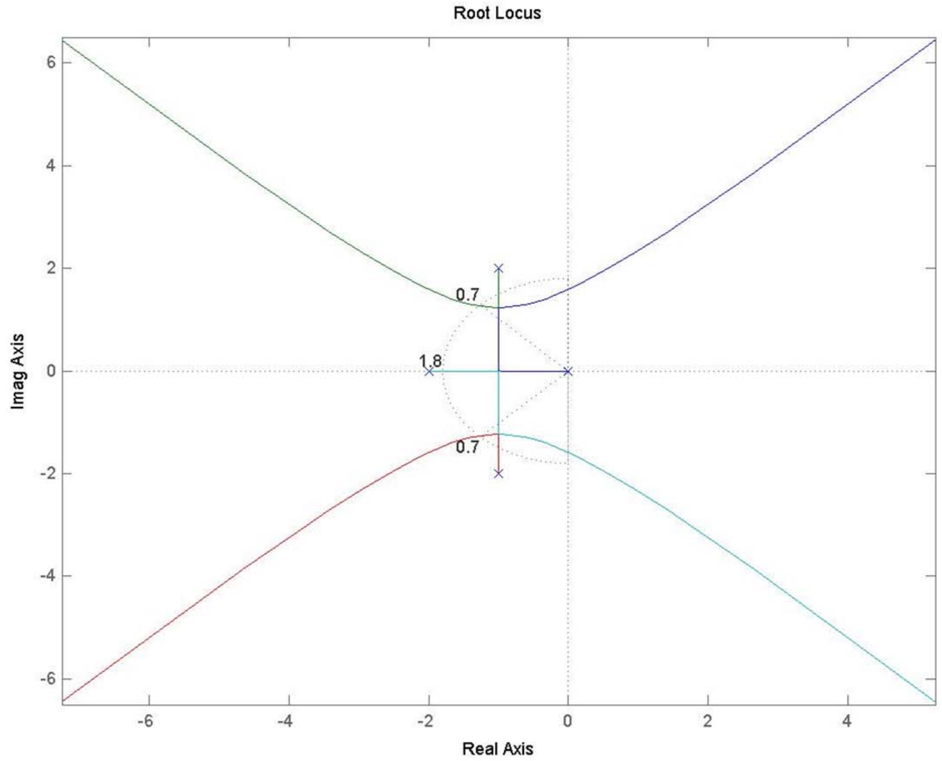
  $$K=1/|G(s)|$$



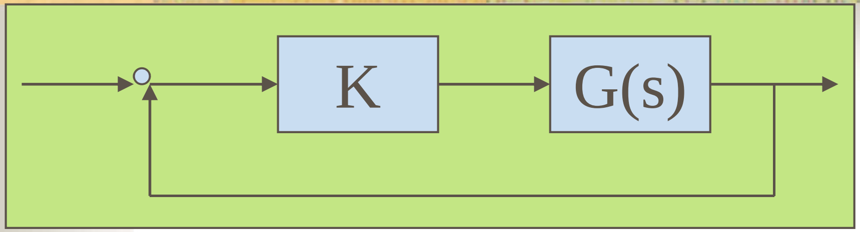
Control Design Using Root Locus (II)

- Command sgrid(Zeta,Wn) to plot lines of constant damping ratio Zeta and natural frequency Wn

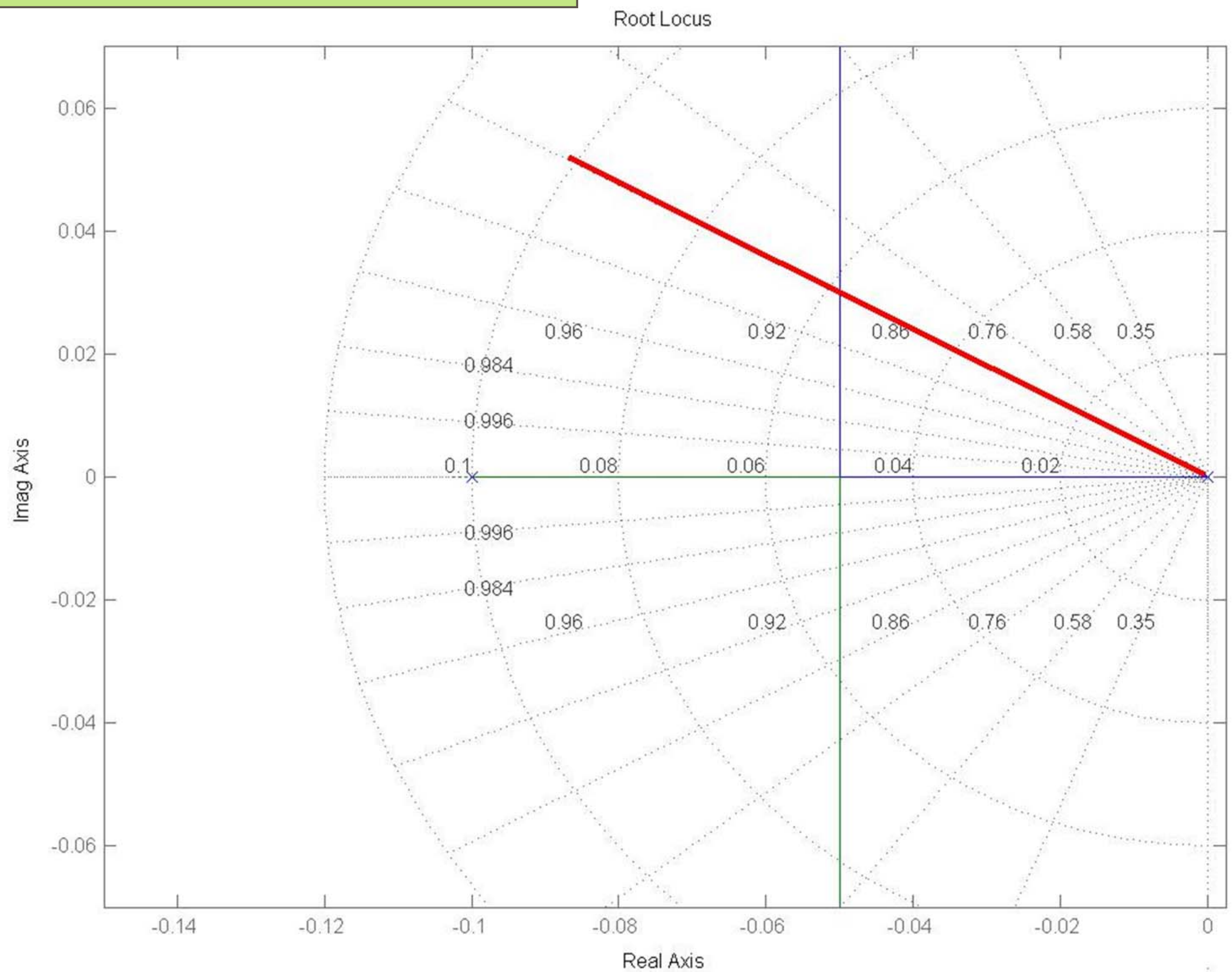
Example:
an overshoot less than 5%
(which means a damping ratio
greater than 0.7) and a rise
time of 1 second (which means
a natural frequency Wn greater
than 1.8).
zeta=0.7; Wn=1.8;
sgrid(zeta, Wn)



Gain controller?



sys=tf(1,[10 1 0]),
rlocus(sys);
sgrid(0.5,0.92)



Root Locus: **Dynamic Compensation**

- **Objective:** If a satisfactory process dynamics can not be obtained by a gain adjustment alone, some modification or compensation of the process dynamics is needed

$$1+KD(s)G(s)=0$$

- **Lead and lag compensations**

Lead Compensation (I)

- **Lead compensation:** acts mainly to lower rise time and decrease the transient overshoot:

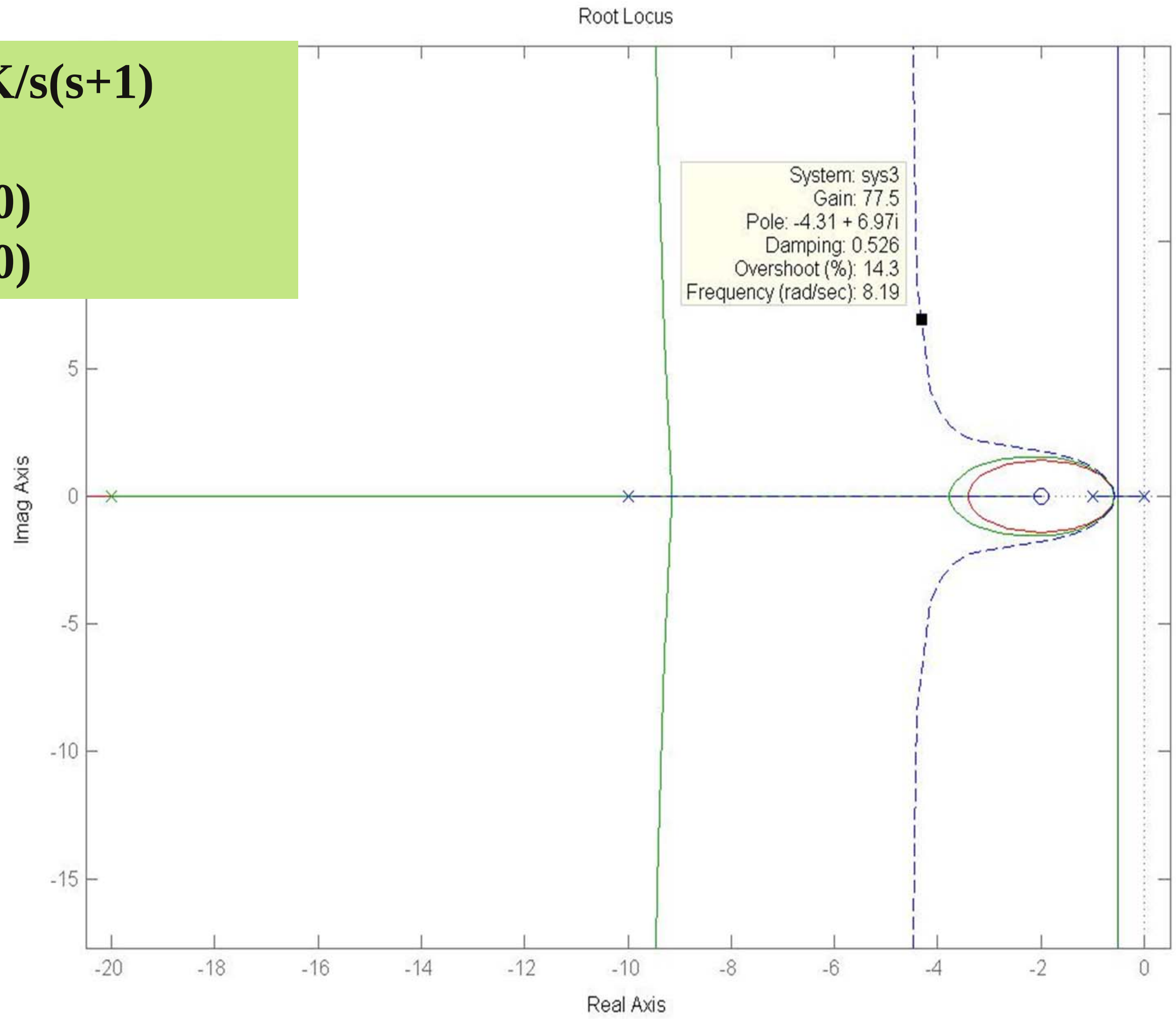
  D(s)=(s+z)/(s+p)   with z < p

- It moves the locus to the left and typically improves the system damping

- **Zero and pole selections (empirical rule)**
  - The zero is placed in the neighborhood of the closed-loop ω_n as determined by rise-time or settling-time requirements
  - The pole is located at s distance 3 to 20 times the value of the zero location

Lead Compensation (II)

Example: G(s)=K/s(s+1)
D1(s)=s+2;
D2(s)=(s+2)/(s+20)
D3(s)=(s+2)/(s+10)



Lead Compenstor: **Antenna Control Example**

- **Step 1: S**elect the lead compensator as: **D(s)=(s+0.92)/(s+10)**

  sysDG=tf(1,[10 1 0])*tf([1 0.92],[1 10]);

- **Step 2**: Draw the rootlocus to determine the **K**

  rlocus(sysDG), sgrid(0.5,0.92)

- **Step 3**: Construct the close-loop system and check the requirements are satisfied or not

  syscl=feedback(K*sysDG,1); figure; step(syscl), grid

- **Step 4**: If the requirements can not be satisfied, iterate the design procedure

Do your design using the root locus method!

Lag Compensation (I)

Lag compensator

- acts mainly to improve the steady-state accuracy:

  D(s)=(s+z)/(s+p) with z > p

- **Lag compensator** approximates PI control

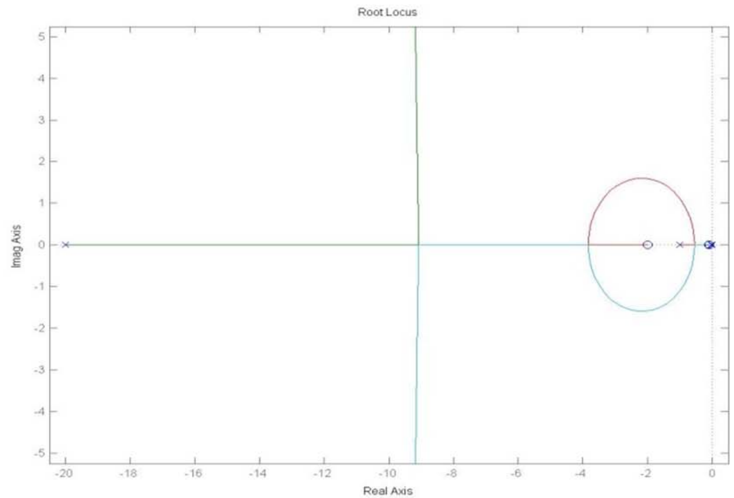
- **Zero and pole selection**
- Select a pole near s=0, and also select the zero nearby, such that the dipole does not significantly interfere the response
- The lag compensation usually degrades stability

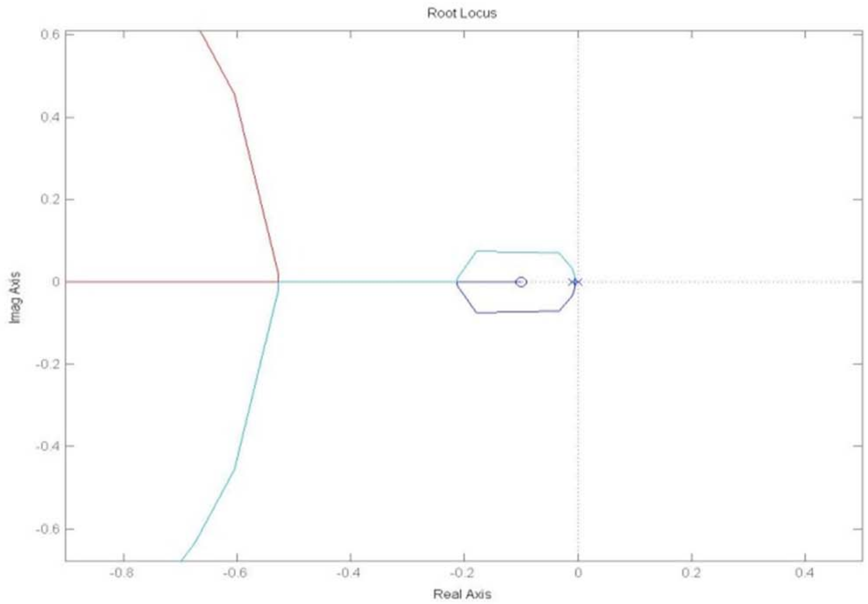
Lag Compensation (II)

Example:

- The effect of zerosis to move the locus to the left
- The effect of the poles is to press the locus toward the right

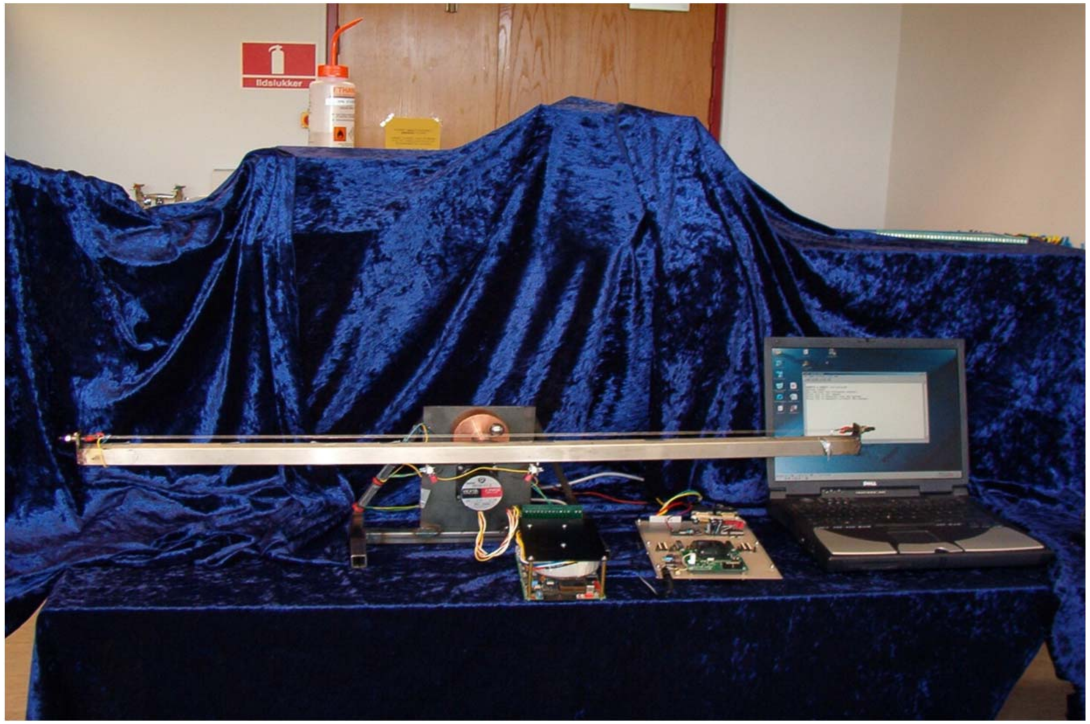
Example: G(s)=K/s(s+1)
D1(s)=(s+2)/(s+20) →K=31
D2(s)=(s+0.1)/(s+0.01)





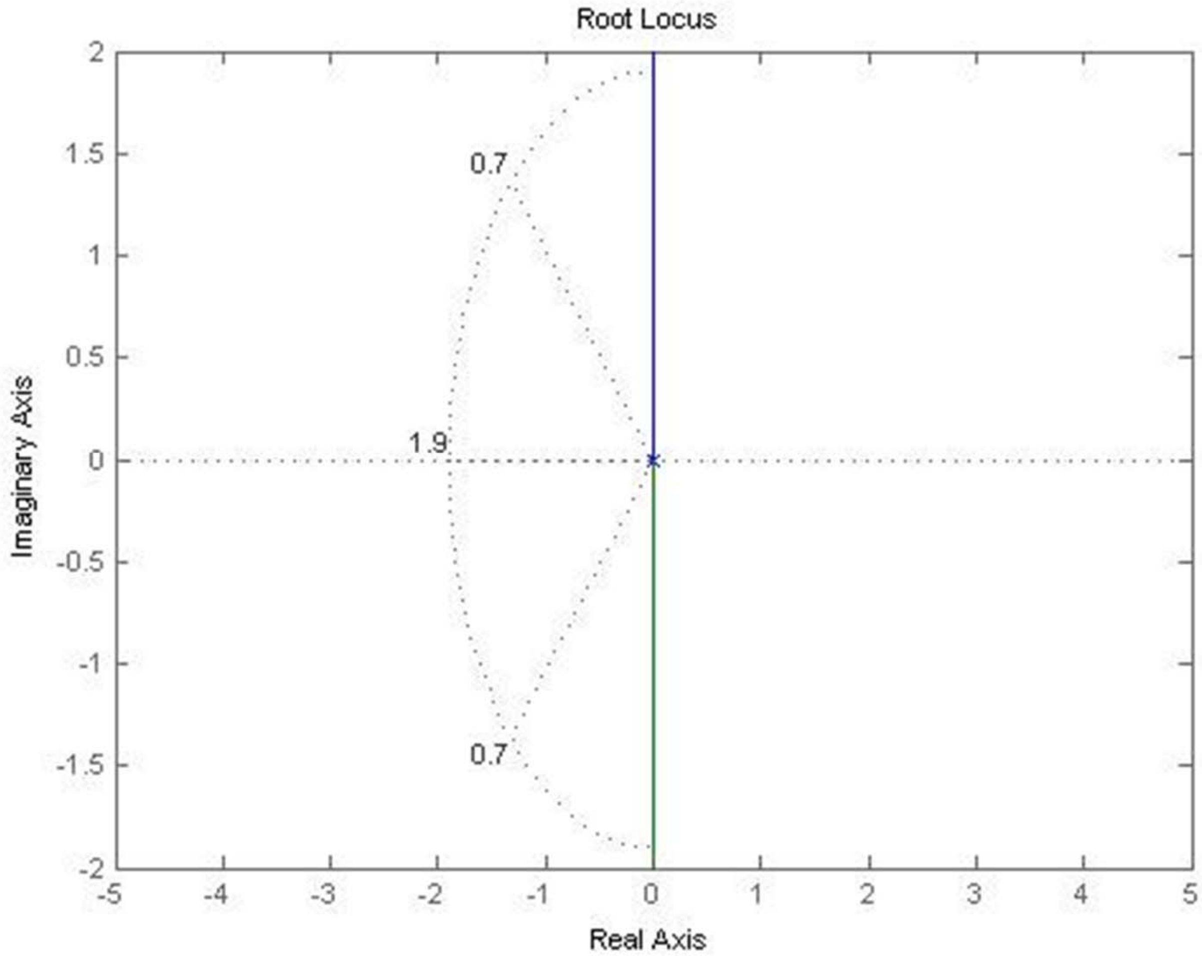
Real Case Study: Beam & Ball

- **System:** A ball rolls along the track of a beam that is pivoted at some position.
- **Objective:** To steadily place the ball at any given position along the track
- **Strategy:** To control the track angle through the control of a servo motor



Example: **Open-Loop Analysis**

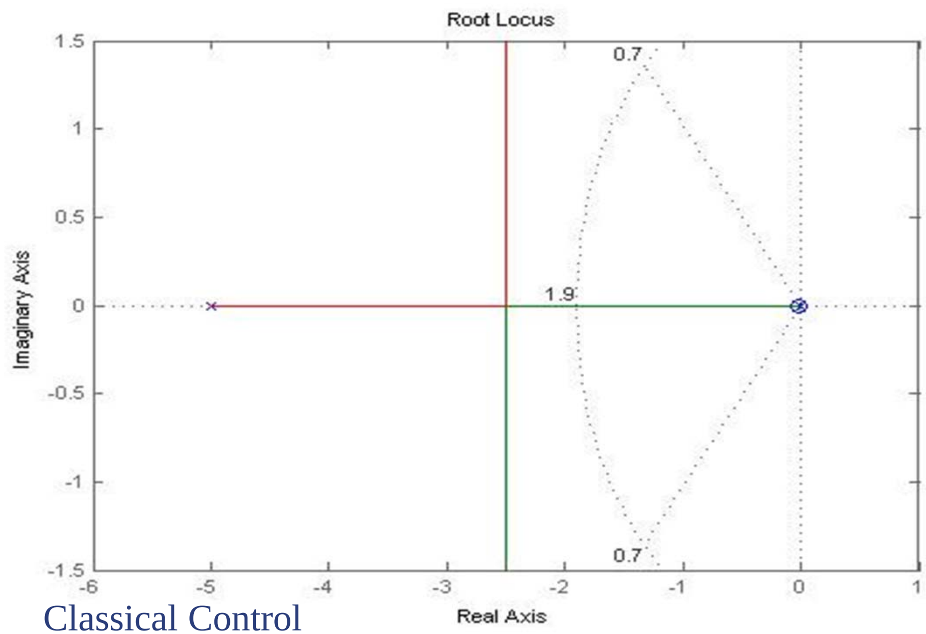
```
m = 0.111;
R = 0.015;
g = -9.8;
L = 1.0;
d = 0.03;
J = 9.99e-6;
K=(m*g*d)/(L*(J/R^2+m));
num = [-K];
den = [1 0 0];
rlocus(num,den);
sgrid(0.70, 1.9);
 axis([-5 5 -2 2])
```



Example: **Lead Compensator**

- A first order lead compensator tends to shift the root locus into the left-hand plane.
- We position the zero near the origin to cancel out one of the poles. The pole of our compensator will be placed to the left of the origin to pull the root locus further into the left-hand plane.

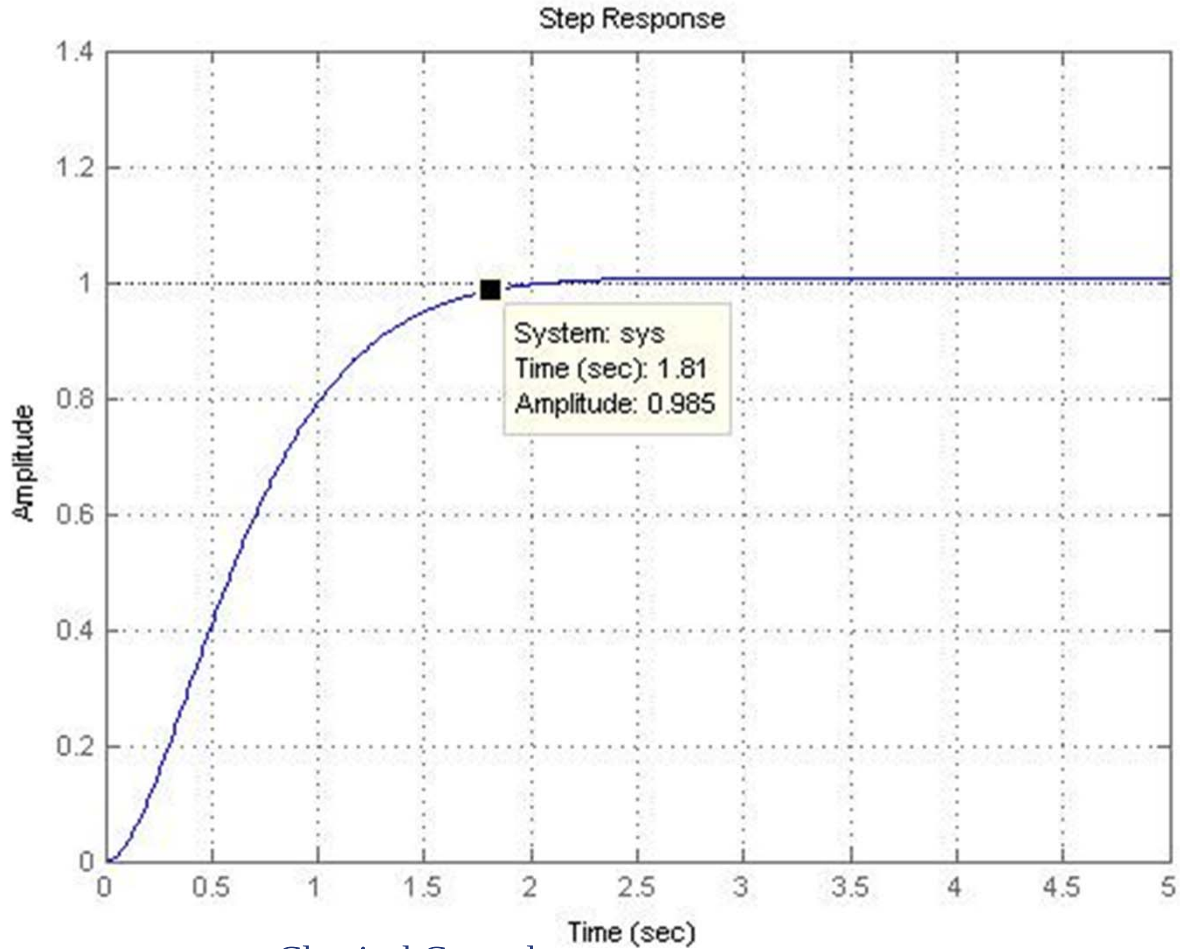
```
zo = 0.01; po = 5;
 numlead = [1 zo];
denlead = [1 po];
numl = conv(num,numlead);
denl = conv(den,denlead);
rlocus(numl,denl);
sgrid(0.70, 1.9)
```



Example: **Finding a Gain K**

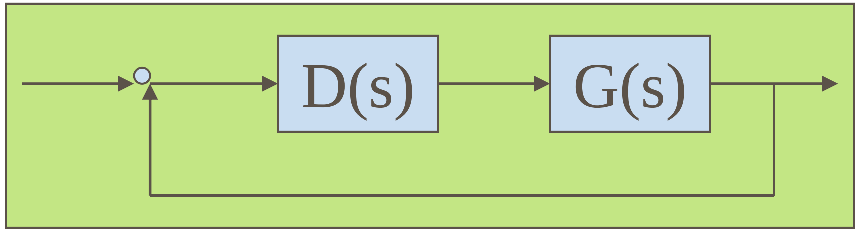
- [kc,poles]=rlocfind(numl,denl)

```
kc = 35.0253;
numl2 = kc*numl;
[numcl,dencl] =
cloop(numl2,denl);
 t=0:0.01:5;
figure
step(1*numcl,dencl,t)
```



Exercise

- Question 5.2 on FC page.321;
- Consider a DC motor control using a PI controler



Where the motor is modeled as $G(s)=K/(\tau s+1)$ and PI controller is $D(s)=K_p(T_i s+1)/T_i s$, with parameters $K=30$, $\tau=0.35$, $T_i=0.041$. Through the root locus method determine the largest vaule of K_p such that $\xi=0.45$

- Try to use the root locus method to design a lead compensator for the examplifed attenna system.