# MM4 System's Poles and Feedback Characteristics

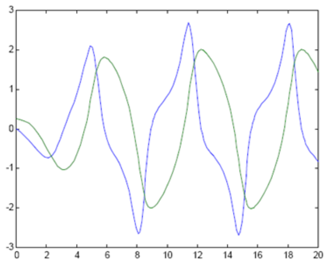
Readings:

- Section 3.3 (response & pole locations, p.118-126);
- Section 4.1 (basic properties of feedback, p.167-179);
- Extra readings (feedback characterisitcs)

# What have we talked in **MM3**?

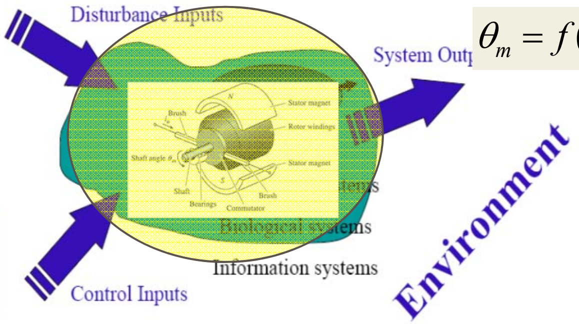
- The reponse analysis
- Performance specification
- Numerical simulation

# MM3: Time Response Analysis (I)

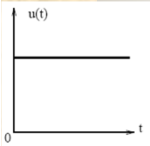


"help ode23" to find out all the other options.

d(t)=0




$$\theta_m = f(\theta_m(0), \dot{\theta}_m(0), v_a, T_e, T_f)$$




Typical input u(t)

Time response y(t)

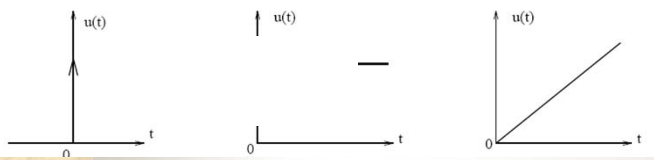
Laplace Trans

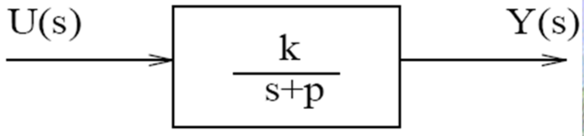
$$\theta_m(0) = \alpha, \quad \dot{\theta}_m(0) = \beta$$

Inv Laplace T.

$$Y(s) = G(s)U(s)$$

$$G(s) = \frac{\sum_{i=0}^{m} b_i s^i}{\sum_{i=0}^{n} a_i s^i}$$




# MM3: Time Response Analysis (II)

- Typical inputs: impulse, step and ramp signals
- 1st, 2nd and high-order (LTI) systems

| Test signal | $u(t)$ | $U(s)$ |
|---|---|---|
| Impulse | $\delta(t)$ | 1 |
| Step | 1 | $1/s$ |
| Ramp | $t$ | $1/s^2$ |

$G(s)=\dfrac{k}{s+p}$, pole : $-p$, time constant : $\dfrac{1}{p}$

$G(s)=\dfrac{c}{\tau s+1}$, pole : $-\dfrac{1}{\tau}$, time constant : $\tau$

time domain :

$g(t)=ke^{-pt}$ or $g(t)=\dfrac{c}{\tau}e^{-\frac{1}{\tau}t}$

$$G(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}.$$

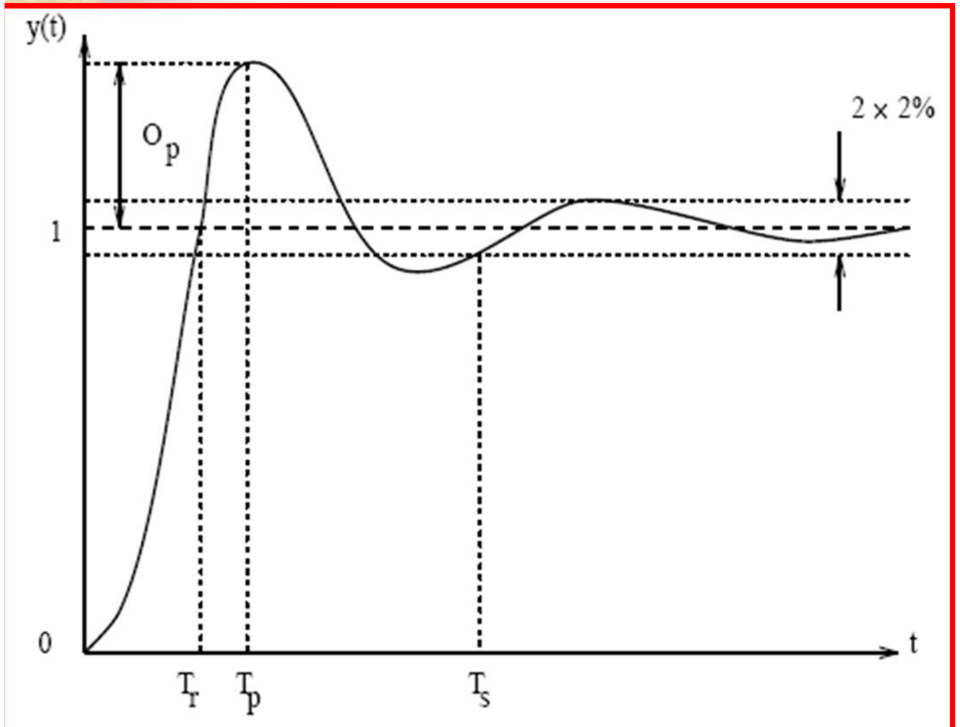
- $\xi$ – *damping ratio*, a dimensionless factor
- $\omega_n$ – *natural frequency* with unit rad/s

**Time response = excitation response + initial condition response (free response)**

$$\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi},\ \omega_d = \omega\sqrt{1-\xi^2}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

# MM3: Performance Specification




$$t_r \cong \frac{1.8}{\omega_n}$$

$$t_s \cong \frac{4.6}{\zeta\omega_n} \cong \frac{4.6}{\sigma}$$

$$M_p \cong \begin{cases} 5\%, & \zeta = 0.7 \\ 16\%, & \zeta = 0.5 \\ 35\%, & \zeta = 0.3 \end{cases}$$

$$t_p \cong \frac{\pi}{\omega_d}, \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

Rise time $T_r = \dfrac{\pi - \phi}{\omega_d}$

Peak time $T_p = \dfrac{\pi}{\omega_d}$

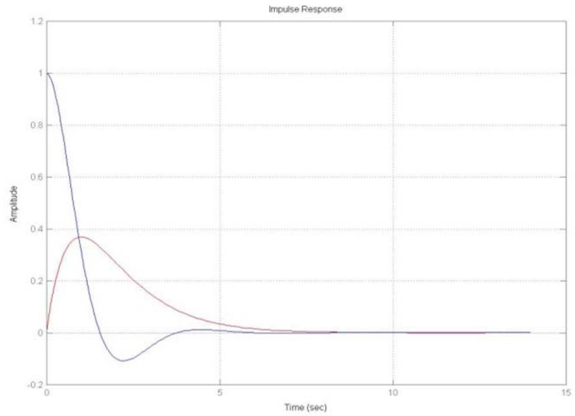
Settling time $T_s \approx \dfrac{4}{\xi\omega_n}$

Overshoot $O_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$

*Steady-state error* $e_{ss}$: difference between input & output as $t \to \infty$

# MM3: Numerical Simulation

- Impulse response: **impulse(sys)**
- Step response: **step(sys)**
- **ltiview(sys)**
- **Subplot(m,n,1)**




**EXAMPLE:**

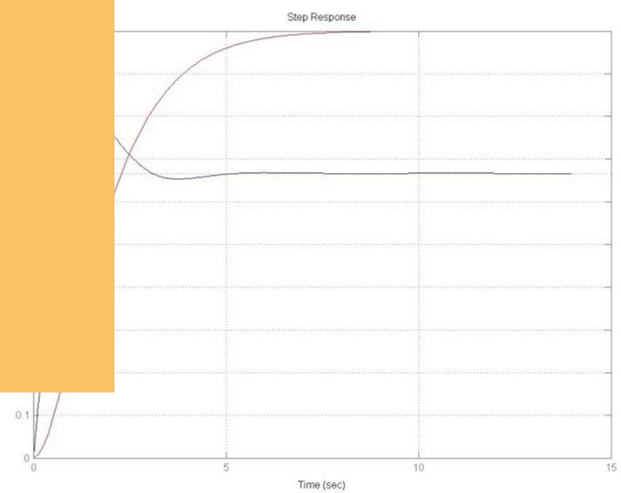
sys1: Sys2:

num1=[1]; num2=[1 2];

den1=[1 2 1]; den2=[1 2 3];

impulse(tf(num1,den1),'r',tf(num2,den2),'b')

step(tf(num1,den1),'r',tf(num2,den2),'b')



# Goals for this lecture (MM4)

- **System poles vs. time responses**
  - **Poles and zeros**
  - **Time responses vs. Pole locations**

- Feedback characteristics
  - Characteristics
  - A simple feedback design

- Block diagram decomposition (simulink)

# System Poles

- First-order system

$$G(s)=\frac{k}{s+p}, \qquad \text{pole}: \ -p, \quad G(s)=\frac{c}{\tau s+1}, \qquad \text{pole}: \ -\frac{1}{\tau}$$

- Second-order system

$$G(s)=\frac{\omega_n{}^2}{s^2+2\xi\omega_n+\omega_n{}^2},$$

$$\text{poles}: \quad p_{1,2}=-\xi\omega_n\pm\omega_n\sqrt{\xi^2-1}$$

$$\text{real (different) poles}: \ p_{1,2}=-\xi\omega_n\pm\omega_n\sqrt{\xi^2-1}, \quad \text{if } \xi>1$$

$$\text{real (identical) poles}: \ p_{1,2}=-\xi\omega_n, \quad \text{if } \xi=1$$

$$\text{complex poles}: \ p_{1,2}=-\xi\omega_n\pm j\omega_n\sqrt{1-\xi^2}, \quad \text{if } 0<\xi<1$$

$$\text{complex poles}: \ p_{1,2}=\pm j\omega_n, \quad \text{if } \xi=0$$

- High-order system

$$G(s)=\frac{\sum_{i=0}^{m} b_i s^i}{\sum_{i=0}^{n} a_i s^i}=\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

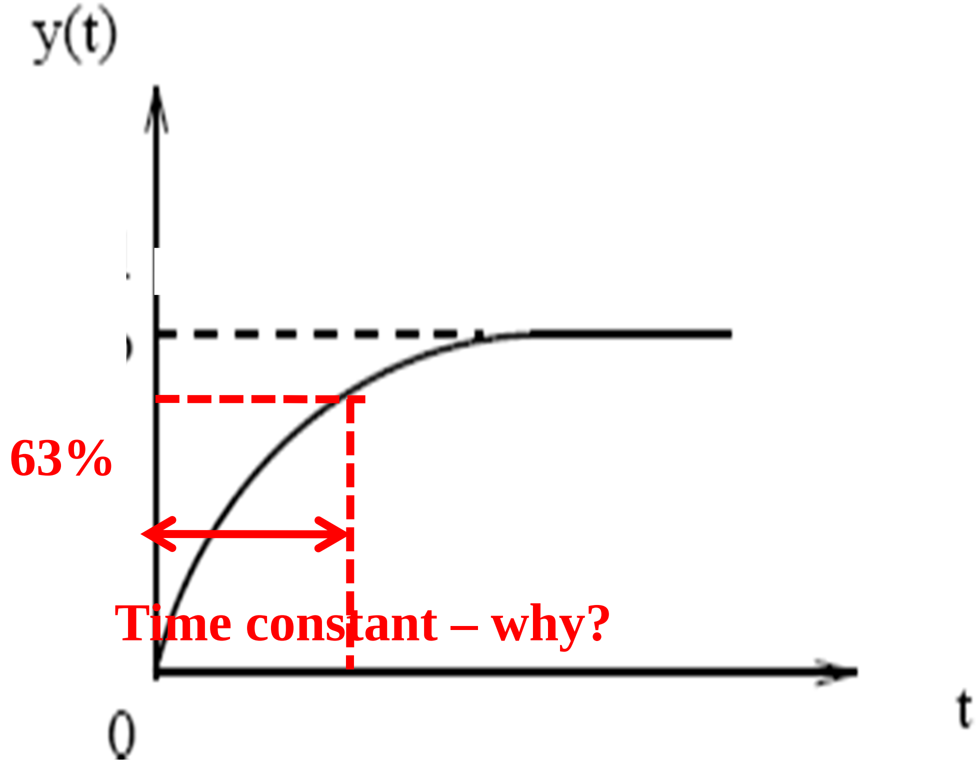
# Pole vs Time Response: **First-order System (I)**

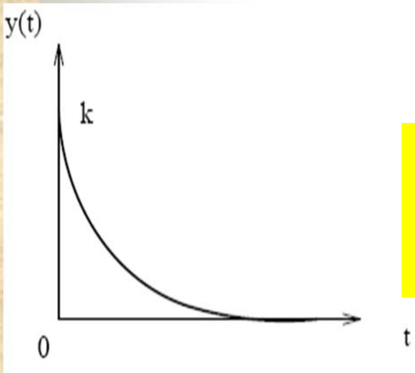
$$G(s) = \frac{1}{\tau s+1}, \quad \text{assume} \quad \tau > 0$$

$$\text{pole:} \quad -\frac{1}{\tau}, \quad \text{time constant: } \tau,$$

$$\text{Impulse response:} \quad y(t) = L(\frac{1}{\tau s+1}) = e^{-\frac{1}{\tau}t}$$

$$\text{Step response:} \quad y(t) = L(\frac{1}{s(\tau s+1)}) = 1 - e^{-\frac{1}{\tau}t}$$



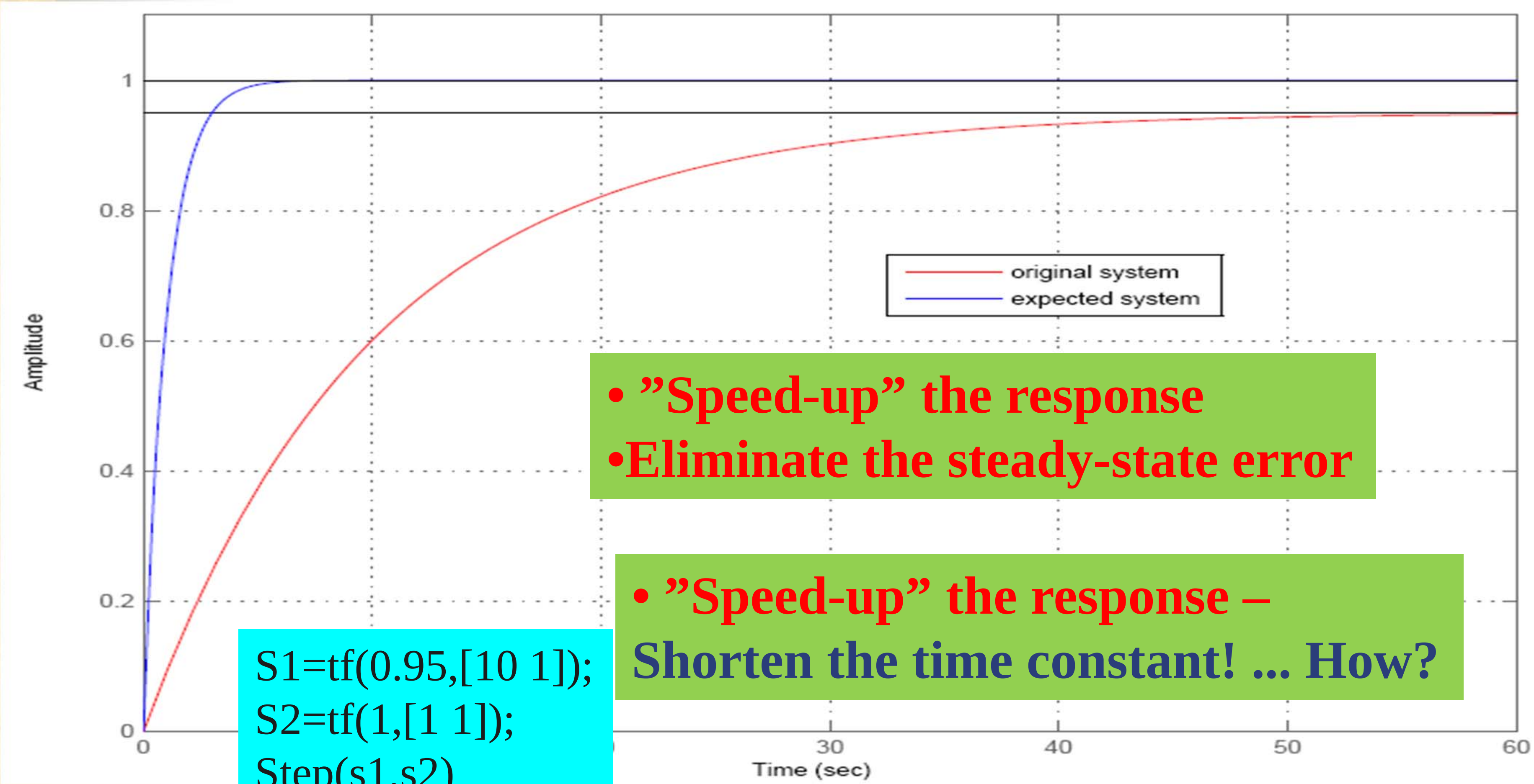




**Time response is determined by the time constant**

**System pole is the negative of inverse time constant**

# Pole vs Time Response: **First-order System (II)**

- An design problem – how to use this knowledge

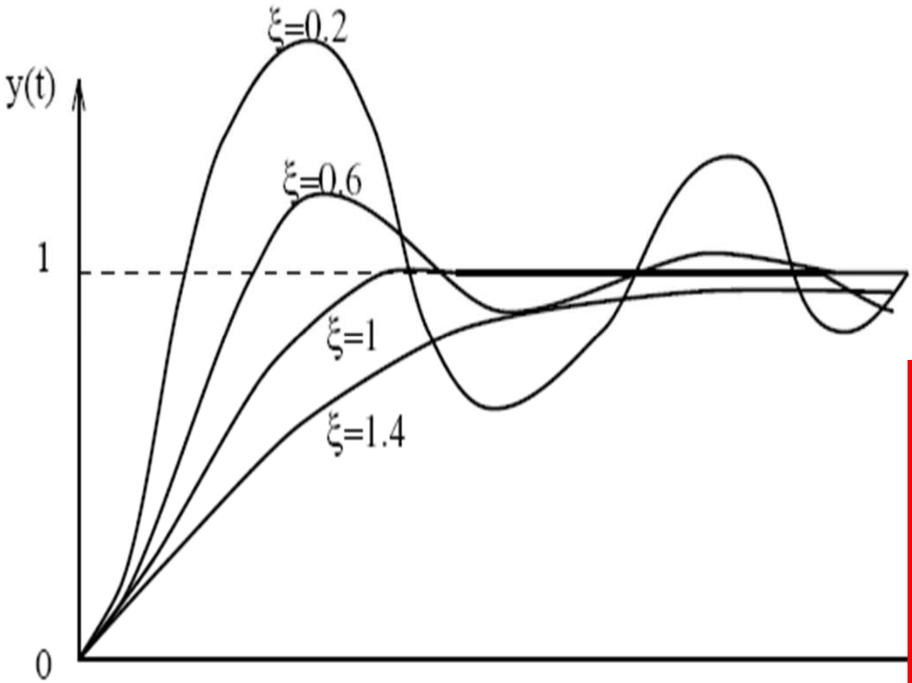



- **"Speed-up" the response**
- **Eliminate the steady-state error**

- **"Speed-up" the response – Shorten the time constant! ... How?**

```
S1=tf(0.95,[10 1]);
S2=tf(1,[1 1]);
Step(s1,s2)
```

# Time Response: **2nd-order System (I)**




$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}.$$

- $\xi$ – *damping ratio*, a dimensionless factor
- $\omega_n$ – *natural frequency* with unit rad/s

| | |
|---|---|
| $\xi = 0$ | Undampted |
| $\xi < 1$ | Underdampted |
| $\xi = 1$ | Critically damped |
| $\xi > 1$ | Overdamped |

Summary: The response

- converges to 1 due to $e^{-\xi\omega_n}$;
- oscillates at frequency $\omega_d$ if $0 < \xi < 1$;
- has no oscillation if $\xi \geq 1$;
- possesses larger amplitude of the initial response period for smaller $\xi$;
- equals $y(t) = 1 - \cos\omega_n t$ for the extreme case: $\xi = 0$

# Time Response: **2nd-order System (II)**

$$G(s)=\frac{\omega_n^{2}}{s^2+2\xi\omega_n+\omega_n^{2}},\quad \text{assume } \omega_n>0,\quad \xi\ge 0$$

poles: $p_{1,2}=-\xi\omega_n\pm\omega_n\sqrt{\xi^2-1}$

real(different) poles: $p_{1,2}=-\xi\omega_n\pm\omega_n\sqrt{\xi^2-1}$, if $\xi>1$

real(identical) poles: $p_{1,2}=-\xi\omega_n$, if $\xi=1$

complex poles: $p_{1,2}=-\xi\omega_n\pm j\omega_n\sqrt{1-\xi^2}$, if $0<\xi<1$

complex poles: $p_{1,2}=\pm j\omega_n$, if $\xi=0$

$$G(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}.$$

- $\xi$ – *damping ratio*, a dimensionless factor
- $\omega_n$ – *natural frequency* with unit rad/s

mmary: The response

converges to 1 due to $e^{-\xi\omega_n}$;

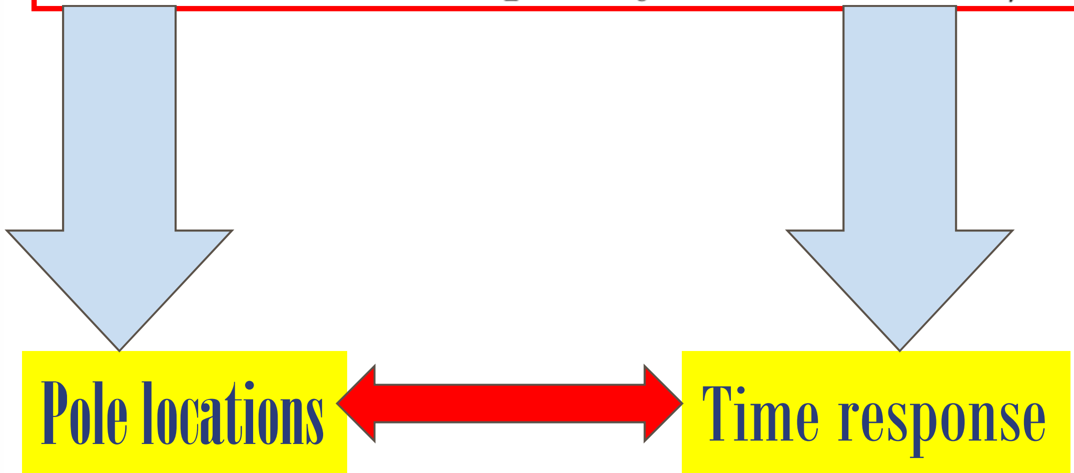
- oscillates at frequency $\omega_d$ if $0<\xi<1$;
- has no oscillation if $\xi\ge 1$;
- possesses larger amplitude of the initial response period for smaller $\xi$;
- equals $y(t)=1-\cos\omega_n t$ for the extreme case: $\xi=0$

| | |
|---|---|
| $\xi=0$ | Undampted |
| $\xi<1$ | Underdampted |
| $\xi=1$ | Critically damped |
| $\xi>1$ | Overdamped |

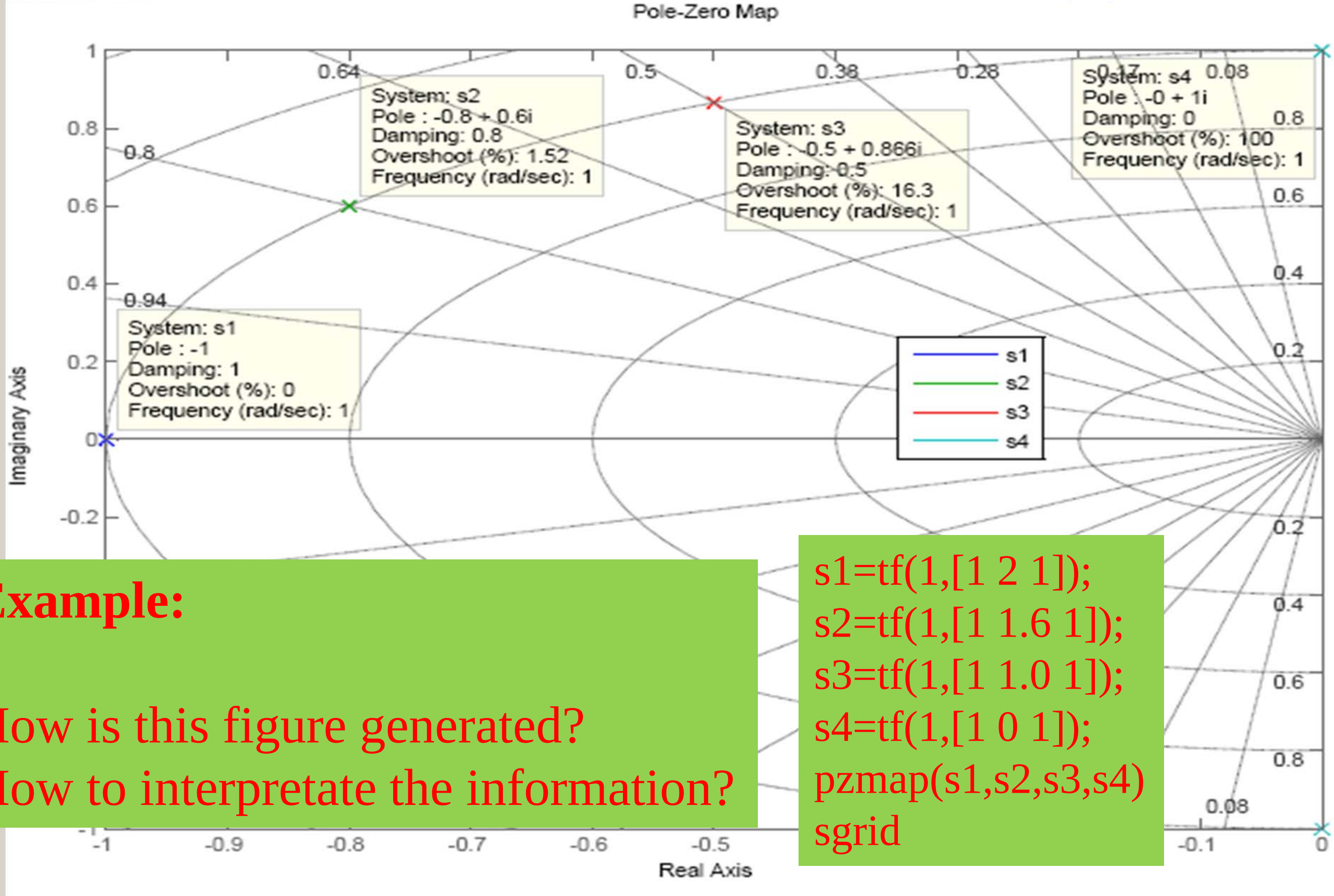
# 2nd-order System: **Poles vs Performance**

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}.$$

- $\xi$ – *damping ratio*, a dimensionless factor
- $\omega_n$ – *natural frequency* with unit rad/s



# 2nd-order System – **Pole Locations (I)**

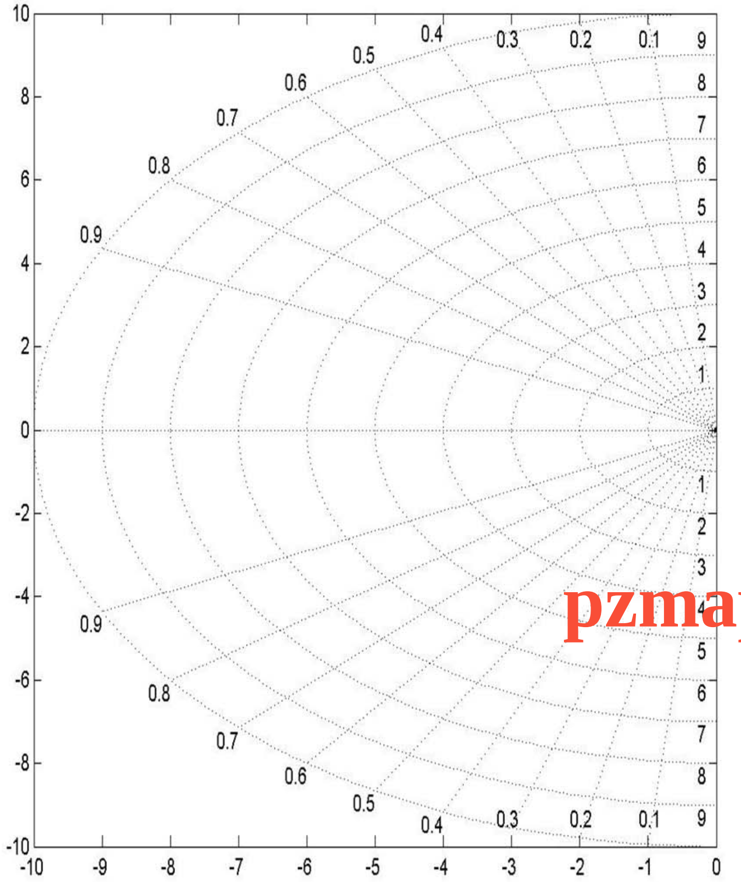



**Example:**

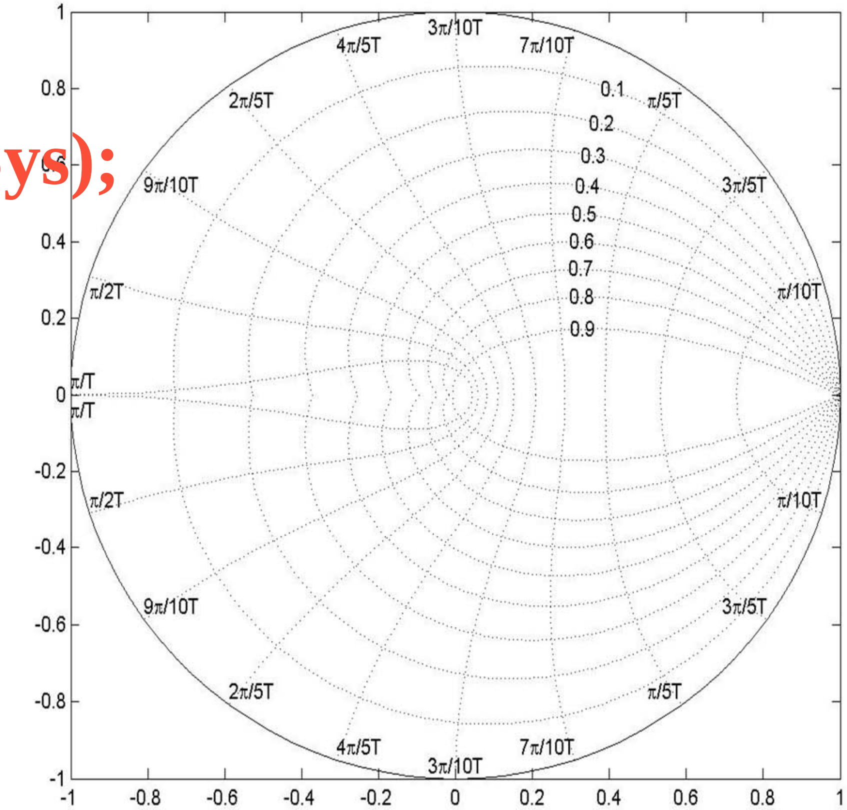
How is this figure generated?
How to interpretate the information?

```
s1=tf(1,[1 2 1]);
s2=tf(1,[1 1.6 1]);
s3=tf(1,[1 1.0 1]);
s4=tf(1,[1 0 1]);
pzmap(s1,s2,s3,s4)
sgrid
```

**SGRID** generates a grid over an existing continuous s-plane root locus or pole-zero map.  Lines of constant damping ratio (zeta) and natural frequency (Wn) are drawn.




**pzmap(Sys);**




**ZGRID** generates a grid over an existing discrete z-plane root locus or pole-zero map. Lines of constant damping factor (zeta) and natural frequency (Wn) are drawn in within the unit Z-plane circle.

# 2nd-order System – **Pole Locations (II)**

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}, \quad \text{assume } \omega_n > 0, \quad \xi \geq 0$$

poles: $p_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$

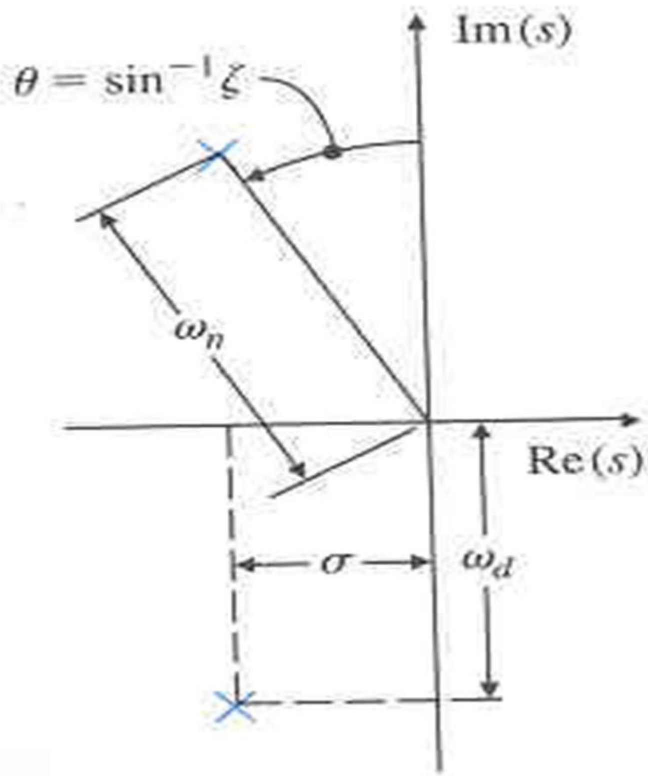
real(different) poles: $p_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$, if $\xi > 1$

real(identical) poles: $p_{1,2} = -\xi\omega_n$, if $\xi = 1$

complex poles: $p_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1 - \xi^2}$, if $0 < \xi < 1$

complex poles: $p_{1,2} = \pm j\omega_n$, if $\xi = 0$

**FIGURE 3.11**
*s*-plane plot for a pair of complex poles

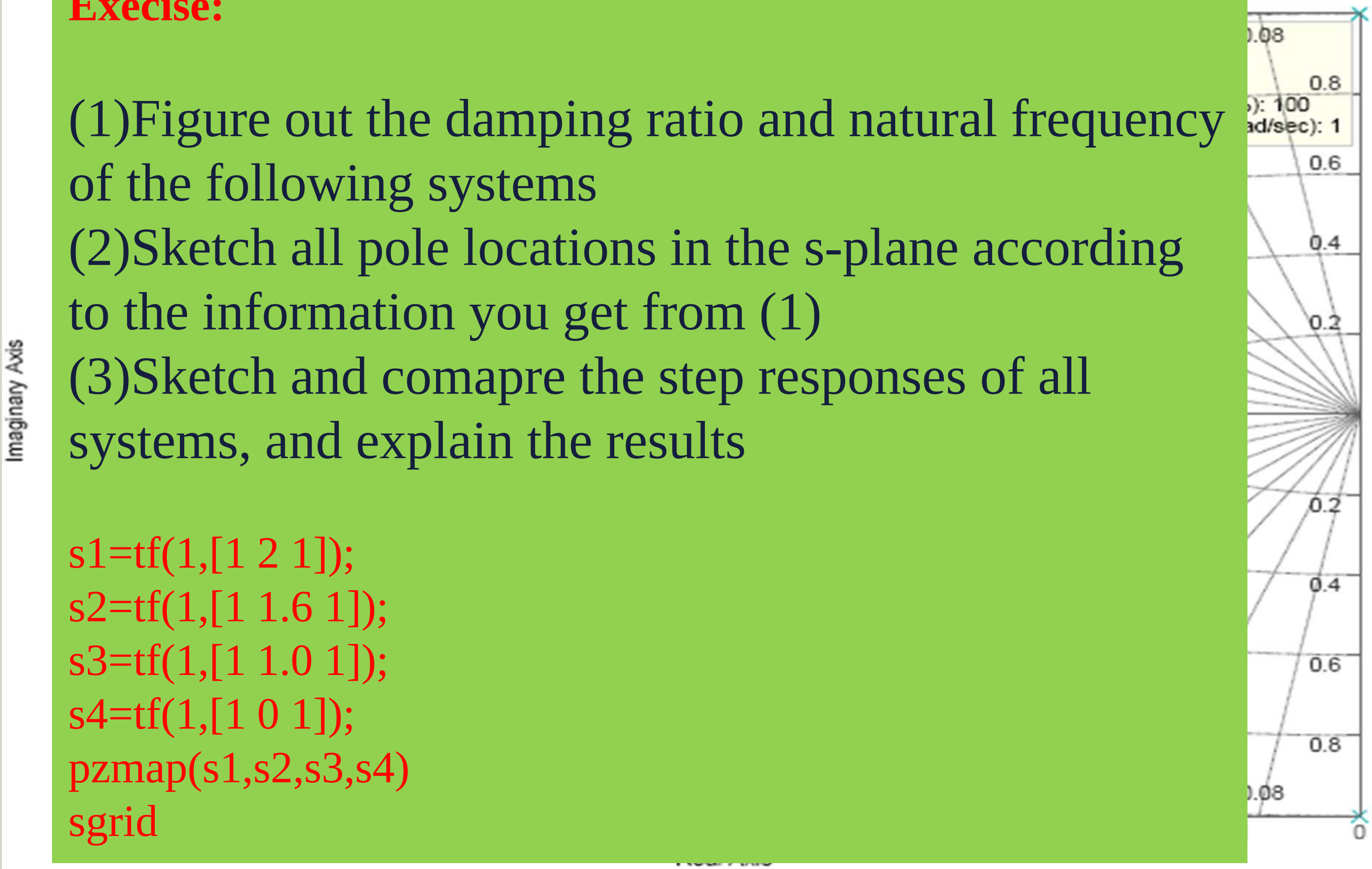


# 2nd-order System – **Pole Locations (III)**

**Execise:**

(1)Figure out the damping ratio and natural frequency of the following systems
(2)Sketch all pole locations in the s-plane according to the information you get from (1)
(3)Sketch and comapre the step responses of all systems, and explain the results

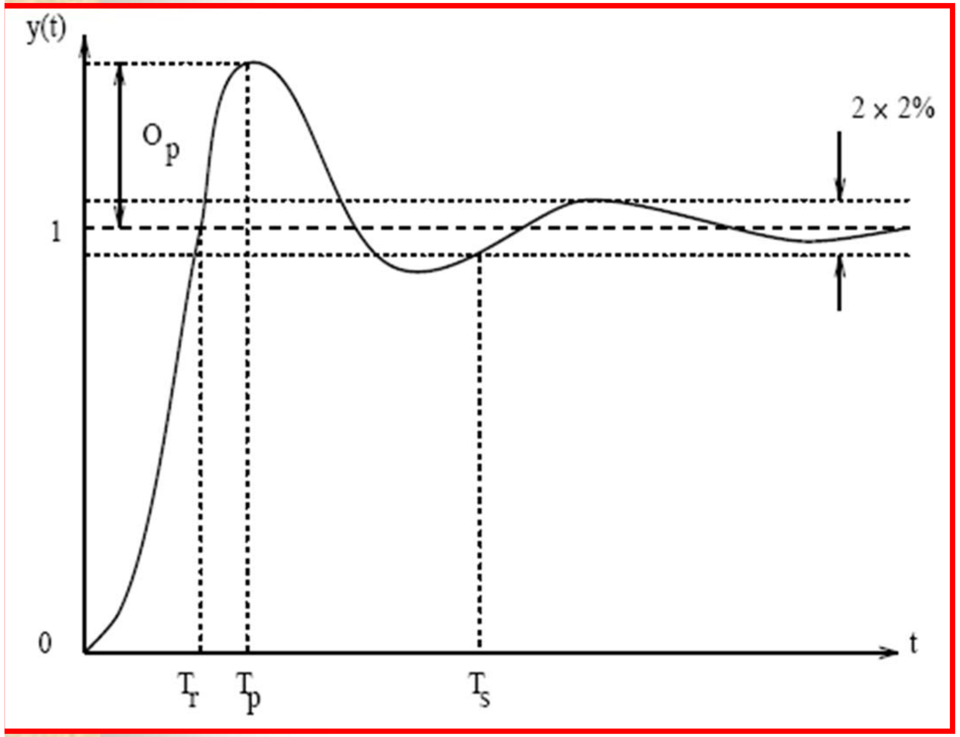
```
s1=tf(1,[1 2 1]);
s2=tf(1,[1 1.6 1]);
s3=tf(1,[1 1.0 1]);
s4=tf(1,[1 0 1]);
pzmap(s1,s2,s3,s4)
sgrid
```



$$\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}, \ \omega_d = \omega\sqrt{1-\xi^2}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

# MM3: Performance Specification




$$t_r \cong \frac{1.8}{\omega_n}$$

$$t_s \cong \frac{4.6}{\zeta\omega_n} \cong \frac{4.6}{\sigma}$$

$$M_p \cong \begin{cases} 5\%, & \zeta = 0.7 \\ 16\%, & \zeta = 0.5 \\ 35\%, & \zeta = 0.3 \end{cases}$$

$$t_p \cong \frac{\pi}{\omega_d}, \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

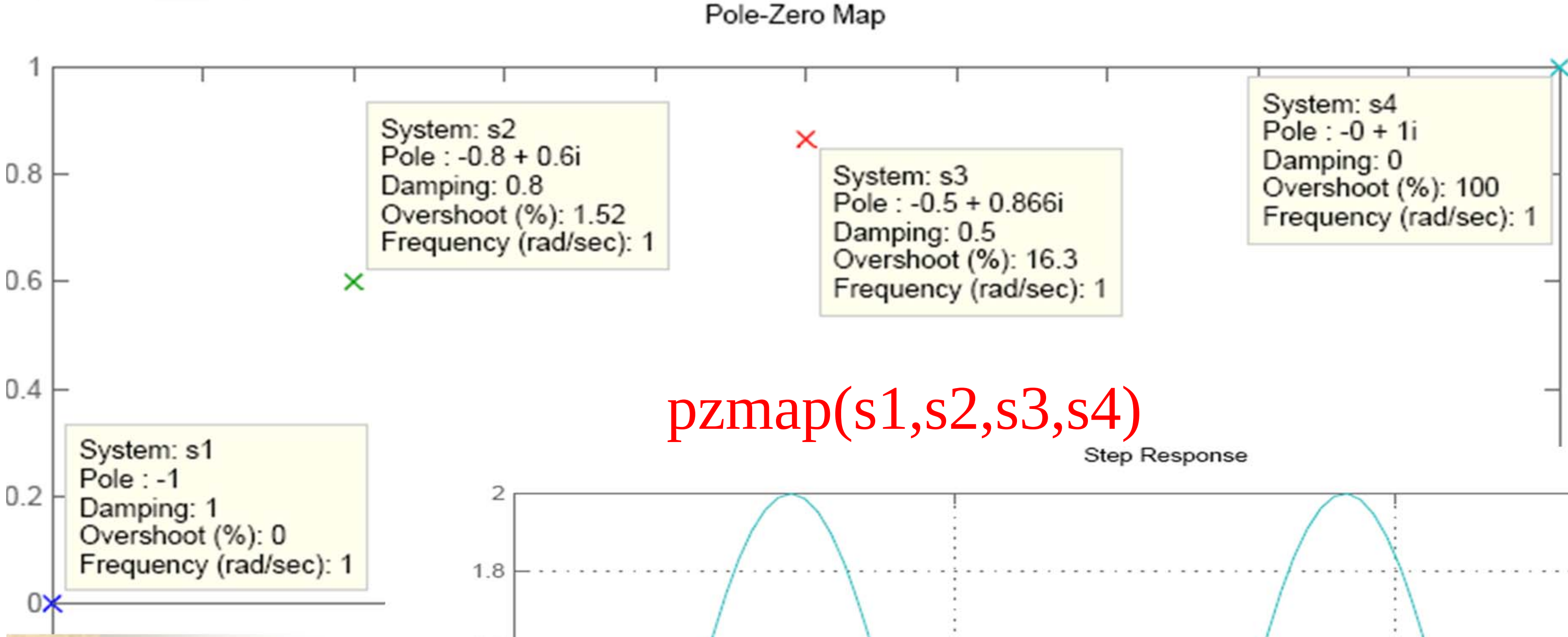
Rise time $T_r = \dfrac{\pi - \phi}{\omega_d}$

Peak time $T_p = \dfrac{\pi}{\omega_d}$

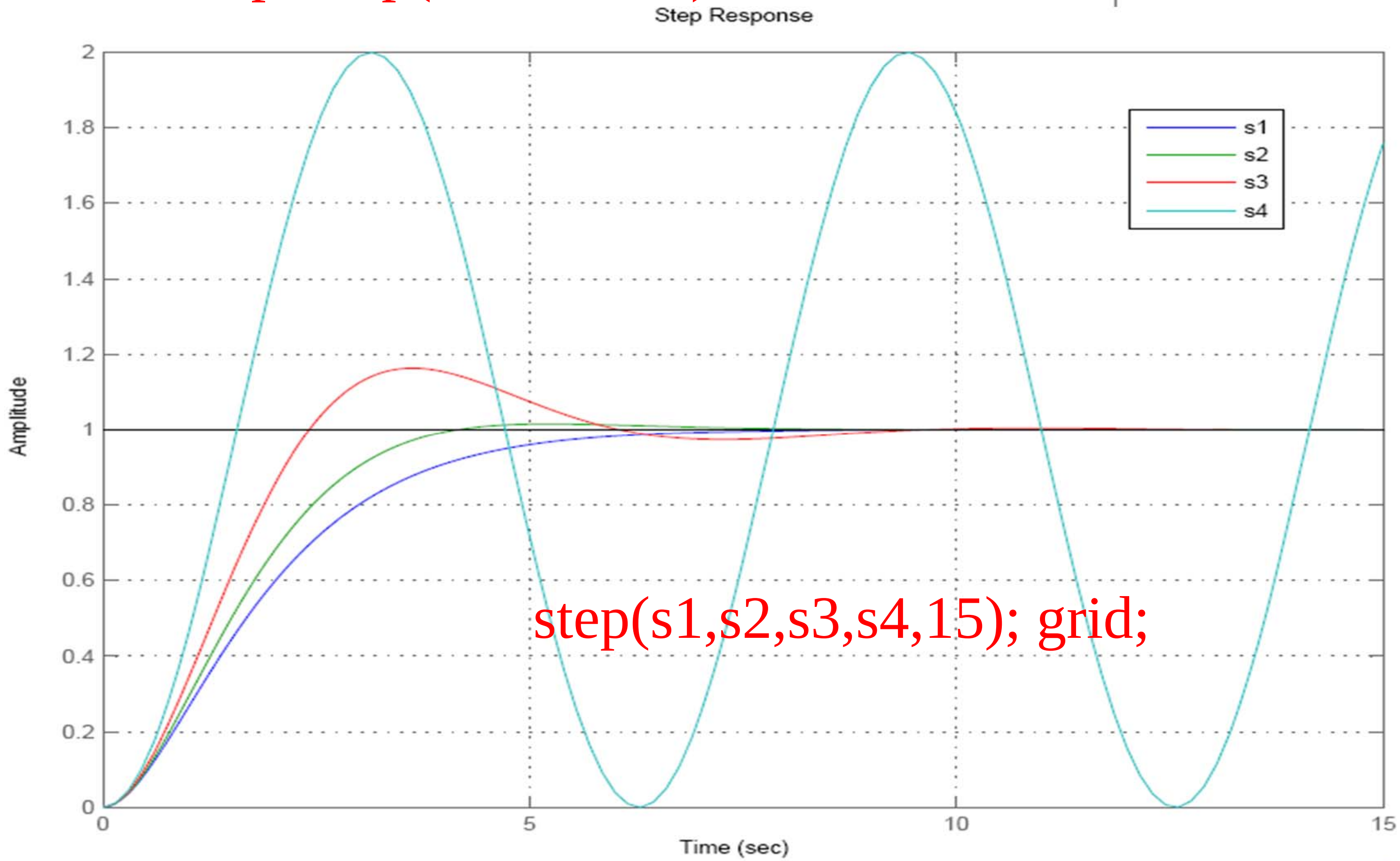
Settling time $T_s \approx \dfrac{4}{\xi\omega_n}$

Overshoot $O_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$

*Steady-state error* $e_{ss}$: difference between input & output as $t \to \infty$

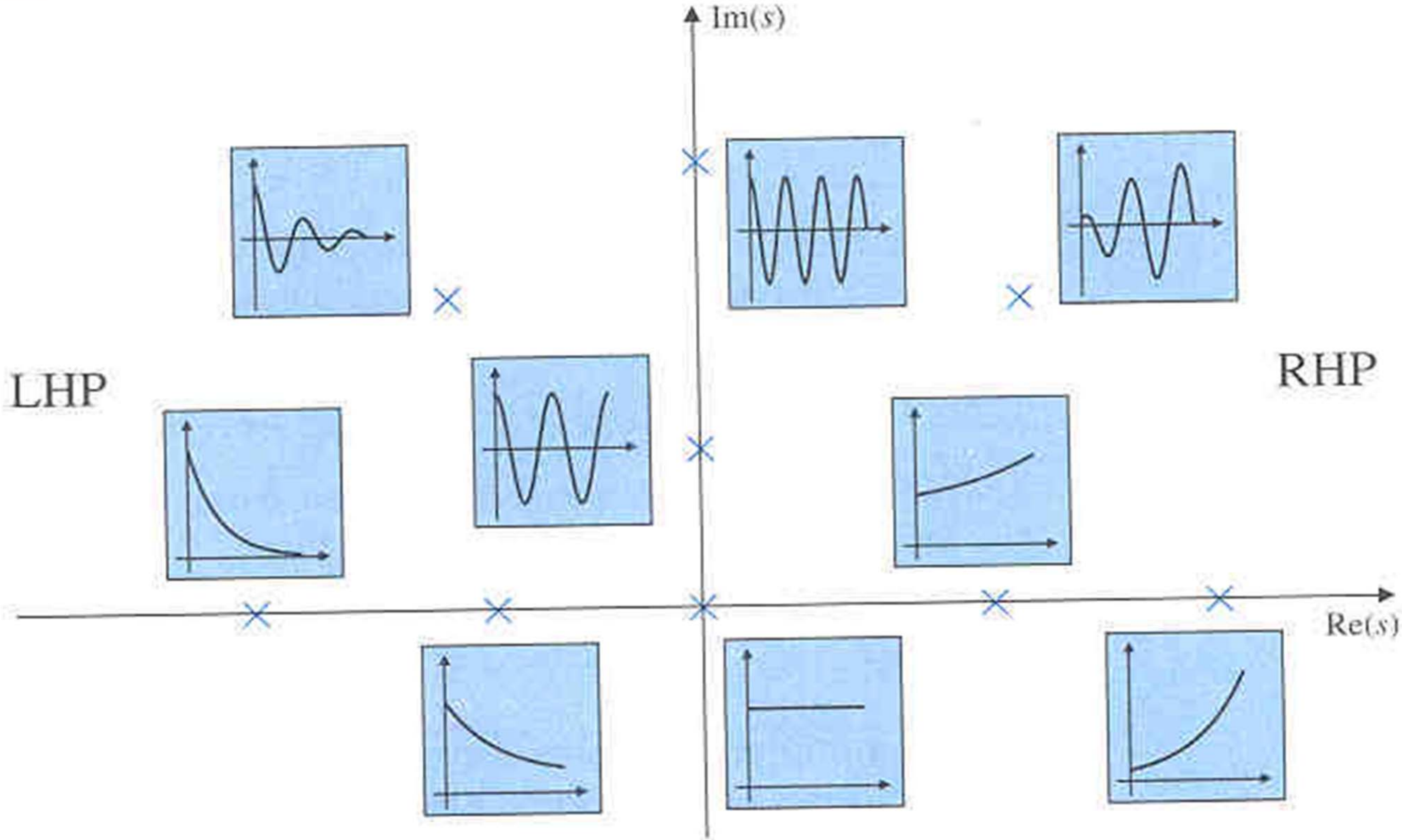



pzmap(s1,s2,s3,s4)




step(s1,s2,s3,s4,15); grid;

# Summary of Pole vs Performance (I)



# Summary of Pole vs Performance (II)

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}.$$

- $\xi$ – *damping ratio*, a dimensionless factor
- $\omega_n$ – *natural frequency* with unit rad/s

$$\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi},\ \omega_d = \omega\sqrt{1-\xi^2}$$

Rise time $T_r = \dfrac{\pi - \phi}{\omega_d}$

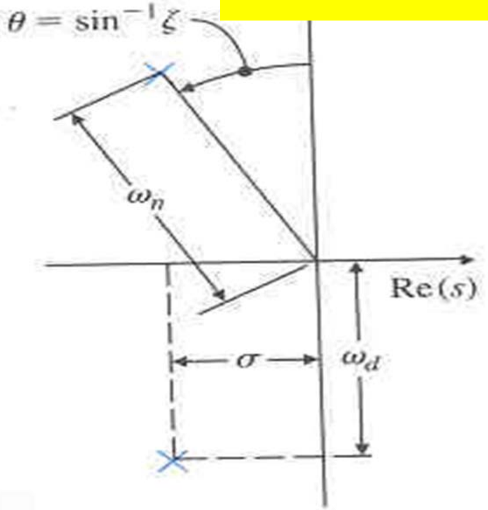
Peak time $T_p = \dfrac{\pi}{\omega_d}$

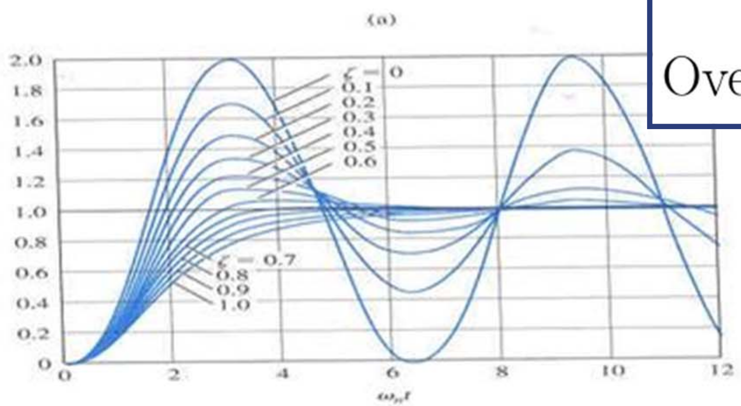
Settling time $T_s \approx \dfrac{4}{\xi\omega_n}$

Overshoot $O_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$

**Pole locations** ⟷ **Time response**

FIGURE 3.11
s-plane plot for ... of
complex poles



# Goals for this lecture (MM4)

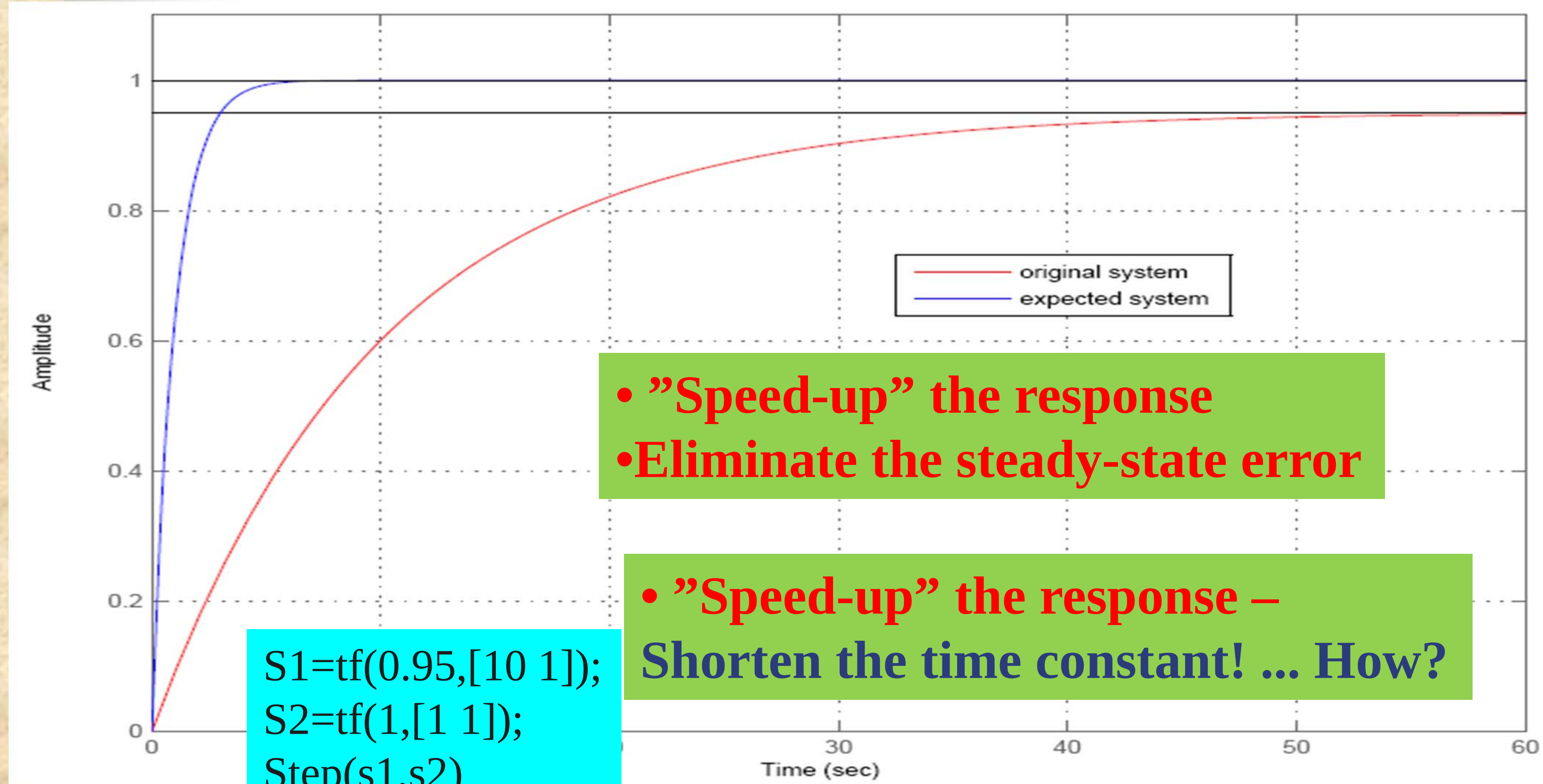
- **System poles vs. time responses**
  - **Poles and zeros**
  - **Time responses vs. Pole locations**
- **Feedback characteristics**
  - **Characteristics**
  - **A simple feedback design**
- Block diagram decomposition (simulink)

# See chapter 2 of Goodwin et al

Material can be downloaded from course webpage

# Revisit of example : First-order System (I)

- An design problem – how to use this knowledge



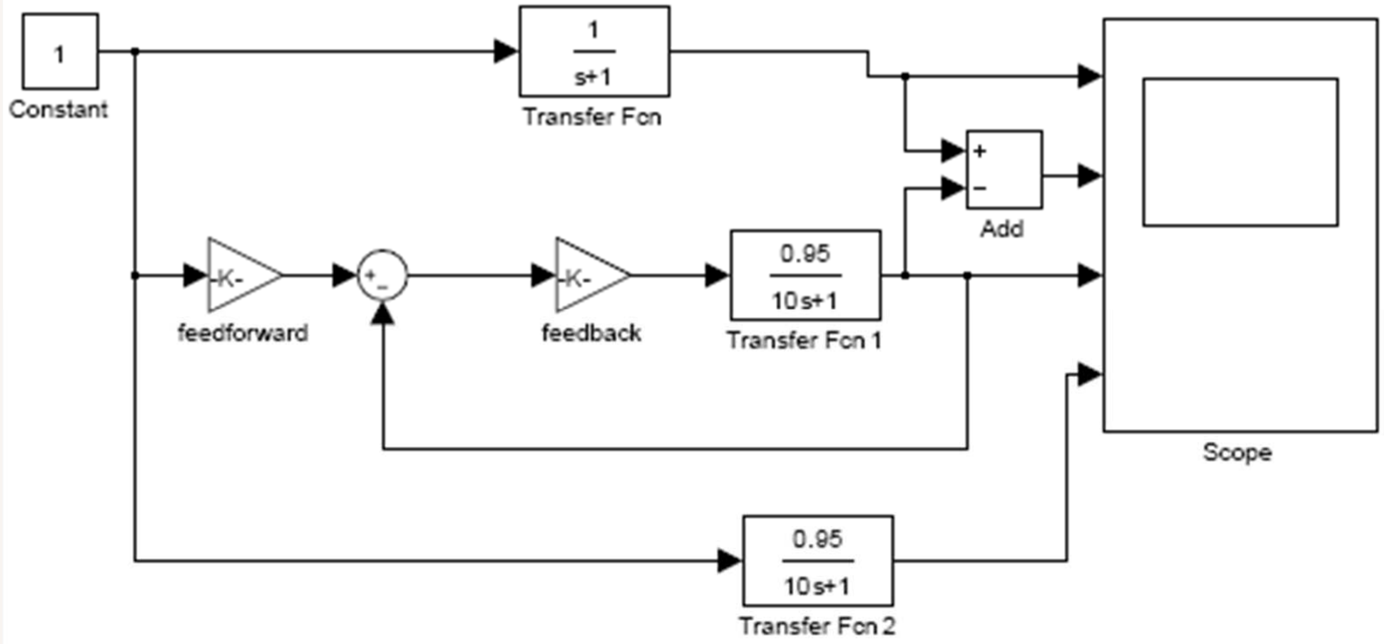

**• "Speed-up" the response**
**•Eliminate the steady-state error**

**• "Speed-up" the response –**
**Shorten the time constant! ... How?**

```
S1=tf(0.95,[10 1]);
S2=tf(1,[1 1]);
Step(s1,s2)
```

# Revisit of example : First-order System (II)

- A design solution




Download feedbackexample.mdl

# Summary of Feedback Characteristics

- System errors can be made less sensitive to disturbance with feedback than they are in open-loop systems
- In feedback control, the error in the controlled quantity is less sensitive to variations in the system gain/parameters
- Design tradeoff between gain and disturbance

# Goals for this lecture (MM4)

- System poles vs. time responses
  - Poles and zeros
  - Time responses vs. Pole locations

- Feedback characteristics
  - Characteristics
  - A simple feedback design

- Block diagram decomposition (simulink)

**See blackboard**