MM6 PID Controllers

Readings:

- Section 4.2 (the classical three-term controllers, p.179-196 except subsection 4.2.5);
- Extra reading materials

What have we talked in **MM5**?

- Stability analysis
- Steady-state errors
- Effects of zeros & additional poles

MM5 : BIBO Stability

- A system is said to have **bounded input-bounded output (BIBO) stability** if every bounded input results in a bounded output (regardless of what goes on inside the system)
- The continuous (LTI) system with impuse response **h(t)** is BIBO stable if and only if **h(t)** is absolutely integrallable
- All system poles locate in the left half s-plane - **asymptotic internal stability**
- **Routh Criterion:** For a stable system, there is no changes in sign and no zeros in the first column of the Routh array

MM5 : Steady-State Error

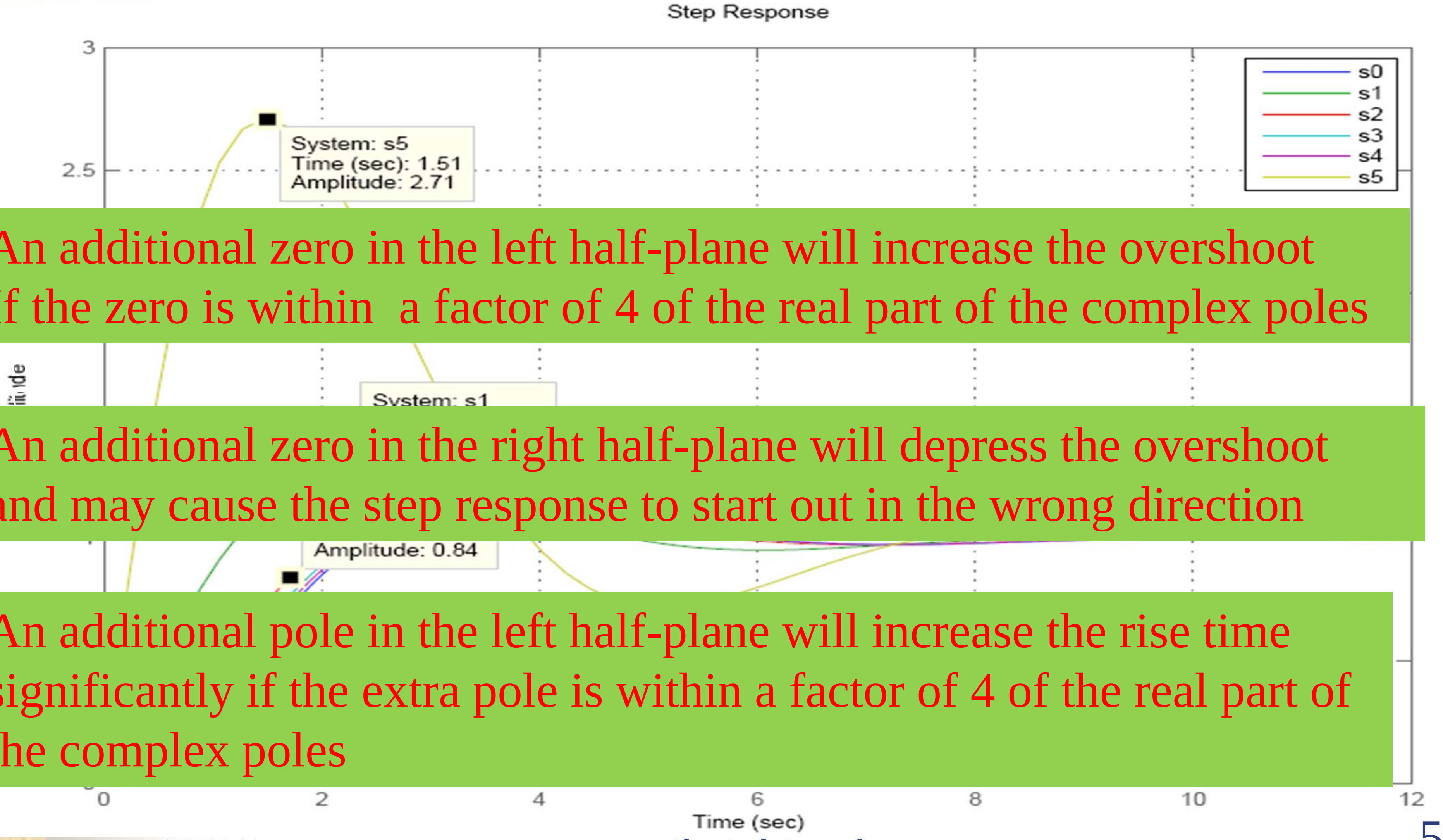
- **Objective:** to know whether or not the response of a system can approach to the reference signal as time increases
- **Assumption:** The considered system is stable
- **Analysis method:** Transfer function + final-value Theorem

$$
\begin{aligned}
e(\infty) &= \lim_{s\to 0} s(R(s)-Y(s)) = \lim_{s\to 0} s(R(s)-G(s)R(s)) \\
&= \lim_{s\to 0} s(1-G(s))R(s), \quad R(s)=\frac{1}{s} \\
&= \lim_{s\to 0}(1-G(s)) = 1-G(0)
\end{aligned}
$$

DC-Gain ($G(0)$)

- **Position-error constant** $K_p = \lim_{s\to 0} G_o(s)$
- **Velocity constant** $K_v = \lim_{s\to 0} sG_o(s)$
- **Acceleration constant** $K_a = \lim_{s\to 0} s^2 G_o(s)$

MM5 : Effect of Additional Zero & Pole



An additional zero in the left half-plane will increase the overshoot
If the zero is within a factor of 4 of the real part of the complex poles

An additional zero in the right half-plane will depress the overshoot and may cause the step response to start out in the wrong direction

An additional pole in the left half-plane will increase the rise time significantly if the extra pole is within a factor of 4 of the real part of the complex poles

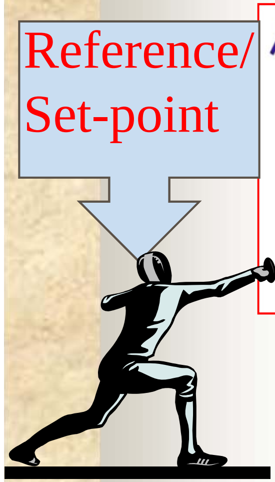
Goals for this lecture (MM6)

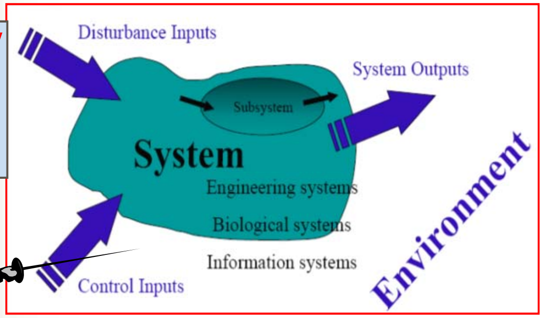
- **Definition characterisitc of PID control**
  - **P- controller**
  - **PI- controller**
  - **PID controller**

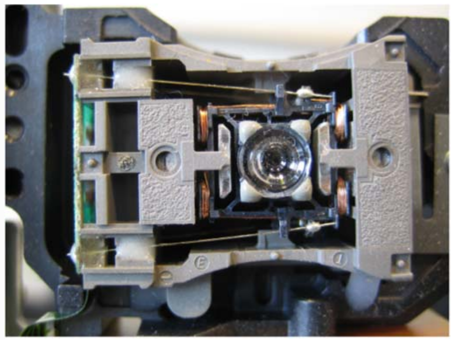
- Ziegler-Nichols tuning methods
  - Quarter decay ratio method
  - Ultimate sensitivity method

Control objectives

Control is a process of causing a system (output) variable to conform to some desired status/value (**MM1**)







Control Objectives

- **Stable (MM5)**
- **Quick responding (MM3, 4)**
- **Adequate disturbance rejection**
- **Insensitive to model & measurement errors**
- **Avoids excessive control action**
- **Suitable for a wide range of operating conditions**

(extra readings: Goodwin's lecture)

Feedback Control Characteristics(MM4)

Control Objectives

- Stable
- Quick responding
- Adequate disturbance rejection
- Insensitive to model & measur. errors
- Avoids excessive control action
- A wide operating range

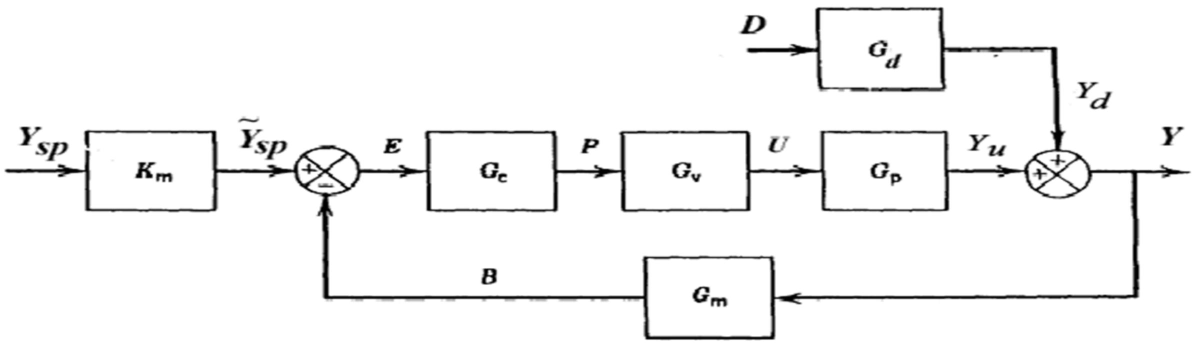
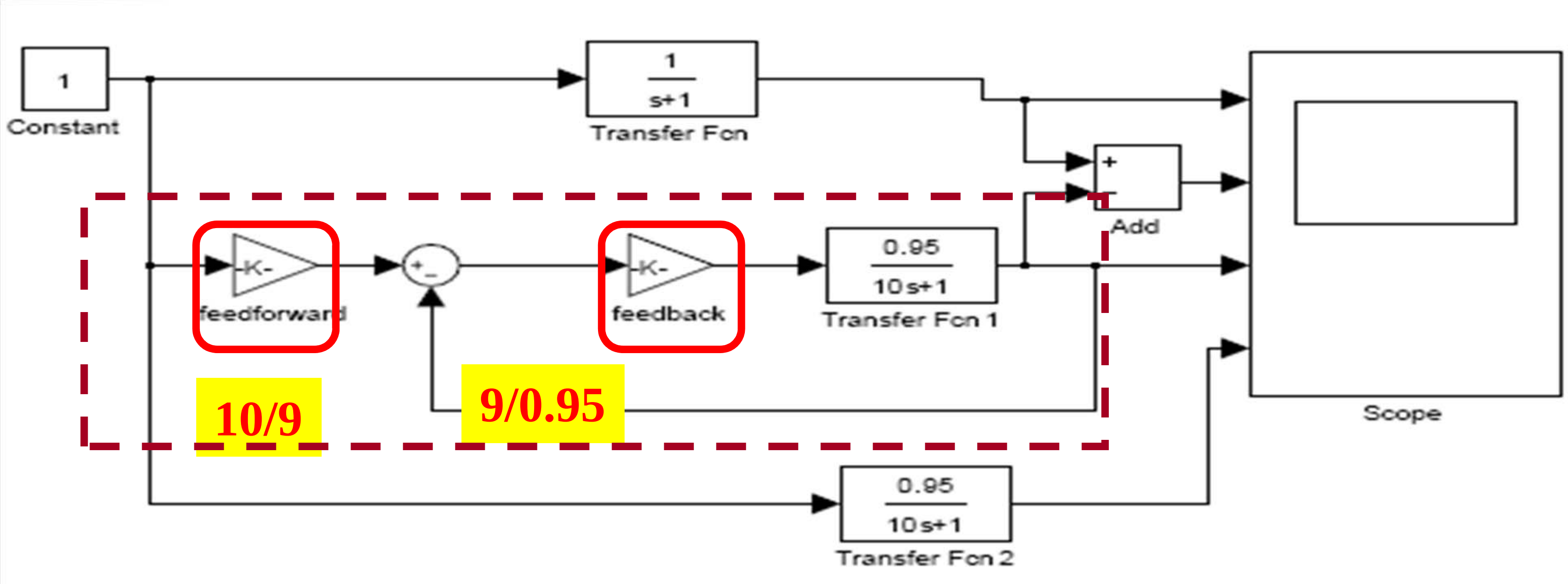


Figure 12.1. Block diagram for a standard feedback control system.

- System errors can be made less sensitive to **disturbance** with feedback than they are in open-loop systems
- In feedback control, the error in the controlled quantity is less sensitive **to variations in the system gain/parameters**
- Design tradeoff between gain and disturbance

Recall example in MM5:

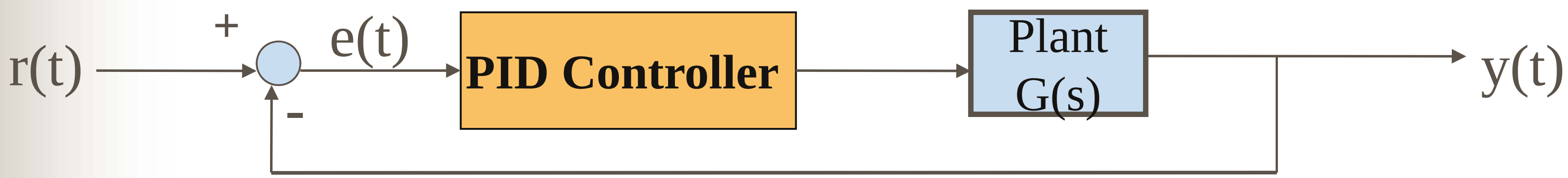


- "**Speed up**" the original system by using feedback (**P-**) control
- Eliminate the **steady-state error** by using feedforward gain

Definition of PID Controllers

- **PID Means:**
  - **P**: Proportional (control) $u(t) = Ke(t)$
  - **I** : Integral (control) $u(t) = \frac{K}{T_I}\int_{t_0}^{t} e(\tau)d\tau$
  - **D**: Derivative (control) $u(t) = KT_D\dot{e}(t)$

- **PID Control System Structure: cascade control**



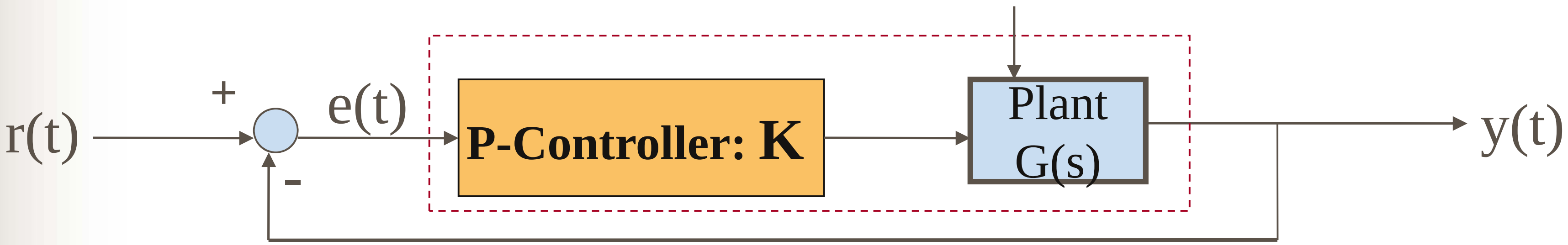
PID Control: **Proportional Control (I)**

- **Control Structure**

Time Domain: $u(t) = Ke(t)$

Frequency Domain: $D(s) = \dfrac{U(s)}{E(s)} = K$

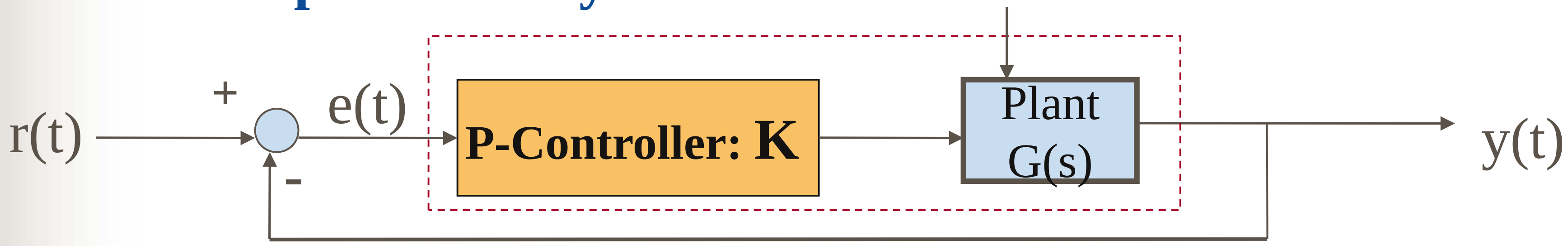
- **Closed-loop Control System**



$$G_{cl}(s) = \frac{D(s)G(s)}{1+D(s)G(s)} = \frac{KG(s)}{1+KG(s)}$$

PID Control: **Proportional Control (II)**

- **Closed loop Control System**



- **Advantage:** **a simple controller (–amplifier)**
- **Disadvantages:**
  - Steady state offset/error problem
  - (type-0, -1, -2 systems –MM5)
  - Disturbance rejection problem

unity feedback sys

$$K_p = \lim_{s \to 0} G_o(s) \qquad e_{ss} = \frac{1}{1+K_p}$$

$$K_v = \lim_{s \to 0} sG_o(s) \qquad e_{ss} = \frac{1}{K_v}$$

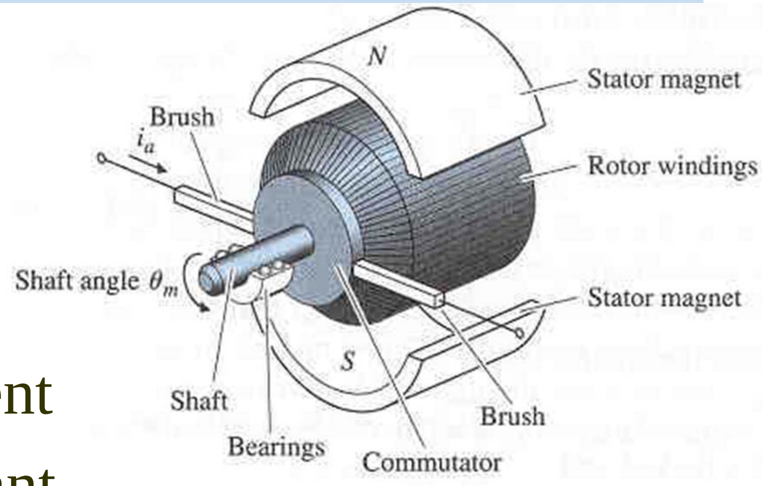
$$K_a = \lim_{s \to 0} s^2 G_o(s) \qquad e_{ss} = \frac{1}{K_a}$$

Example: Speed Control of a DC Motor

- Working mechanism of a DC motor

$$T = K_t i_a$$

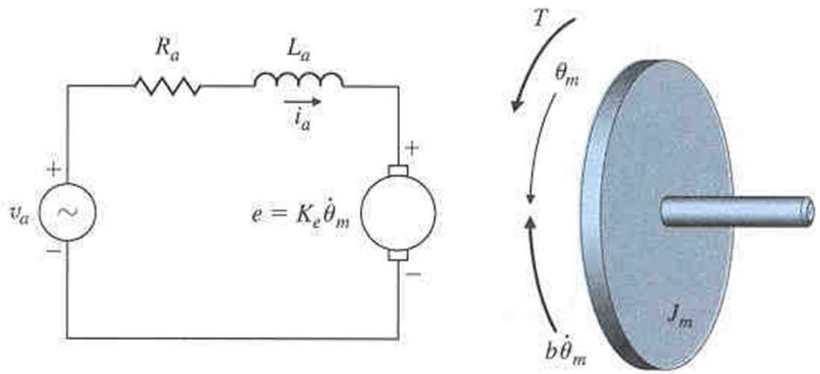
$$e = K_e \dot{\theta}_m$$



K_t torque constant **i_a** armature current

K_e electromotive force (emf) constant

- Differential equation description



$$J_m \ddot{\theta}_m + b\dot{\theta}_m = K_t i_a$$

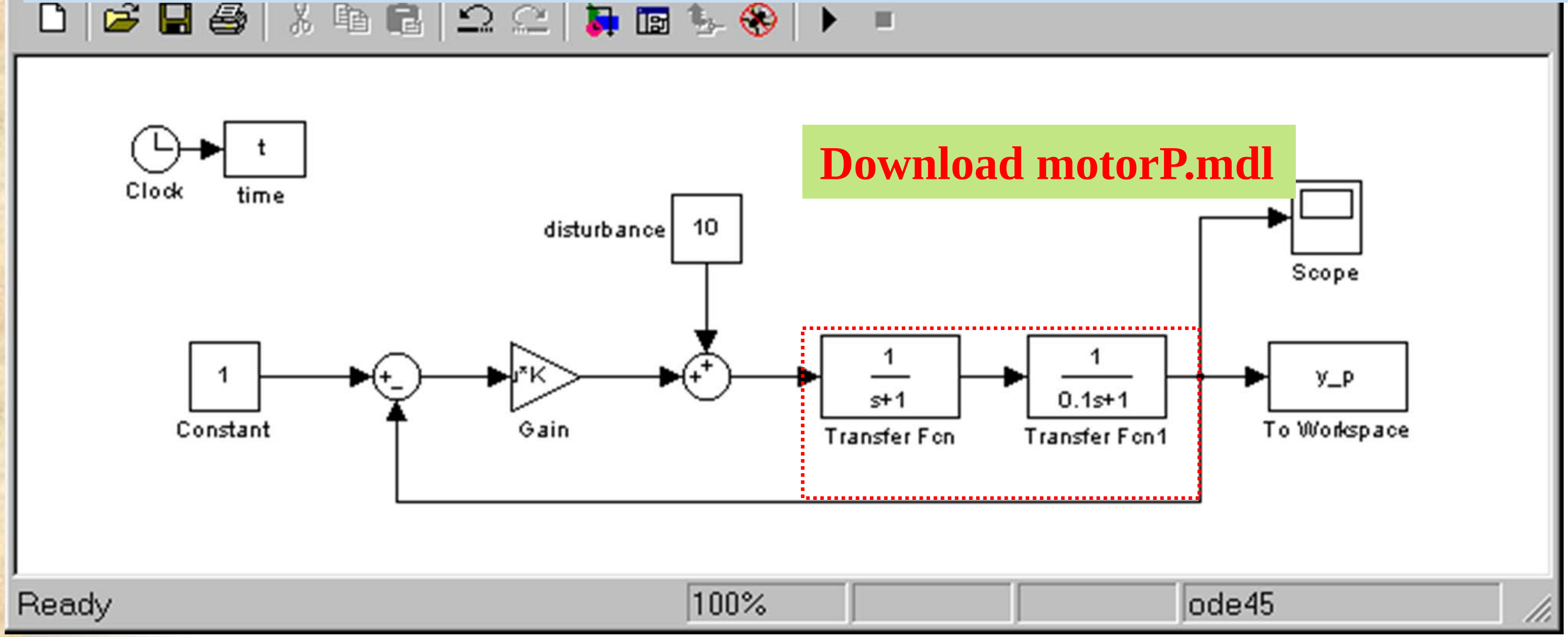
$$L_a \dot{i}_a + R_a i_a = v_a - K_e \dot{\theta}_m$$

simplified :

$$J_m \ddot{\theta}_m + \left(b + \frac{K_t K_e}{R_a}\right)\dot{\theta}_m = \frac{K_t}{R_a} v_a$$

See FC p.47-49

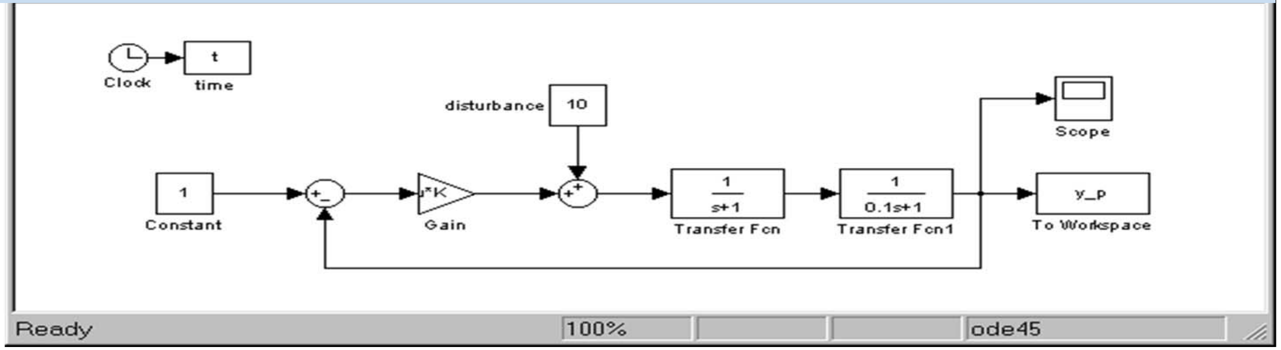
P-Control for the DC Motor (I)



Download motorP.mdl

- Poles of the original system?
- Image the step response (undamped, underdamped, critially damped or over damped)
- Type of this original system?

P-Control for the DC Motor (II)



Tune gain K value, what we can observe:

- Larger gain leads to quicker response, but with larger oscillation
- There is always steady-state error. This error decreases as K value increases
- Effect of disturbance is always existing in the response, which causes extra steady-state error
- The effect of disturbance can be reduced by increasing K value

introduce one more degree-of-freedom ...

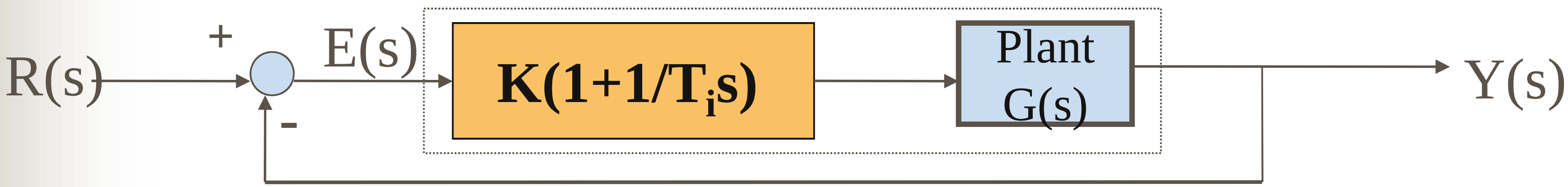
PID Control: **PI Control (I)**

- **Control Structure** T_I – integral/reset time

$$\text{Time Domain: } u(t) = K(e(t) + \frac{1}{T_I}\int_{t_0}^{t} e(\tau)d\tau)$$

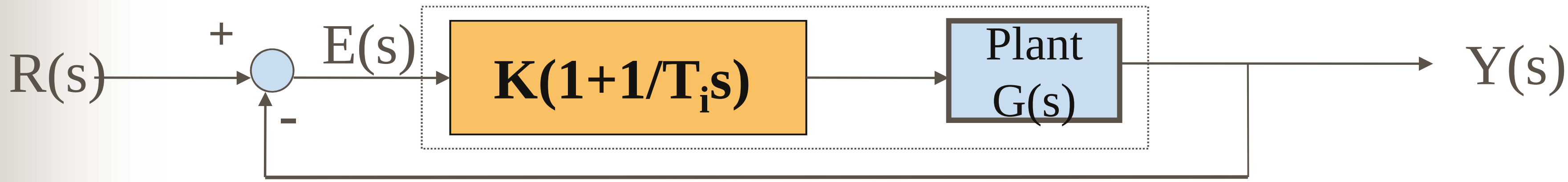
$$\text{Frequency Domain: } D(s) = \frac{U(s)}{E(s)} = K(1+\frac{1}{T_I s})$$

- **Closed loop-Control System**



$$G_{cl}(s) = \frac{D(s)G(s)}{1+D(s)G(s)} = \frac{K(1+\frac{1}{T_I s})G(s)}{1+K(1+\frac{1}{T_I s})G(s)} = \frac{K(T_I s+1)G(s)}{T_I s + K(T_I s+1)G(s)}$$

PID Control: **PI Control (II)**



- **Control-loop transfer function**

- **Advantages:**
  - Eliminate steady state offset/error (**why?**)
  - Good steady-state disturbance rejection (**why?**)

- **How about the transient response?**

Example: PI-Control for the DC Motor

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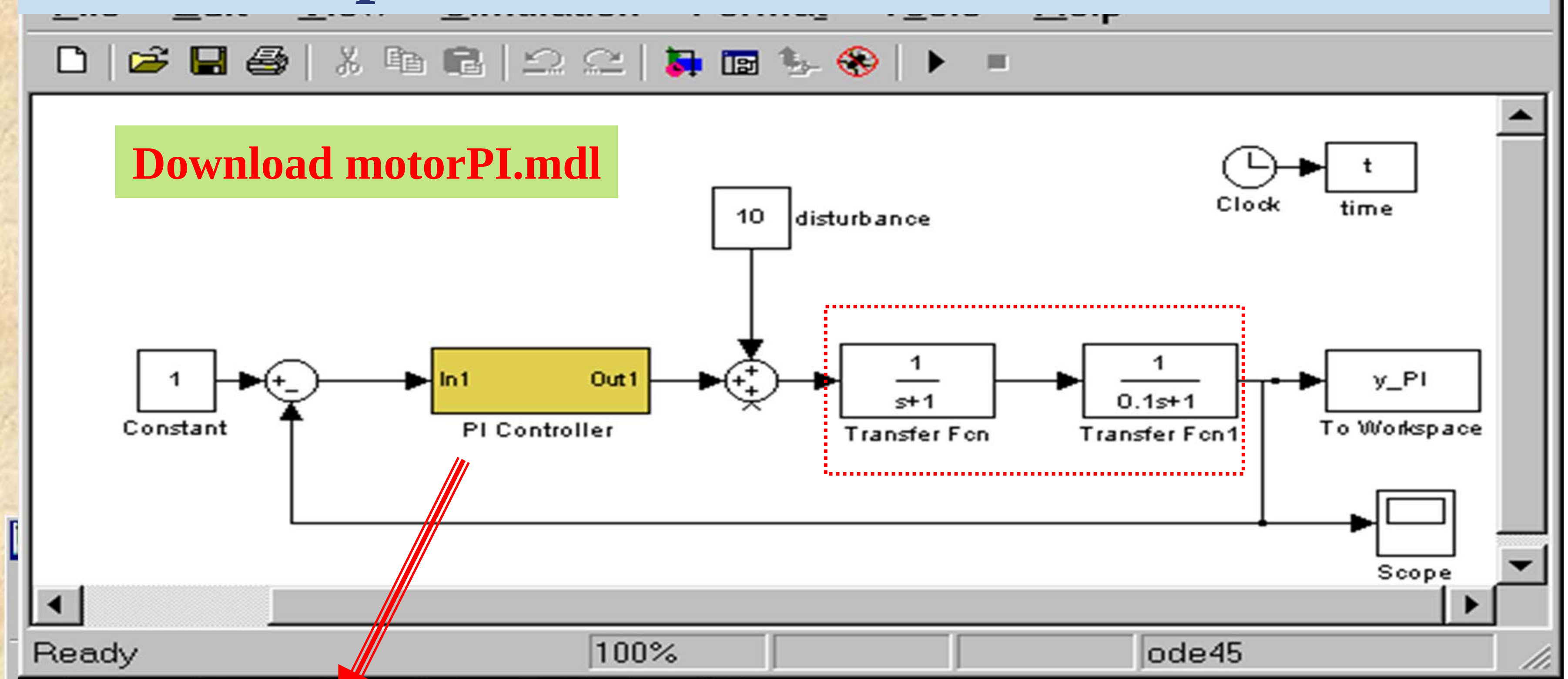
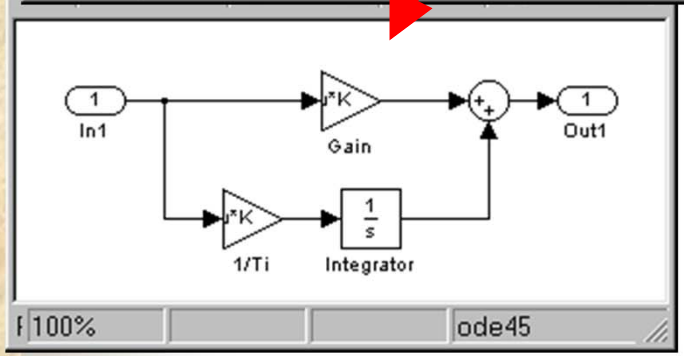
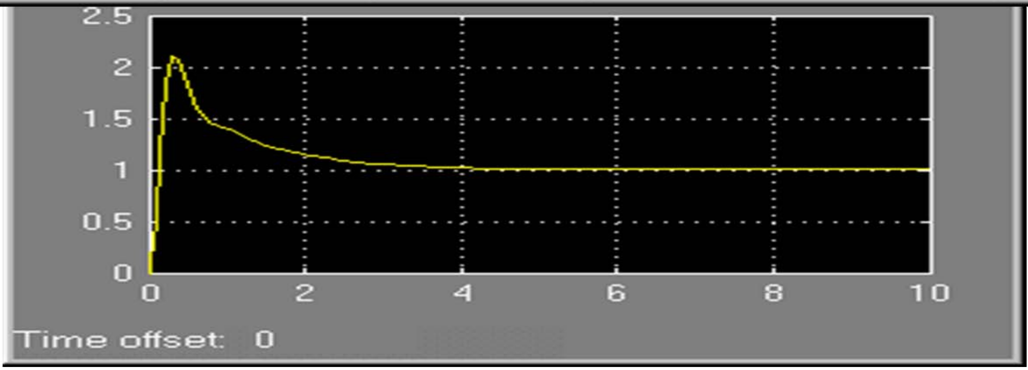






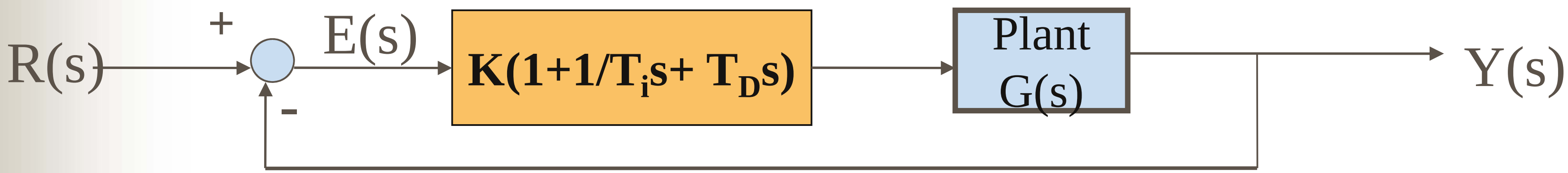
PID Control: **PID Feedback Control (I)**

- **Control Structure** **T_D – Derivative/rate time**

$$u(t) = K\left(e(t) + \frac{1}{T_I}\int_{t_0}^{t} e(\tau)d\tau + T_D\dot{e}(t)\right)$$

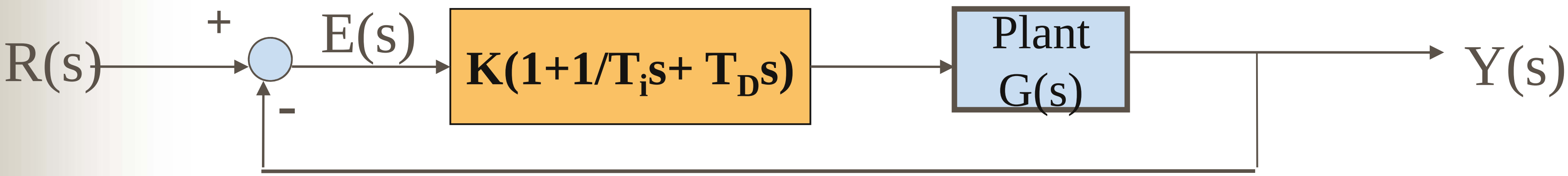
$$D(s) = \frac{U(s)}{E(s)} = K\left(1 + \frac{1}{T_I s} + T_D s\right)$$

- **Closed loop Control System**



$$G_{cl}(s) = \frac{D(s)G(s)}{1 + D(s)G(s)} = \frac{K(T_D T_I s^2 + T_I s + 1)G(s)}{T_I s + K(T_D T_I s^2 + T_I s + 1)G(s)}$$

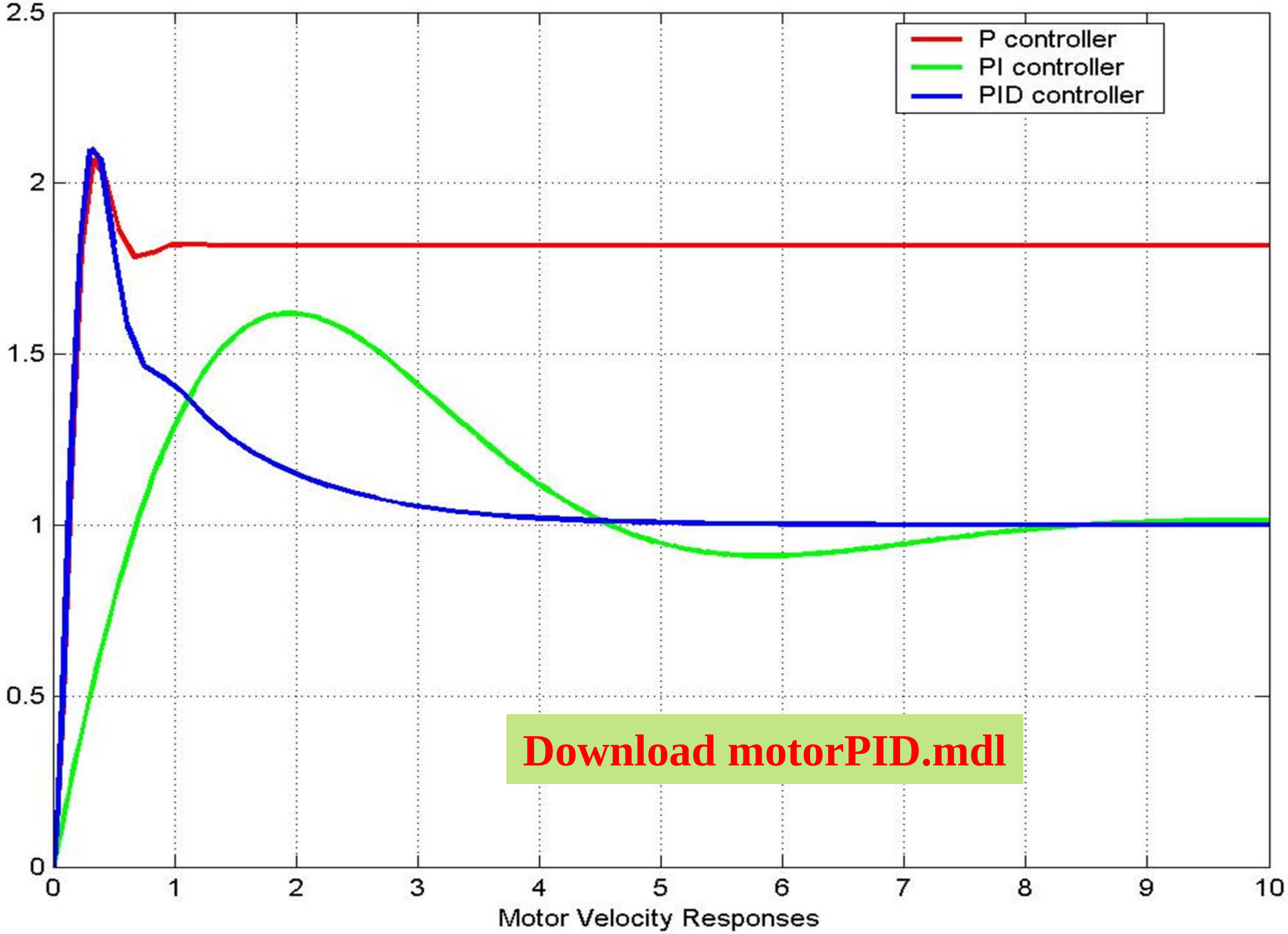
PID Control: **PID Feedback Control (II)**



- **Advantages:**
  - Increase the damping → Improve the stability
  - Good transient and steady disturbance rejection

- **The most popular control technique used in industry!**

PID-Control for the DC Motor



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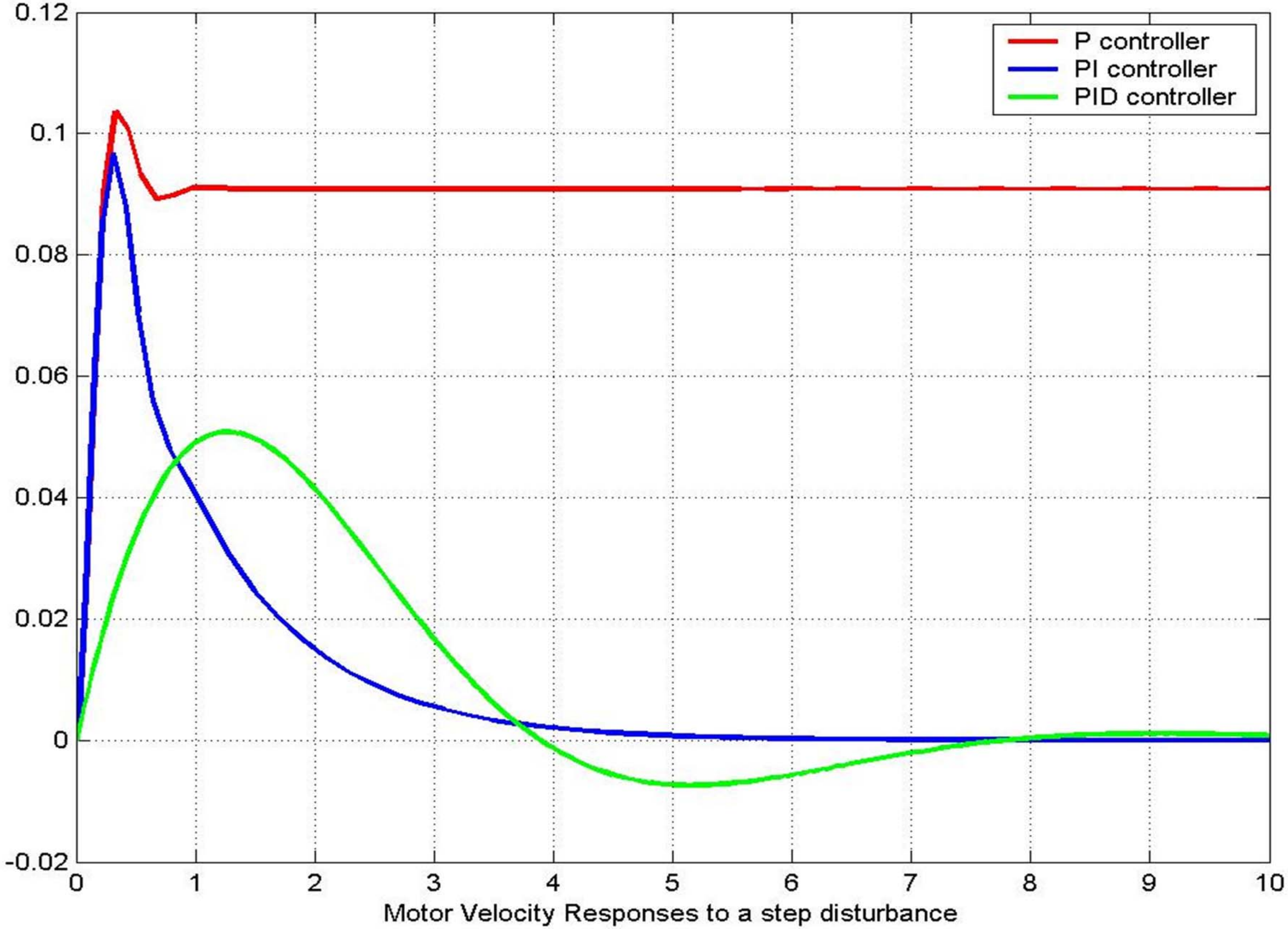
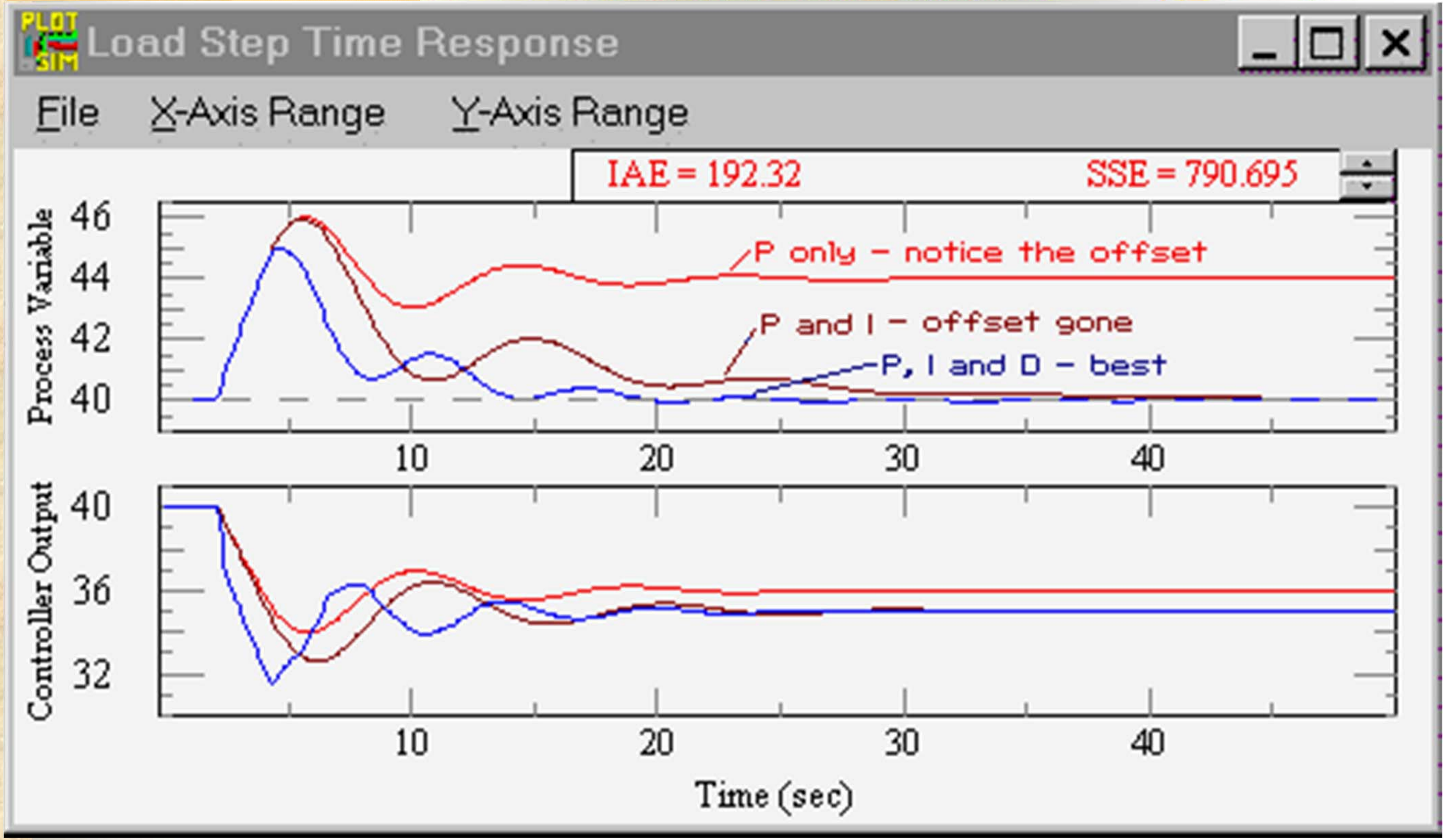
Effect of constant Disturbance





http://www.expertune.com/tutor.html#InitialPID

PID Control: **Characteristic Summary**

- ❑ A proportional controller (Kp) will have the effect of reducing the rise time and will reduce ,but never eliminate, the steady-state error
- ❑ An integral control (Ki) will have the effect of eliminating the steady-state error, but it may make the transient response worse.
- ❑ A derivative control (Kd) will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response.

CL RESPONSE	**RISE TIME**	**OVERSHOOT**	**SETTLING TIME**	**S-S ERROR**
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	Small Change

PID Control: **General Design Procedure**

When you are designing a PID controller for a given system, follow the steps shown below to obtain a desired response.

- Obtain an open-loop response and determine what needs to be improved
- Add a proportional control to improve the rise time
- Add a derivative control to improve the overshoot
- Add an integral control to eliminate the steady-state error
- Adjust each of Kp, Ki, and Kd until you obtain a desired overall response.

http://www.engin.umich.edu/group/ctm/PID/PID.html

Goals for this lecture (MM6)

- **Definition & characterisitc of PID control**
  - **P- controller**
  - **PI- controller**
  - **PID controller**

- **Ziegler-Nichols tuning methods**
  - **Quarter decay ratio method**
  - **Ultimate sensitivity method**

PID Trial Tuning

The "by-guess-and-by-golly" method

1. Enter an initial set of tuning constants from experience. A conservative setting would be a gain of 1 or less and a reset of less than 0.1.

2. Put loop in automatic with process "lined out".

3. Make step changes (about 5%) in setpoint.

4. Compare response with diagrams and adjust.

Ziegler-Nichols Mthods

- These well-known tuning rules were published by Z-N in 1942.

- Z-N controller settings are widely considered to be an "industry standard".

- **Other readings/resources:**

- Control Tutorials for Matlab, http://www.engin.umich.edu/group/ctm/

- Process Control Articles, Software Reviews,

http://www.expertune.com/articles.html

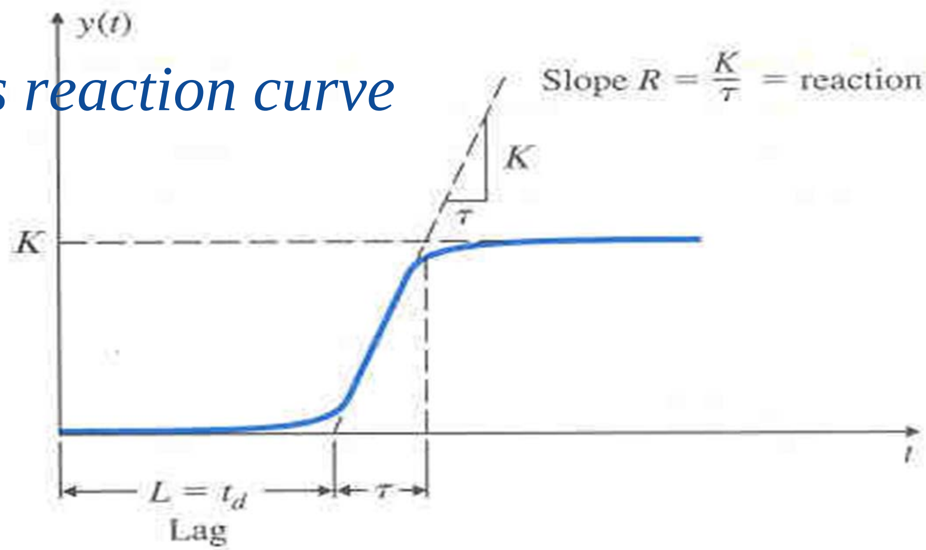
- http://www.jashaw.com/pid/tutorial/

Tuning PID: **Quarter decay ratio method (I)**

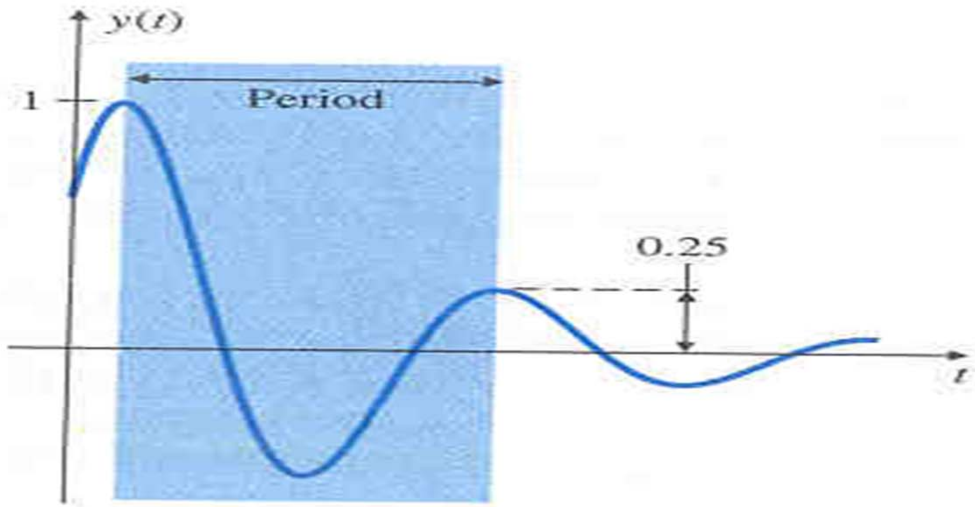
$$u(t) = K(e(t) + \frac{1}{T_I}\int_{t_0}^{t} e(\tau)d\tau + T_D\dot{e}(t) \qquad D(s) = \frac{U(s)}{E(s)} = K(1 + \frac{1}{T_I s} + T_D s)$$

Ziegler-Nichols Quarter decay ratio method:

- Precondition: system is stable
- Open loop tuning method
- Step response: → *Process reaction curve*
- Slope rate **R=K/τ**
- Lag time **L**



Tuning PID: **Quarter decay ratio method (II)**



Exercise 1:
Design a P, PI, PID controller for the DC motor example, According to quarter decay m.

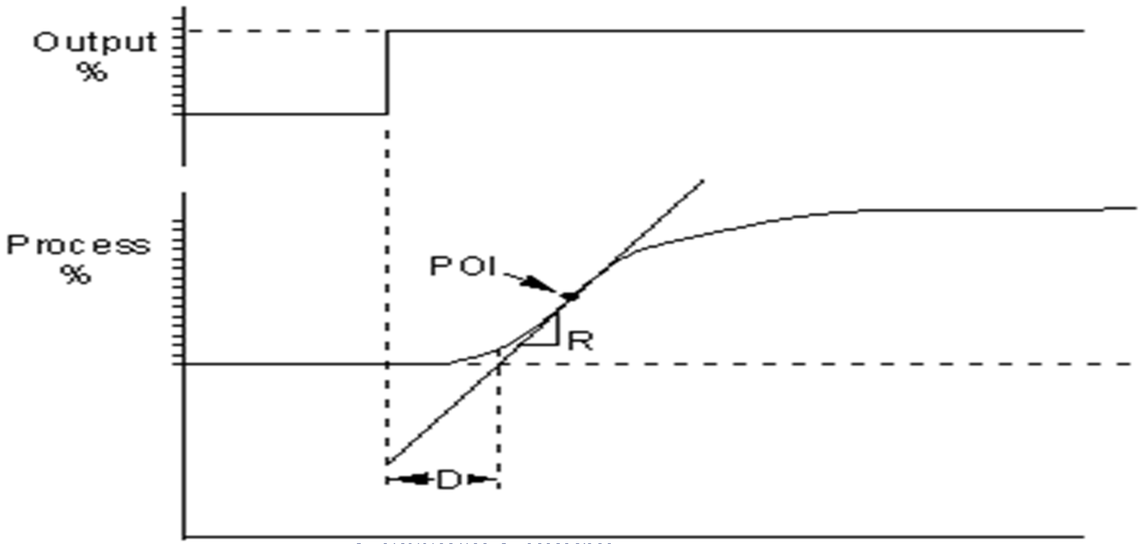
Ziegler–Nichols Tuning for the Regulator $D(s) = K(1 + 1/T_I s + T_D s)$, for a Decay Ratio of 0.25

Type of Controller	*Optimum Gain*
Proportional	$K = 1/RL$
PI	$K = 0.9/RL$, $T_I = L/0.3$
PID	$K = 1.2/RL$, $T_I = 2L$, $T_D = 0.5L$

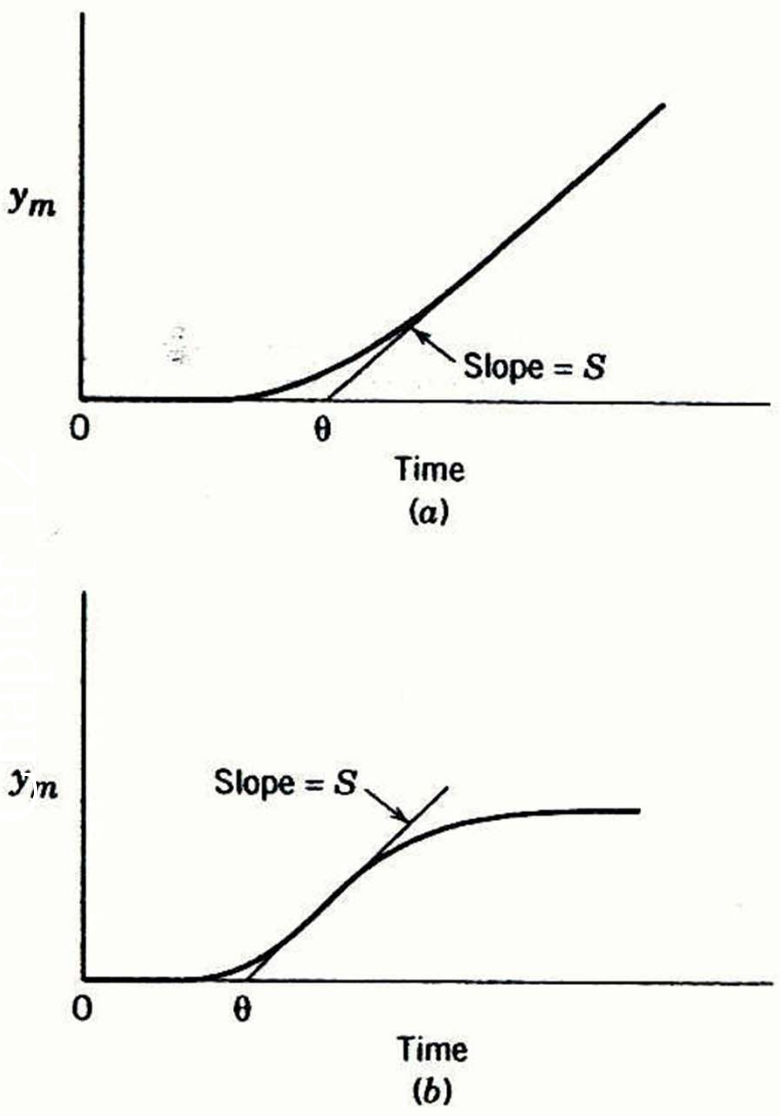
Tuning PID: **Quarter decay ratio method(III)**

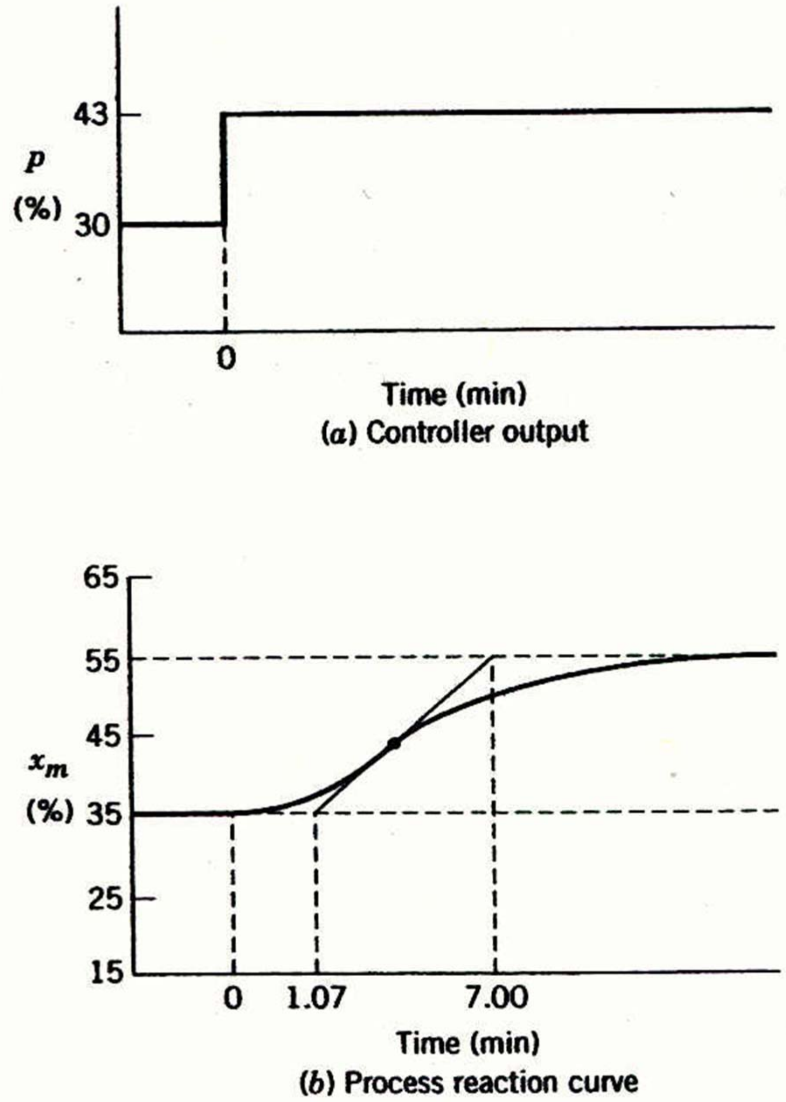
Pratical issues in Quarter decay ratio method:

- Process reaction curve
- X % Change of control output
- R %/min. Rate of change at the point of inflection (POI)
- D min. Time until the intercept of tangent line and original process value



http://www.jashaw.com/pid/tutorial/pid6.html



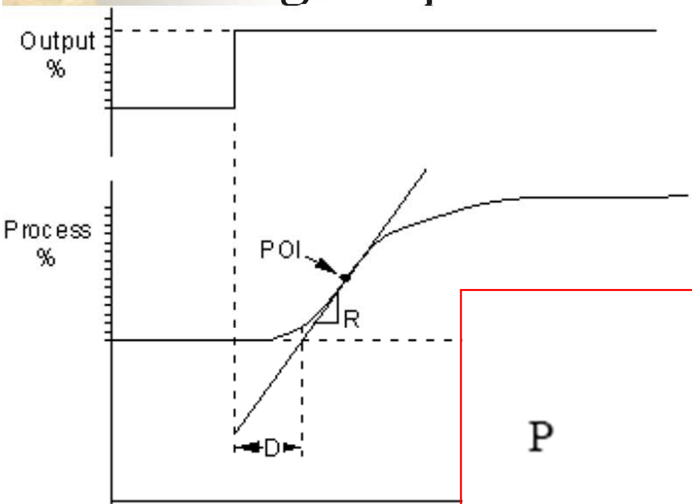


Typical process reaction curves: (a) non-self-regulating process, (b) self-regulating process.

http://www.jashaw.com/pid/tutorial/pid6.html

Tuning PID: **Quarter decay ratio method(IV)**

- X % Change of control output
- R %/min. Rate of change at the point of inflection (POI)
- D min. Time until the intercept of tangent line and original process value



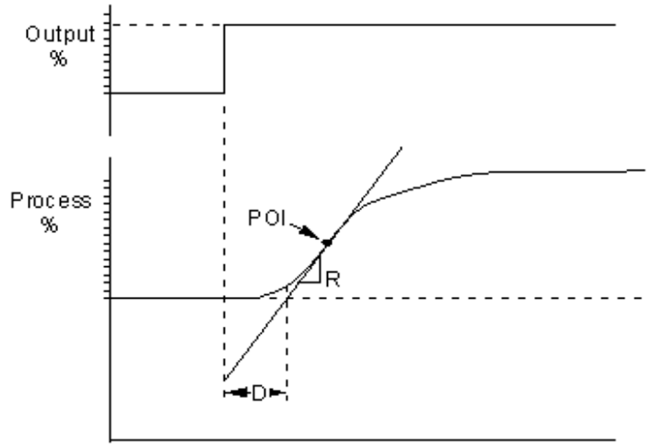
	Gain	Reset	Derivative
P	X/DR	—	—
PI	0.9X/DR	0.3/D	—
PID	1.2X/DR	0.5/D	0.5D

http://www.jashaw.com/pid/tutorial/pid6.html

Tuning PID: **Quarter decay ratio method(V)**

- Another means of determining parameters based on the ZN open loop is: After "bumping" the output, watch for the point of **inflection** and note:

Ti	min	Time from output change to POI
P	%	Process value change at POI
R	%/min	Rate of change at POI
X	%	Change in output



- D is calculated:

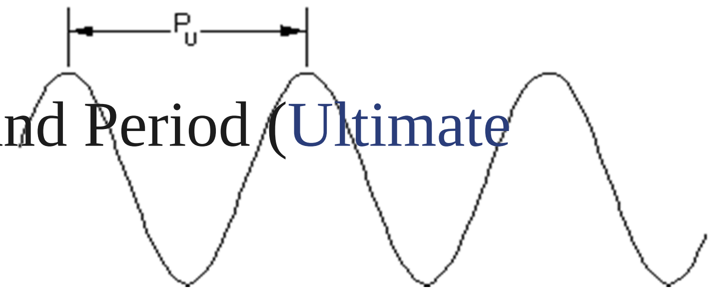
  D=Ti - P/R

- D & X are then used ------------→

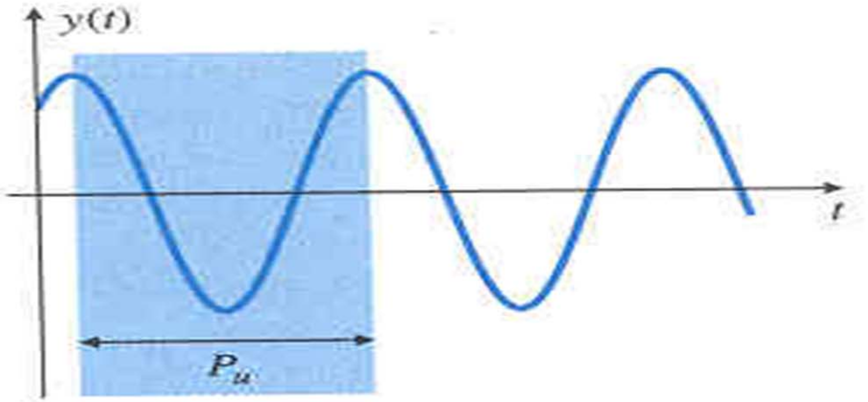
	Gain	Reset	Derivative
P	X/DR	—	—
PI	0.9X/DR	0.3/D	—
PID	1.2X/DR	0.5/D	0.5D

Tuning PID: **Ultimate Sensitivity Method(I)**

- Place controller into automatic with low gain, no reset or derivative.
- Gradually increase gain, making small changes in the setpoint, until oscillations start.
- Adjust gain to make the oscillations continue with a constant amplitude.
- Note the gain (Ultimate Gain, Gu,) and Period (Ultimate Period, Pu.)



Tuning PID: **Ultimate Sensitivity Method(II)**



Closed loop tuning method

Ziegler–Nichols Tuning for the Regulator $D(s) = K(1 + 1/T_I s + T_D s)$, Based on a Stability Boundary

Type of Controller	*Optimum Gain*
Proportional	$K = 0.5K_u$
PI	$\begin{cases} K = 0.45K_u \\ T_I = 1/1.2P_u \end{cases}$
PID	$\begin{cases} K = 0.6K_u \\ T_I = \frac{1}{2}P_u \\ T_D = \frac{1}{8}P_u \end{cases}$

MM6 Exercise

Design a P, PI, PID controller for the following DC motor speed control, According to quarter decay method.

Download ZN_tuning_motor.mdl