# MM9 Frequency Response Analysis (II) – Nyquist Diagram

Readings:

- Section 6.3 (Nyquist stability criterion, page361-375);
- Section 6.4 (stability margins, page 375-383)

# What Have We Talked about in MM8?

- Bode plot analysis
  - How to get a Bode plot
  - What we can gain from Bode plot

- How to use bode plot for design purpose
  - Stability margins (Gain margin and phase margin)
  - Transient performance
  - Steady-state performance

- Matlab functions: bode(), margin()

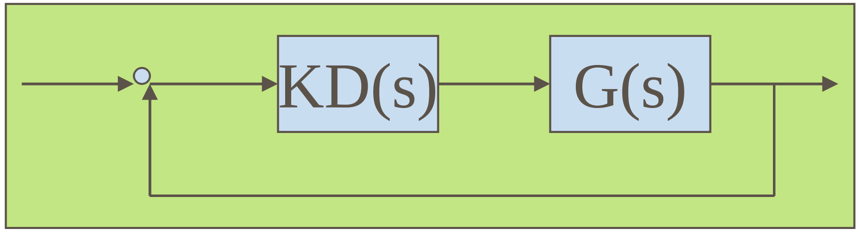
# Goals for this lecture (MM9)

- **A design example based on Bode plot**
  - **Open-loop system feature analysis**
  - **Bode plot based design**

- Nyquist Diagram
  - What's Nyquist diagram?
  - What we can gain from Nyquist diagram

- Matlab functions: nyquist()

# Design Example from MM8:....




- Plant model: G(s)=10/(1.25s+1)
- Requirement:
  - Zero steady state error for step input
  - Maximum overshoot must be less than 40%
  - Settling time must be less than 0.2 secs
- Is it necessary to develop a controler?
- If so, how to develop what kind of controller?

# Analysis of Open-Loop TF (I)

- **Stability -** Stable plant?
  - Bode plot
  - Nyquist plot (MM11)
  - Pole-zero plot
  - Routh criterion
- Software aided analysis

```
Sysp=tf(10,[1.25 1]), ltiview(Sysp)
num = 10; den = [1.25,1];
step(num,den); figure; bode(num, den)
```

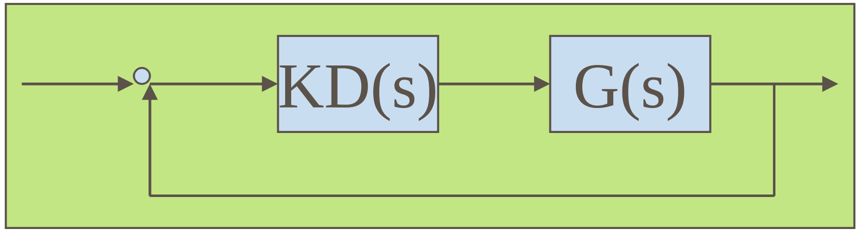
# Analysis of Open-Loop TF (II)

## Open-loop performance

- **Req1:** Zero steady state error for step input?
- **Req2:** Maximum overshoot must be less than 40%?
- **Req3:** Settling time must be less than 0.2 secs?

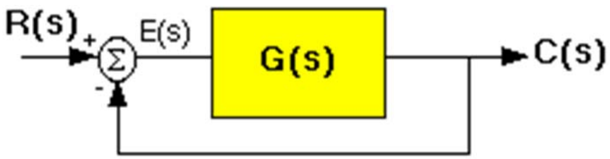
```
num = 10; den = [1.25,1]; step(num,den); figure; bode(num, den)
```

# Analysis of Closed-Loop: **Steady-State Error (I)**



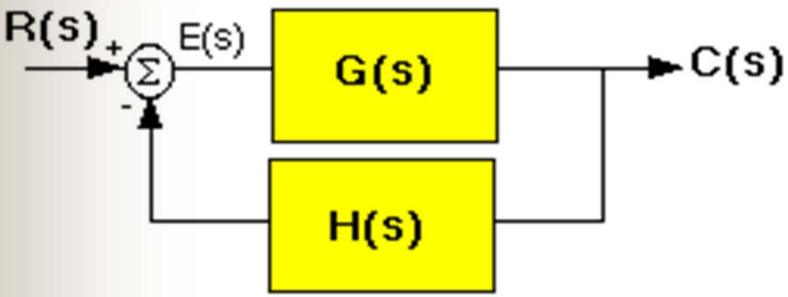

- **Req1: Zero steady state error for step input?**

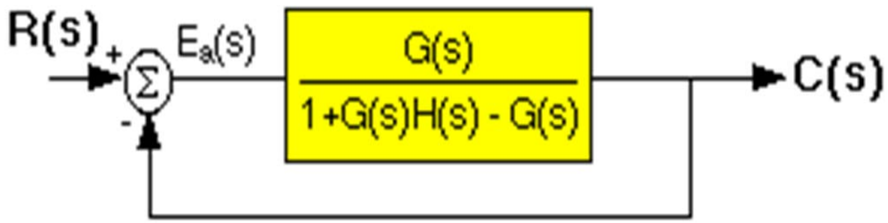
- The steady-state error of the closed-loop system will depend on the type of input (step, ramp, etc) as well as the (open-loop) system type (0, I, or II)



# Revisit of System Types & Steady State Error (MM5)

- Step Input (R(s) = 1/s): $e(\infty) = \dfrac{1}{1 + \lim_{s \to 0} G(s)} = \dfrac{1}{1 + K_p} \Rightarrow K_p = \lim_{s \to 0} G(s)$
- Ramp Input (R(s) = 1/s^2): $e(\infty) = \dfrac{1}{\lim_{s \to 0} sG(s)} = \dfrac{1}{K_v} \Rightarrow K_v = \lim_{s \to 0} sG(s)$
- Parabolic Input (R(s) = 1/s^3): $e(\infty) = \dfrac{1}{\lim_{s \to 0} s^2 G(s)} = \dfrac{1}{K_a} \Rightarrow K_a = \lim_{s \to 0} s^2 G(s)$



# Analysis of Closed-Loop: **Steady-State Error(II)**

- **Plant model**: G(s)=10/(1.25s+1)
- **Type** of the system?
- The steady-state error for step input:

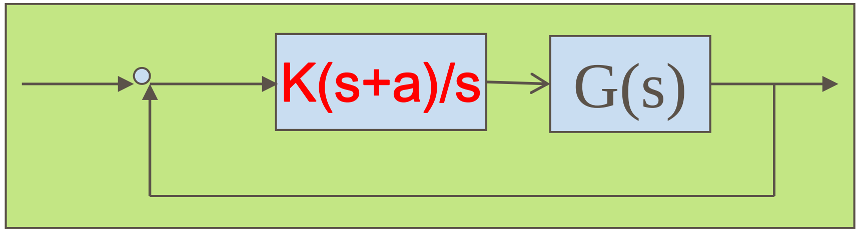
  **e(infty)=1/(1+Kp)=1/(1+10)=0.091**

- Add one integrator to the system, what's the type then?

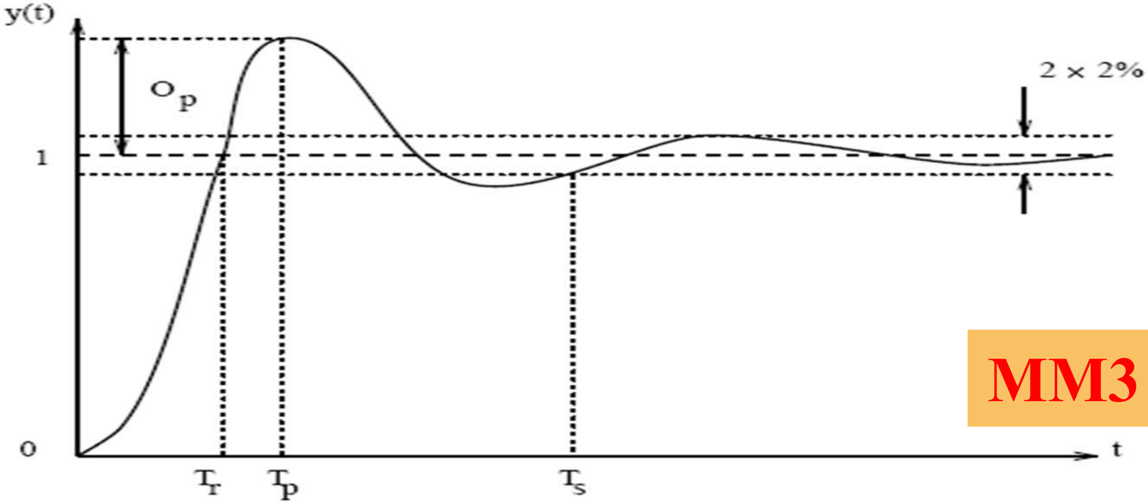
  G(s)=10/**s**(1.25s+1)

- choose a **PI controller** - because it will yield zero steady state error for a step input.
- Also, the PI controller has a zero, which we can place. This gives us additional design flexibility to help us meet our criteria. KD(s)=K(s+a)/s

# Analysis of Closed-Loop: **Transient Response (I)**



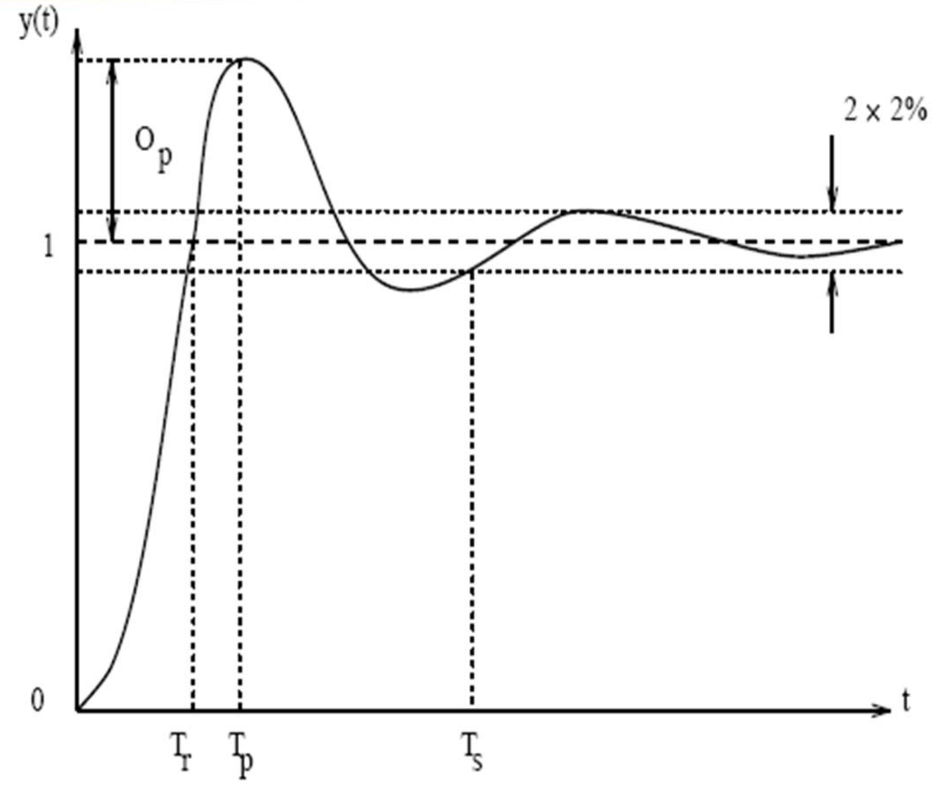

- **Req2: Overshoot** must be less than 40%?
- **Req3: Settling time** must be less than 0.2 secs?




**MM3 lecture**

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

# Revisit of Transient Response Specification(MM3)




$$t_r \cong \frac{1.8}{\omega_n}$$

$$t_s \cong \frac{4.6}{\zeta\omega_n} \cong \frac{4.6}{\sigma}$$

$$M_p \cong \begin{cases} 5\%, & \zeta = 0.7 \\ 16\%, & \zeta = 0.5 \\ 35\%, & \zeta = 0.3 \end{cases}$$

$$t_p \cong \frac{\pi}{\omega_d}, \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

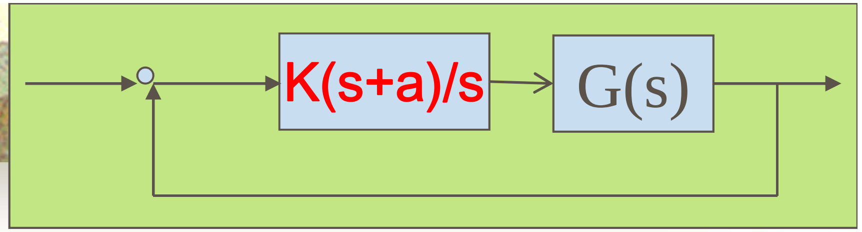
Rise time $T_r = \frac{\pi - \phi}{\omega_d}$

Peak time $T_p = \frac{\pi}{\omega_d}$

Settling time $T_s \approx \frac{4}{\xi\omega_n}$

Overshoot $O_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$

$$\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}, \; \omega_d = \omega\sqrt{1-\xi^2}$$




# Analysis of Closed-Loop: **Transient Response (II)**

- The first thing is to find the damping ratio of the closed-loop system corresponding to a percent **overshoot of 40%**
- the **damping ratio** of the closed-loop system corresponding to this overshoot is approximately **0.28**,
- the **phase margin** of the open-loop system should be approximately (28) **30 degrees**

**MM8:** For second-order systems, the closed-loop damping ratio is approximately equal to the phase margin divided by 100 if the phase margin is between 0 and 60 deg.

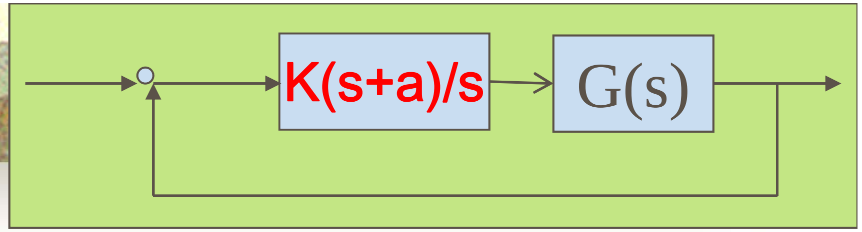
**ξ≈PM/100**

Rise time $T_r = \dfrac{\pi - \phi}{\omega_d}$

Peak time $T_p = \dfrac{\pi}{\omega_d}$

Settling time $T_s \approx \dfrac{4}{\xi \omega_n}$

Overshoot $O_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$




# Analysis of Closed-Loop: **Transient Response (III)**

- The seond thing is to find the bandwidth of the closed-loop system corresponding to a **settling time 0.2 second**
- the **damping ratio** corresponding to 40% overshoot is approximately **0.28**,
- The natural frequency of the closed-loop (bandwidth frequency) should greater than or equal to **71 rad/sec**
- Relationship: **Wgc ≤ Wbw ≤ 2Wgc**

```
num = [10]; den = [1.25, 1]; numPI = [1]; denPI = [1 0
newnum = conv(num,numPI); newden = conv(den,de
margin(newnum, newden); grid
```

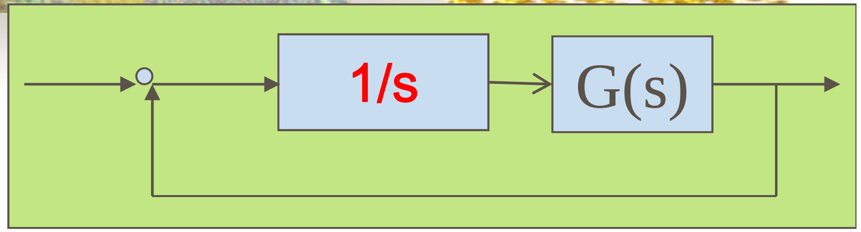
Rise time $T_r = \dfrac{\pi - \phi}{\omega_d}$

Peak time $T_p = \dfrac{\pi}{\omega_d}$

Settling time $T_s \approx \dfrac{4}{\xi \omega_n}$

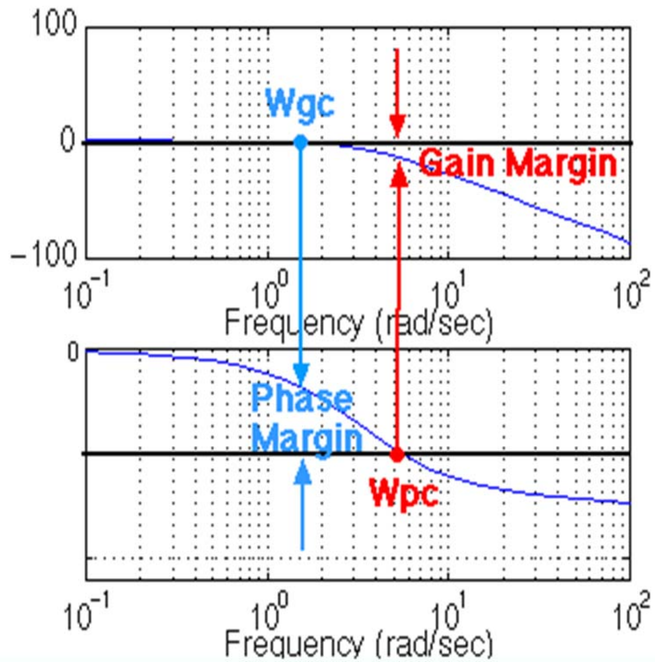
Overshoot $O_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$

# Control Design: **First-Try**




- the **phase margin** of the open-loop system should be approximately (28) **30 degrees**
- the **(gain) crossover frequency** should **Wgc ≥ 71 rad/sec**

```
num = [10];
den = [1.25, 1];
numPI = [1]; denPI = [1 0];
newnum = conv(num,numPI);
newden = conv(den,denPI);
margin(newnum, newden); grid
```



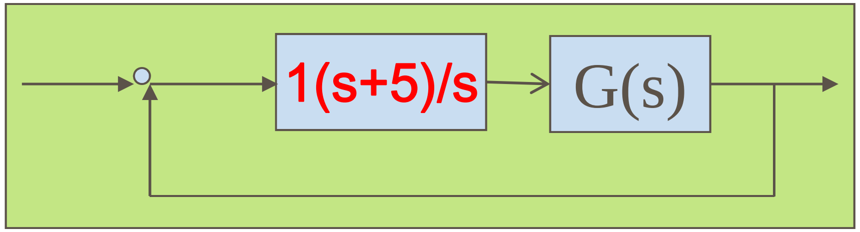

**phase margin and Wgc are too small!**

# Control Design: **Tuning PI controller (I)**

- Add gain and phase with a zero. Let's place the zero at -5 and see what happens

MM5: An additional zero in the left half-plane will increase the overshoot
If the zero is within  a factor of 4 of the real part of the complex poles

```
num = [10]; den = [1.25, 1];
numPI = [1 5]; denPI = [1 0];
newnum =
conv(num,numPI);
newden = conv(den,denPI);
margin(newnum, newden);
grid
```

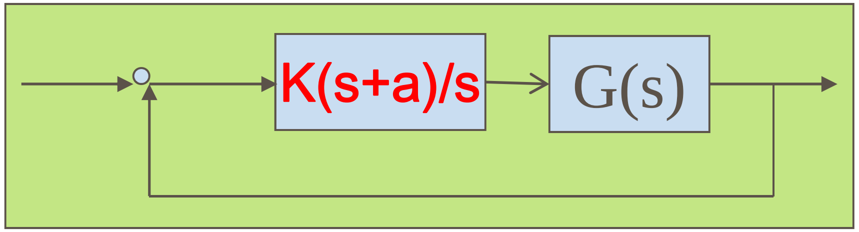


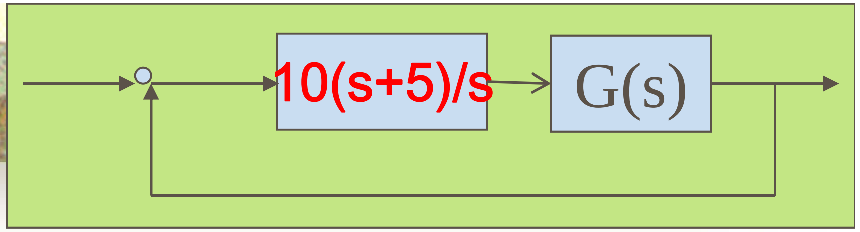
# Control Design: **Tuning PI controller (II)**

- try to get a larger crossover frequency with satisfactory phase margin. Let's try to **increase the gain to 10**

MM8: Adding gain only shifts the magnitude plot up.
Finding the phase margin is simply the matter of finding
the new cross-over frequency and reading off the phase margin

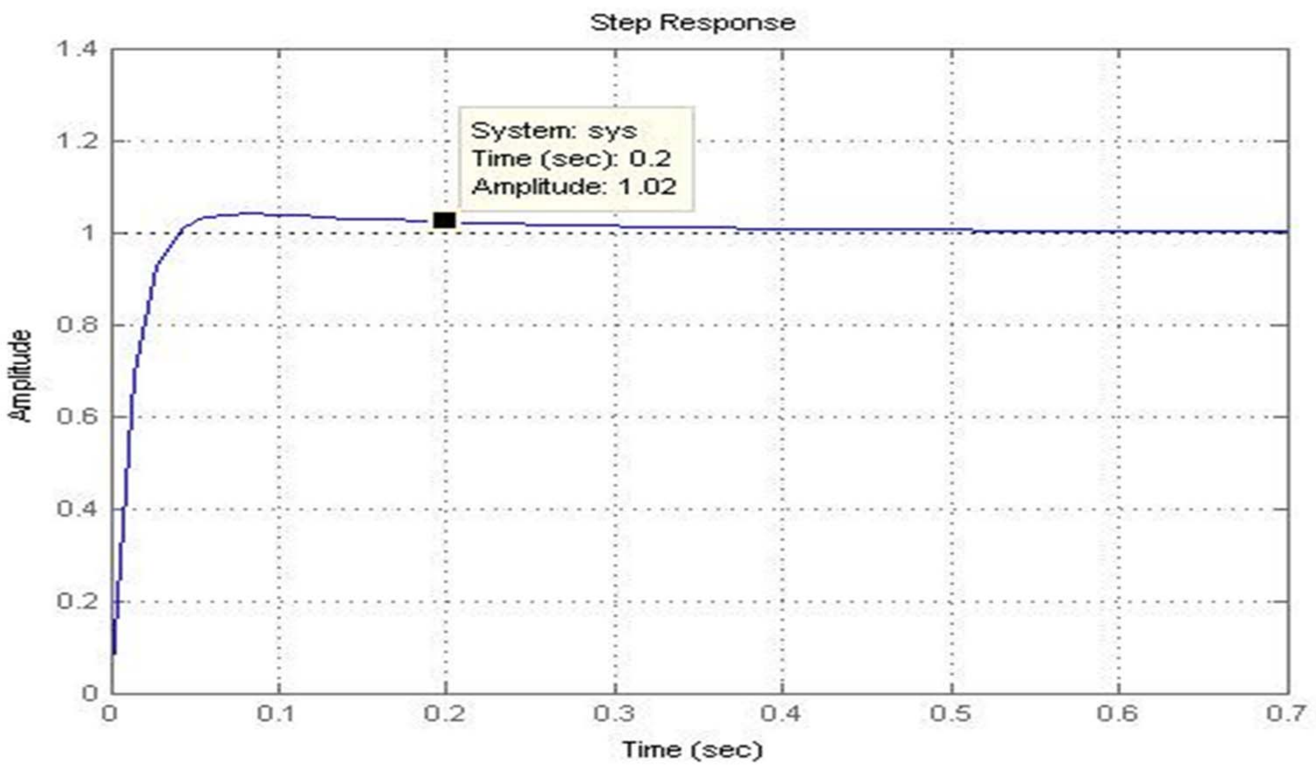
```
num = [10]; den = [1.25, 1];
numPI = 10*[1 5]; denPI = [1 0];
newnum = conv(num,numPI);
newden = conv(den,denPI);
margin(newnum, newden); grid
```






# Validation of Design

- [clnum,clden] =cloop(newnum,newden,-1);
  step(clnum,clden)



# Goals for this lecture (MM9)

- A design example based on Bode plot
  - Open-loop system feature analysis
  - Bode plot based design
- **Nyquist Diagram**
  - **What's Nyquist diagram?**
  - **What we can gain from Nyquist diagram**
- **Matlab functions: nyquist()**

# Nyquist Diagram: **Motivation**

- **Motivation:**

  to predict the stability and performance of a closed-loop system by observing its open-loop system's feautre

- **Benefit:**

  can be used for design purposes regardless of open-loop stability (remember that the Bode design methods assume that the system is stable in open loop)

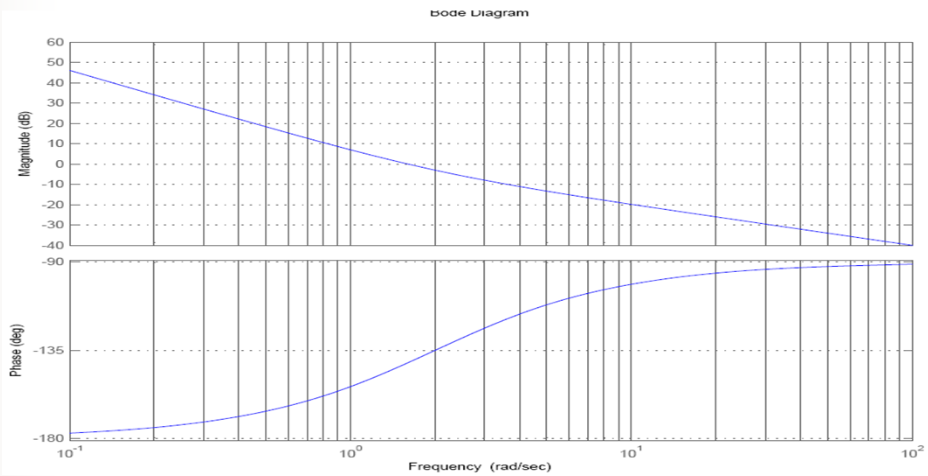
- http://www.engin.umich.edu/group/ctm/freq/nyq.html

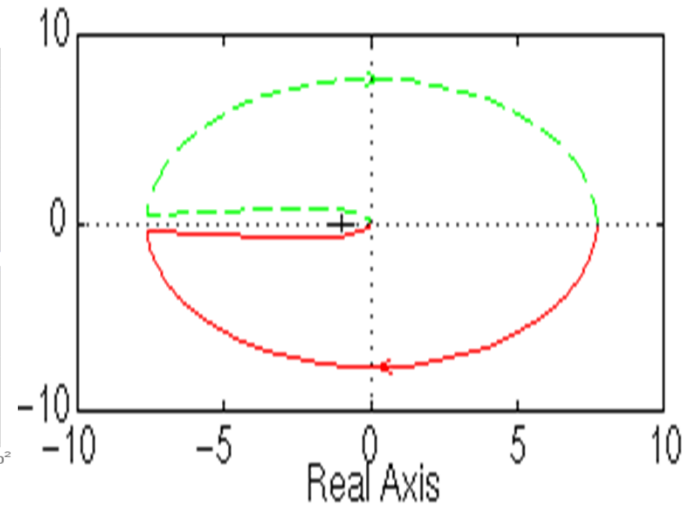
# Nyquist Diagram: **Definition**

The Nyquist diagram is a plot of **G(jΩ)** , where **G(s)** is the open-loop transfer function and **Ω** is a vector of frequencies which encloses the entire right-half plane

$$G(j\Omega) = |G(j\Omega)|\, e^{\sphericalangle G(j\Omega)},$$

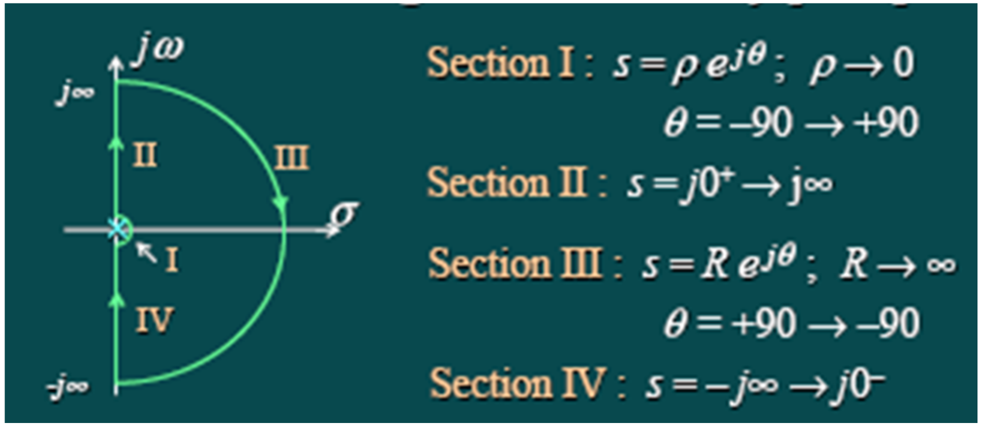
- The Nyquist diagram plots the position its the complex plane , while the Bode plot plots its magnitude and phase separately.



# Nyquist Diagram: **Ploting**

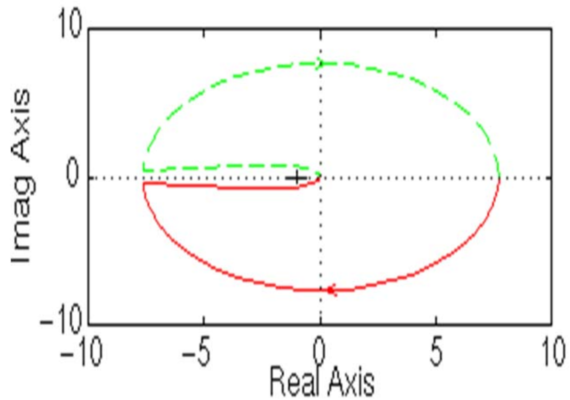
- **Frequency contour**




- if we have open-loop poles or zeros on the jw axis, G(s) will not be defined at those points, and we must loop around them when we are plotting the contour
- **Matlab function:**

nyquist (0.5,[1 0.5])

lnyquist1([1 2], [1 0 0])



# What's the Usefulness of Nyquist Diagram

- Predict the Stability of the closed-loop based on open-loop plot
- Check the stability margins
- Not limited by the open-loop stability

- **How to use that?**

# Nyquist Criterion for Stability

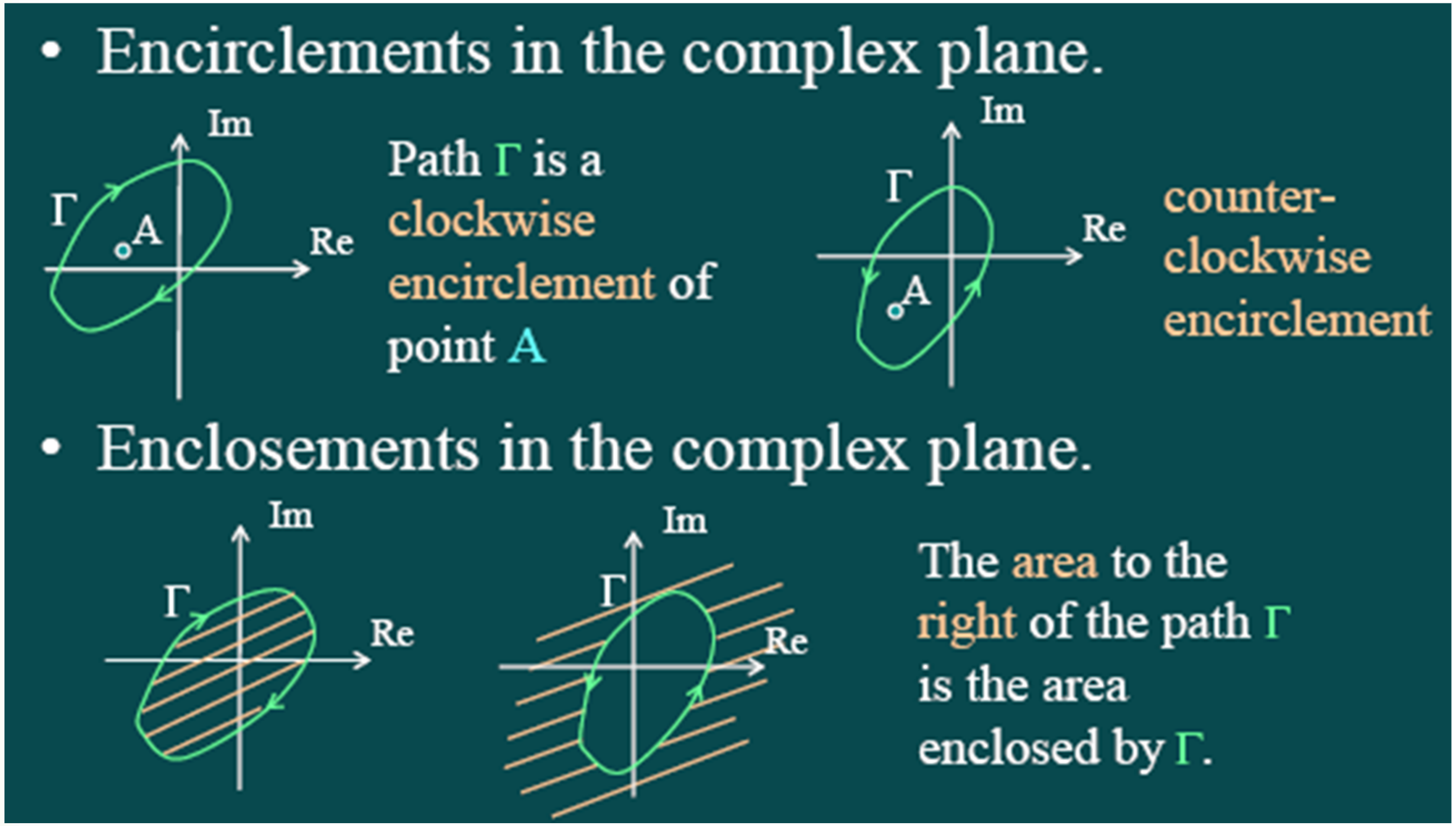
The Nyquist criterion states that:

- P = the number of open-loop (unstable) poles of G(s)H(s)
- N = the number of times the Nyquist diagram encircles –1
  - clockwise encirclements of -1 count as positive encirclements
  - counter-clockwise (or anti-clockwise) encirclements of -1 count as negative encirclements
- Z = the number of right half-plane (positive, real) poles of the closed-loop system
- The important equation:

$$Z = P + N$$

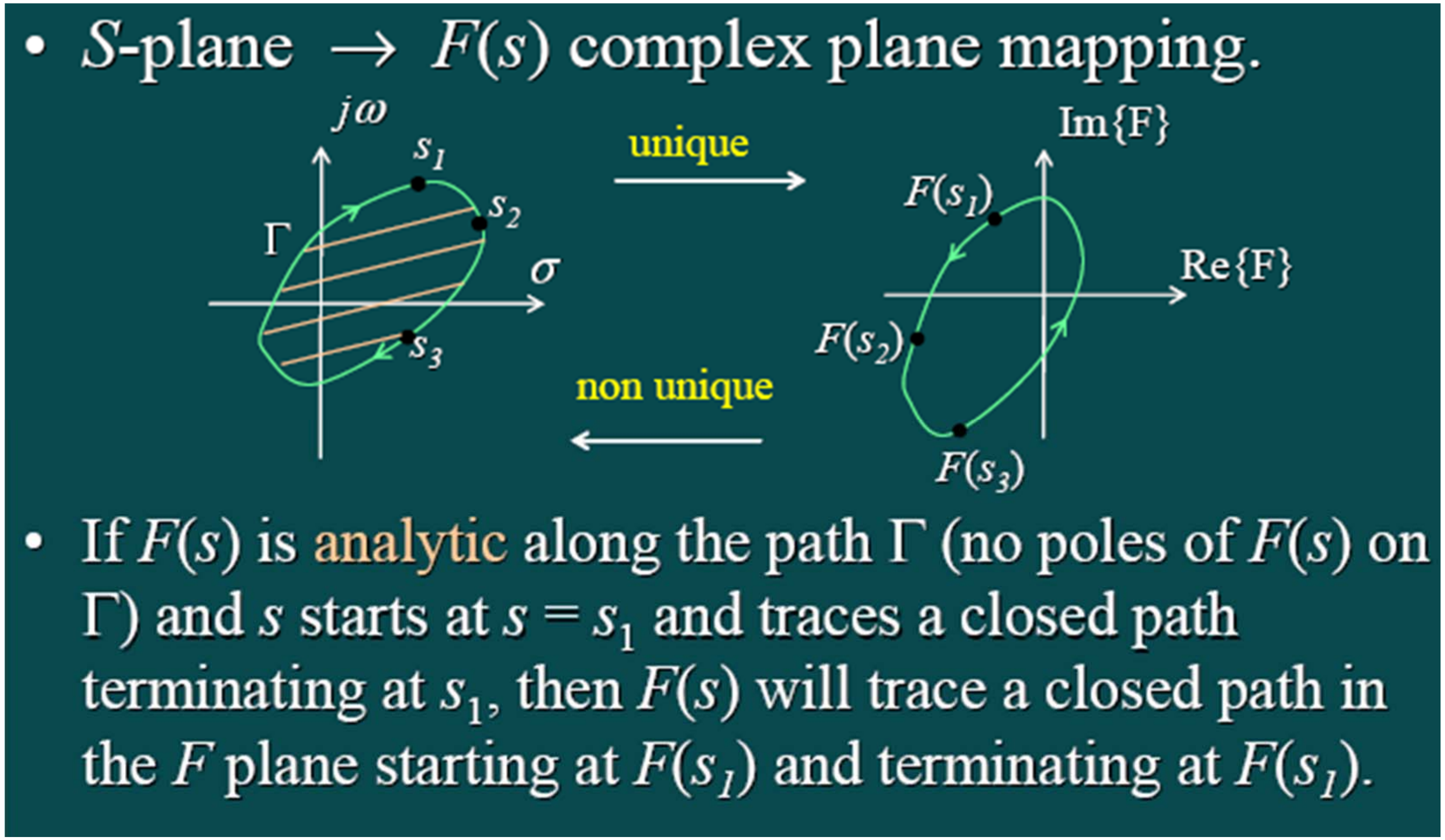
# Cauchy Criterion - Complex Analysis (I)

- when taking a closed contour in the complex plane,



# **Cauchy Criterion** - Complex Analysis (II)

- when taking a closed contour in the complex plane, and mapping it through a complex function G(s)



# **Cauchy Criterion** - Complex Analysis (III)

- when taking a closed contour in the complex plane, and mapping it through a complex function G(s)
- the number of times (N) that the plot of G(s) encircles the origin is equal to the number of zeros of G(s) (Z) enclosed by the frequency contour minus the number of poles of G(s) enclosed by the frequency contour (P).

$$N = Z - P$$

- Encirclements of the origin are counted as positive if they are in the same direction as the original closed contour or negative if they are in the opposite direction.

# **Cauchy Criterion:** for feedback Control (I)

- When studying feedback controls, the closed-loop transfer function:

  **Gcl(s)=G(s)/[1 + G(s)]**

- If 1+ G(s) encircles the origin, then G(s) will enclose the point -1

- Since we are interested in the closed-loop stability, we want to know if there are any closed-loop poles (zeros of 1 + G(s)) in the right-half plane

# **Cauchy Criterion:** for feedback Control (II)

- Remember from the Cauchy criterion that the number **N** of times that the plot of G(s)H(s) encircles **-1** is equal to the number **Z** of zeros of 1 + G(s)H(s) enclosed by the frequency contour minus the number **P** of poles of 1 + G(s)H(s) enclosed by the frequency contour (N = Z - P).
- Keeping careful track of open- and closed-loop transfer functions, as well as numerators and denominators, i.e., :
  - the zeros of 1 + G(s)H(s) are the poles of the closed-loop transfer function
  - the poles of 1 + G(s)H(s) are the poles of the open-loop transfer function.

# Nyquist Criterion for Stability (repeat)

The Nyquist criterion states that:

- P = the number of open-loop (unstable) poles of G(s)H(s)
- N = the number of times the Nyquist diagram encircles –1
  - clockwise encirclements of -1 count as positive encirclements
  - counter-clockwise (or anti-clockwise) encirclements of -1 count as negative encirclements
- Z = the number of right half-plane (positive, real) poles of the closed-loop system
- The important equation:

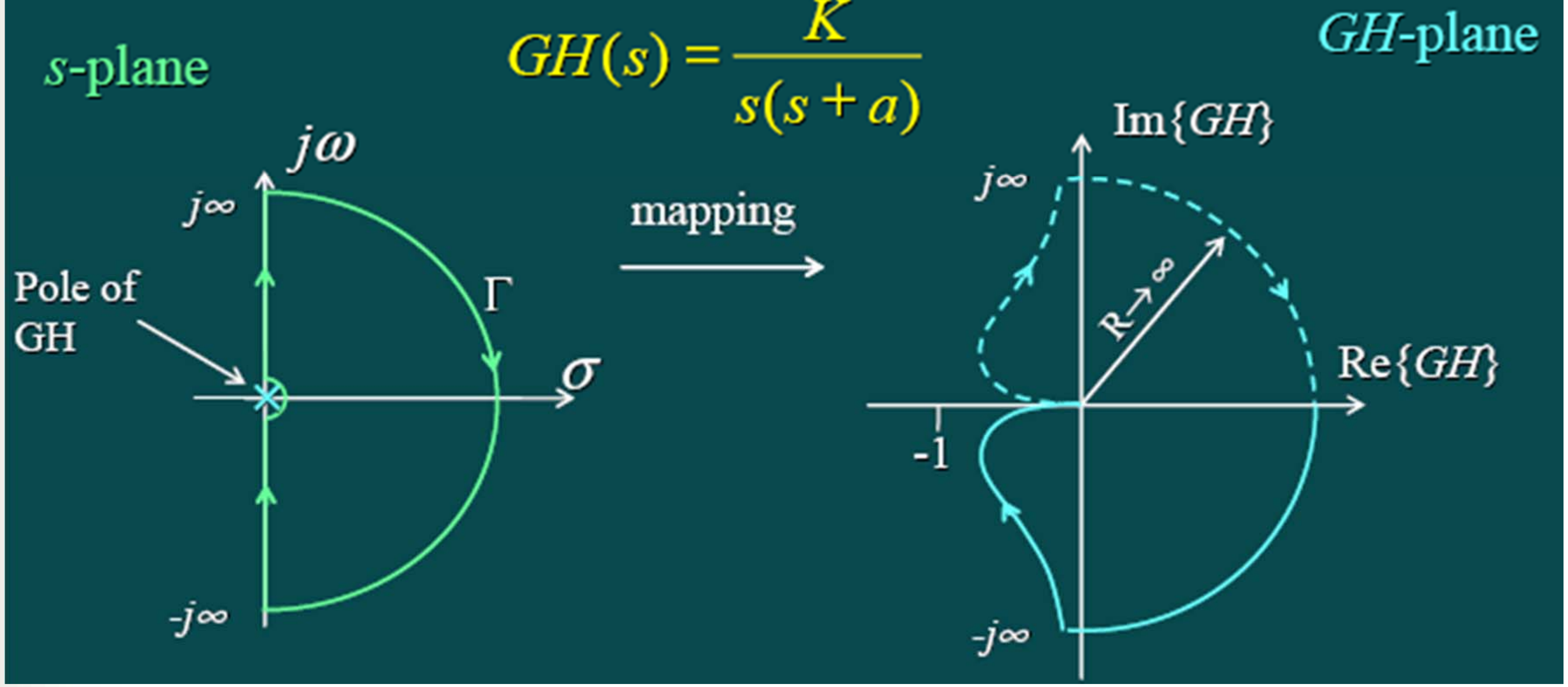
$$Z = P + N$$

# Nyquist Criterion:  A Simple Example (I)

## A Simple Example

- Consider a system with the loop TF

$$GH(s) = \frac{K}{s(s+a)}$$



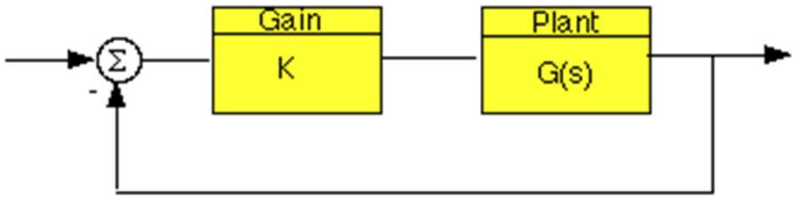
# Nyquist Criterion: A Simple Example (II)

$$GH(s) = \frac{K}{s(s+a)}$$

- The number of encirclements of the –1 point in the *GH*-plane is zero, $N = 0$.
- The number of poles of the loop transfer function *GH*(*s*) in the RHS, is zero ($P = 0$).
  Note: the Nyquist path excludes the pole at the origin.
- Therefore, the number of poles of the closed-loop system = the zeroes of 1 + *GH*(*s*), in the RHS, is $Z = N + P = 0 + 0 = 0$.
- The closed-loop system is stable.

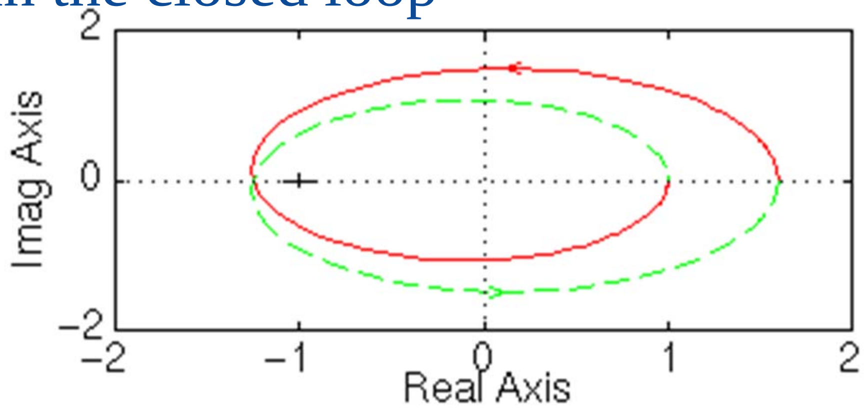
# Nyquist Criterion for Control Design

- The Nyquist criterion states that: if Z = P + N is a positive, nonzero number, the closed-loop system is unstable
- **Example:**



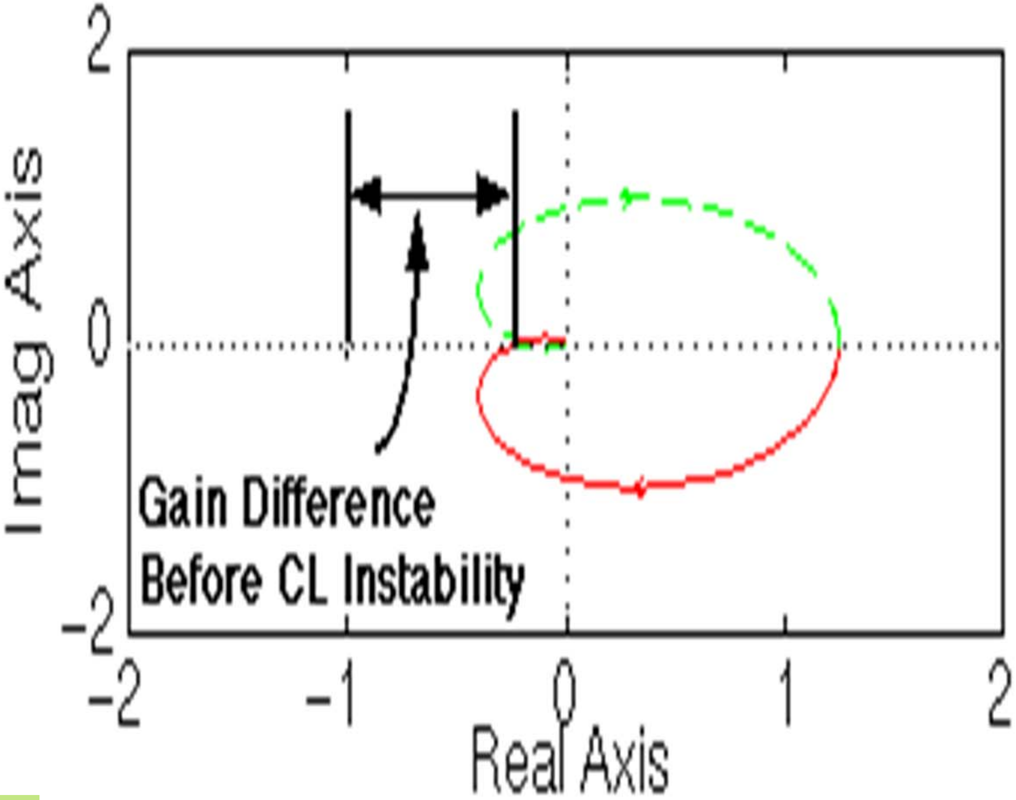

Look for: the range of gains that will make this system stable in the closed loop

```
G=tf([ 1 10 24], [ 1 -8 15])
roots([1 -8 15])
nyquist([ 1 10 24], [ 1 -8 15])
nyquist(20*[ 1 10 24], [ 1 -8 15])
nyquist(0.5*[ 1 10 24], [ 1 -8 15])
```



# Nyquist Diagram – Gain Margin

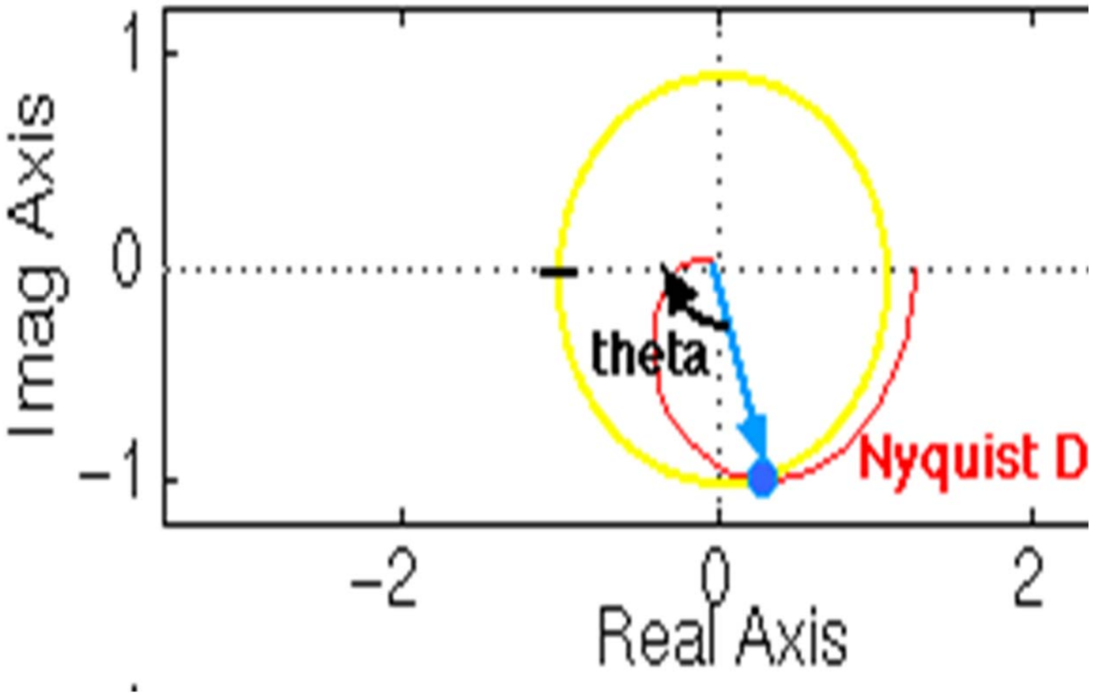
- First of all, let's say that we have a system that is stable if there are no Nyquist encirclements of –1
- the gain margin as the change in open-loop gain expressed in decibels (dB), required at 180 degrees of phase shift to make the system unstable



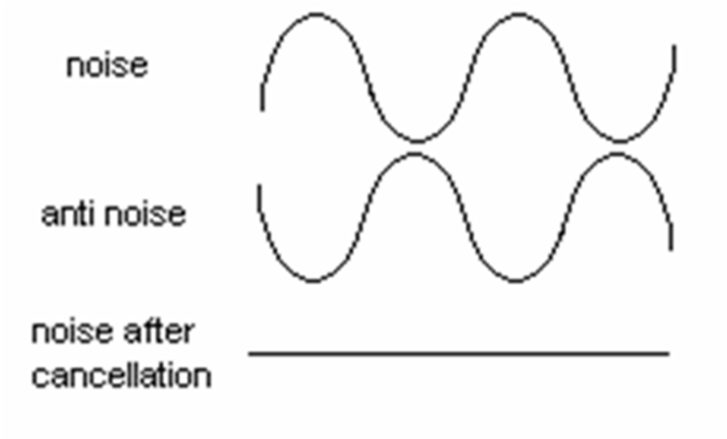

nyquist (50, [1 9 30 40 ])

# Nyquist Diagram – Phase Margin

- First of all, let's say that we have a system that is stable if there are no Nyquist encirclements of –1
- the phase margin as the change in open-loop phase shift required at unity gain to make a closed-loop system unstable.




nyquist (50, [1 9 30 40 ])



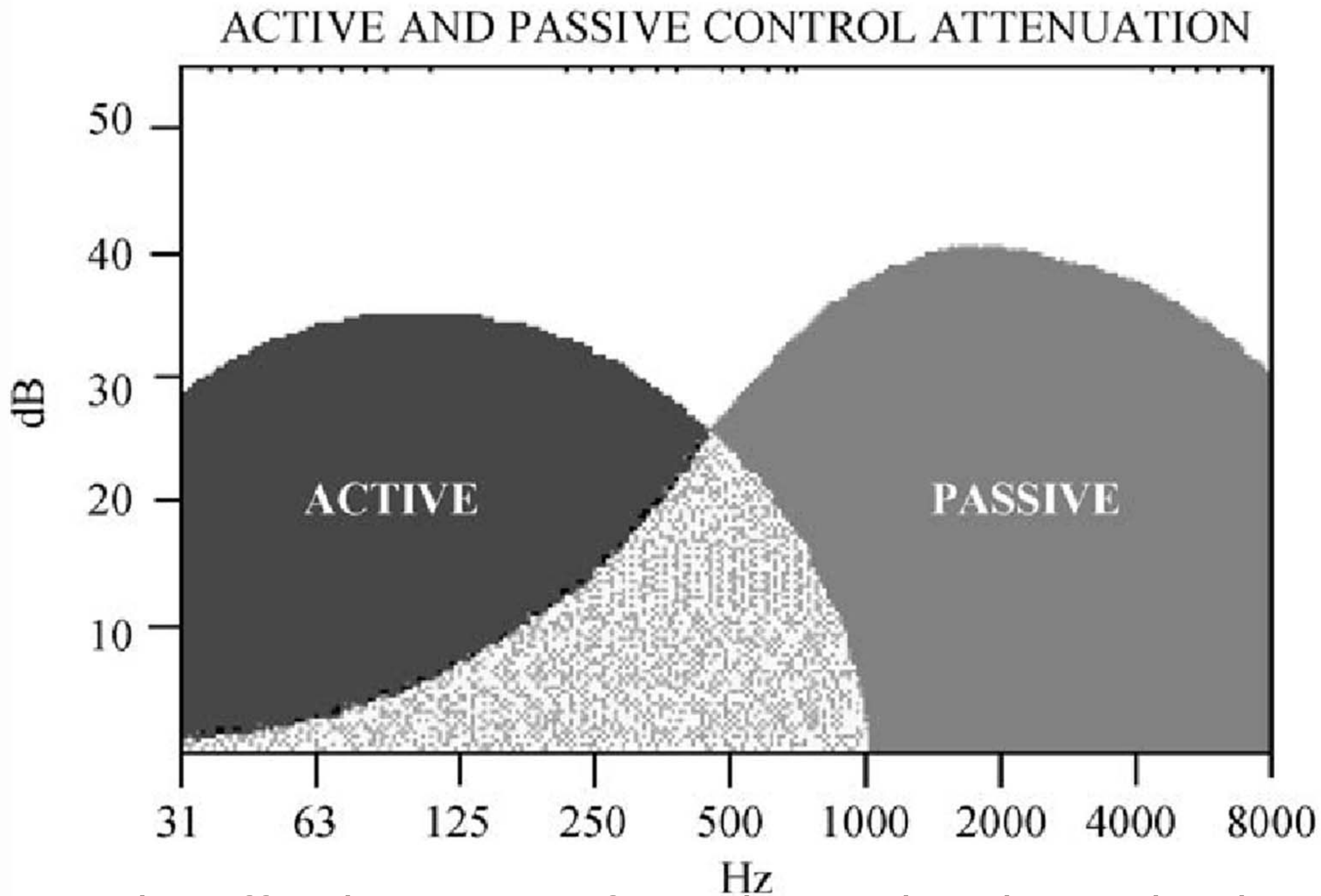

# A Real Case Study:

**Active Noise Reduction**

**-for a High Speed CD-ROM System**

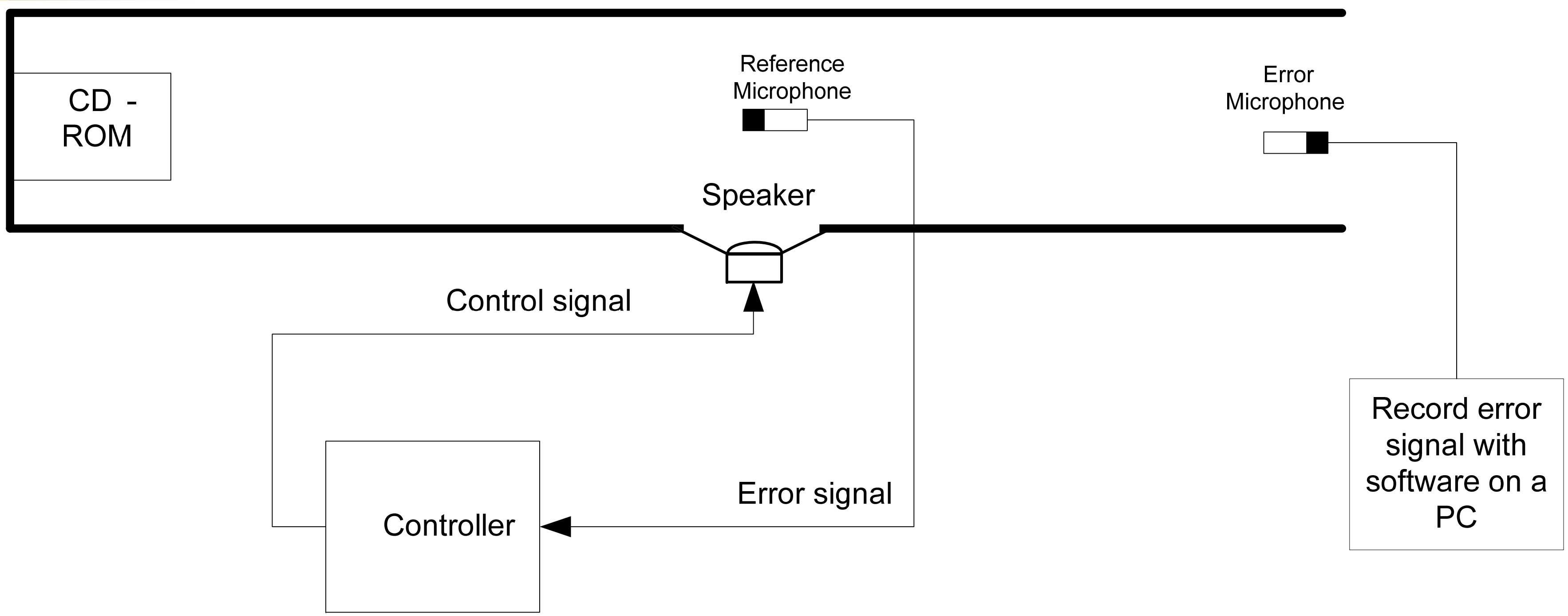
Cooperated with B&O A/s

# Active and Passive Approaches for ANR

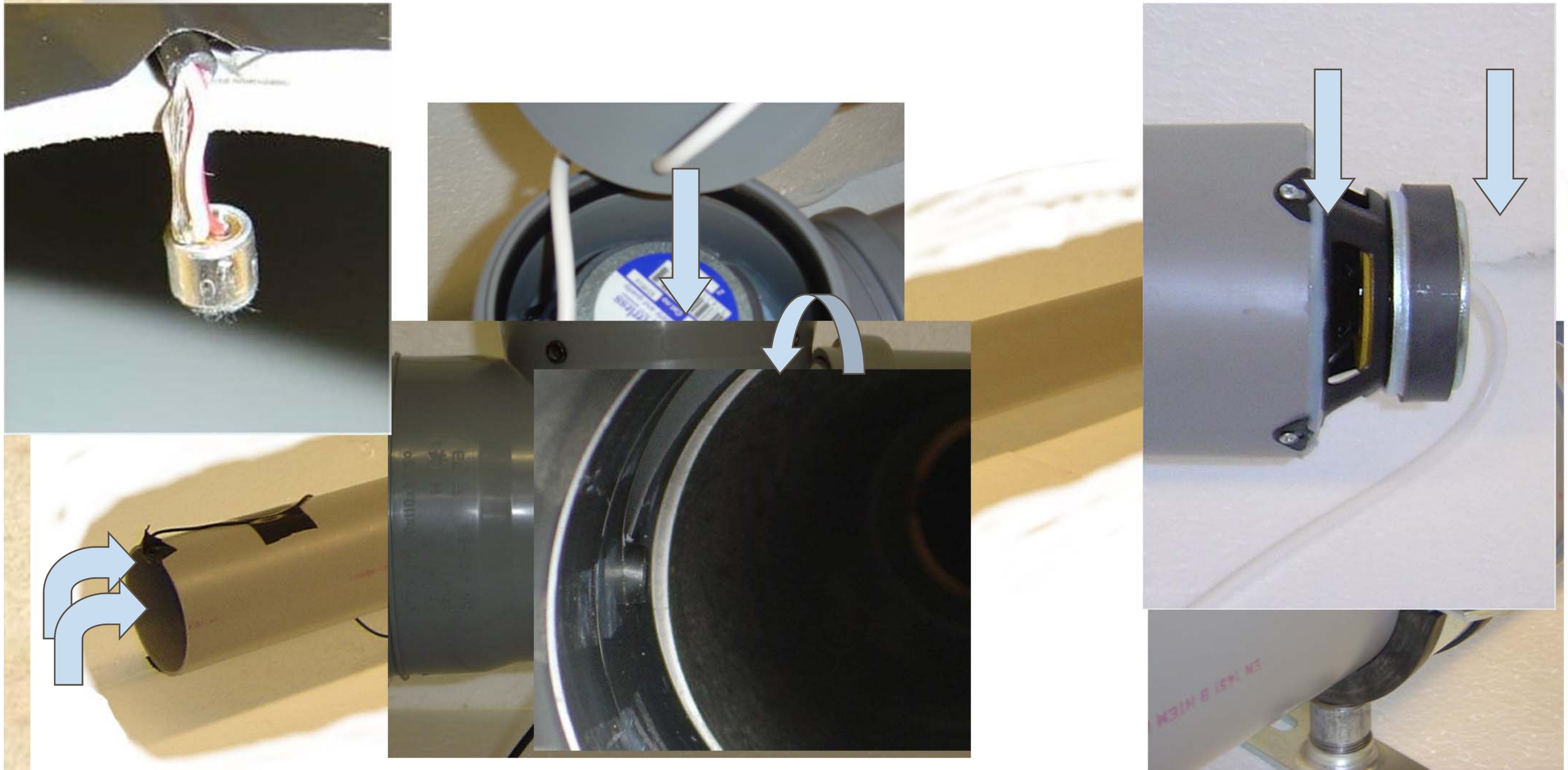



The effective areas of passive and active reduction:

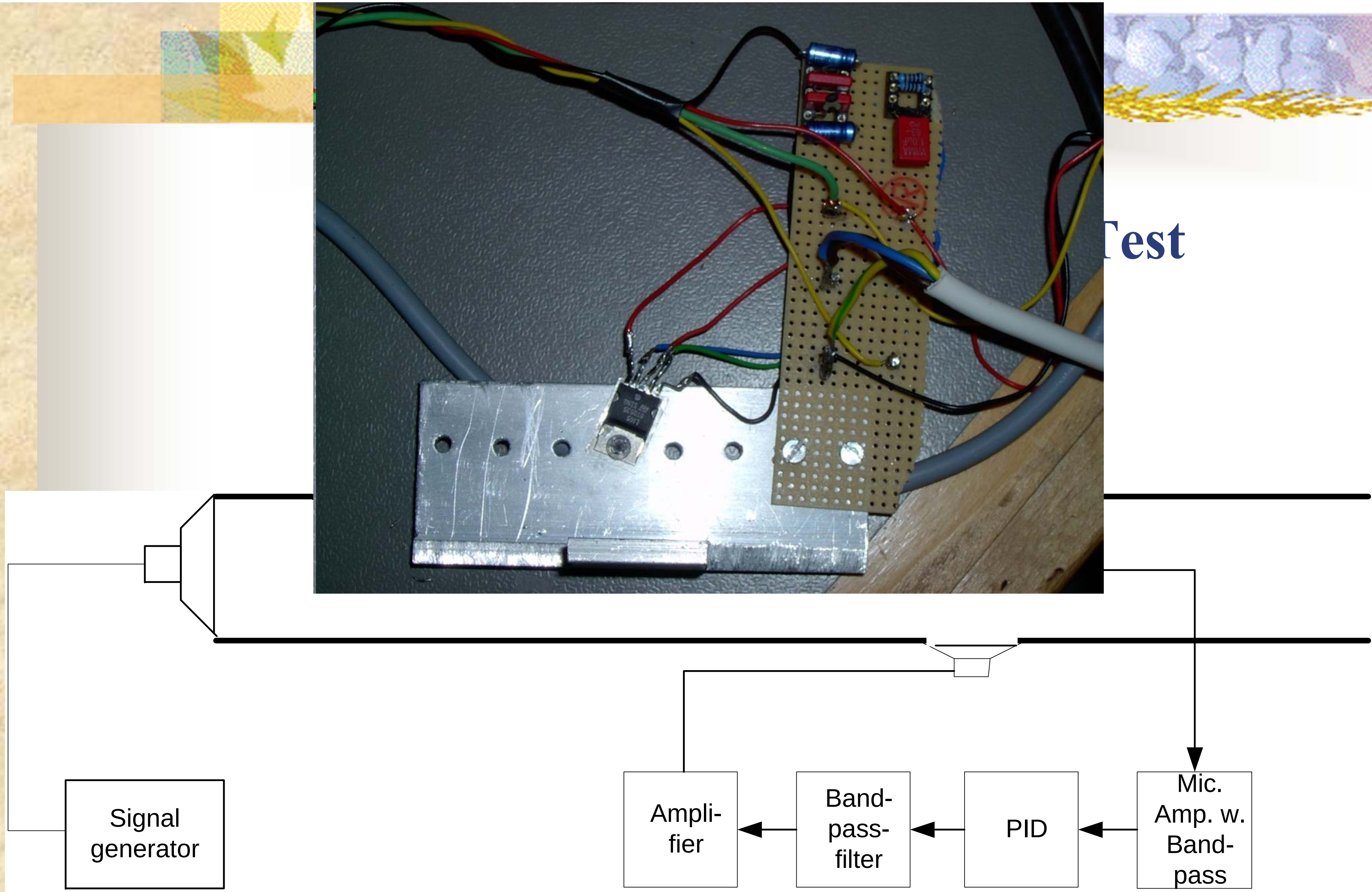
# Feedback ANR



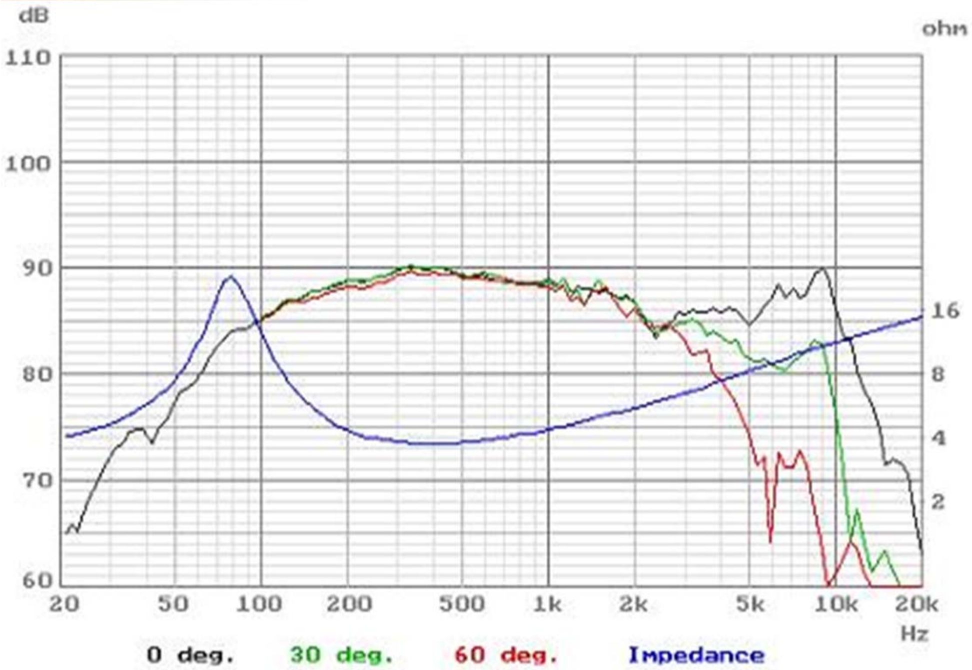
# Testing facility

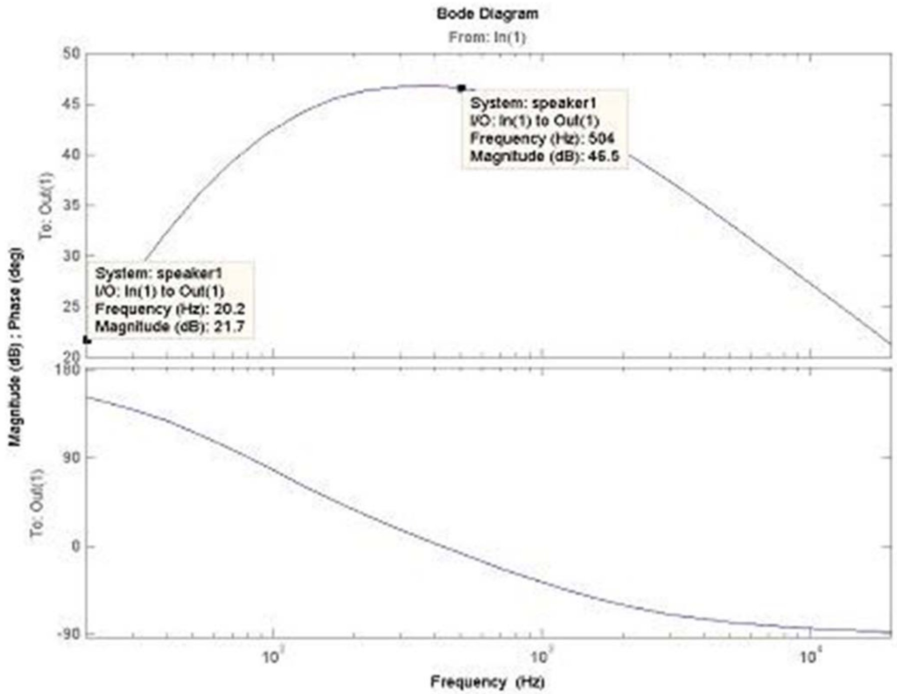


# Test


Signal generator
Ampli-fier
Band-pass-filter
PID
Mic. Amp. w. Band-pass

# **Modeling -** Verification of loudspeaker



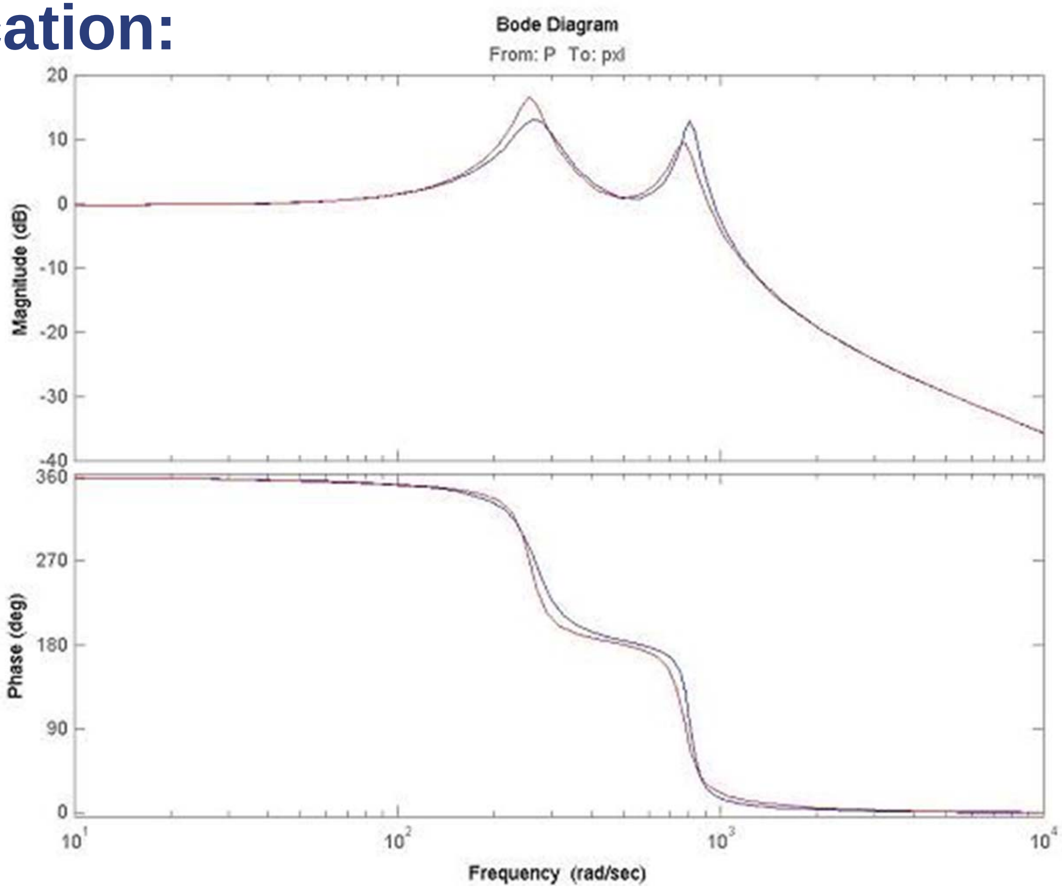




Frequency response from datasheet

Frequency response from bodeplot taken on model

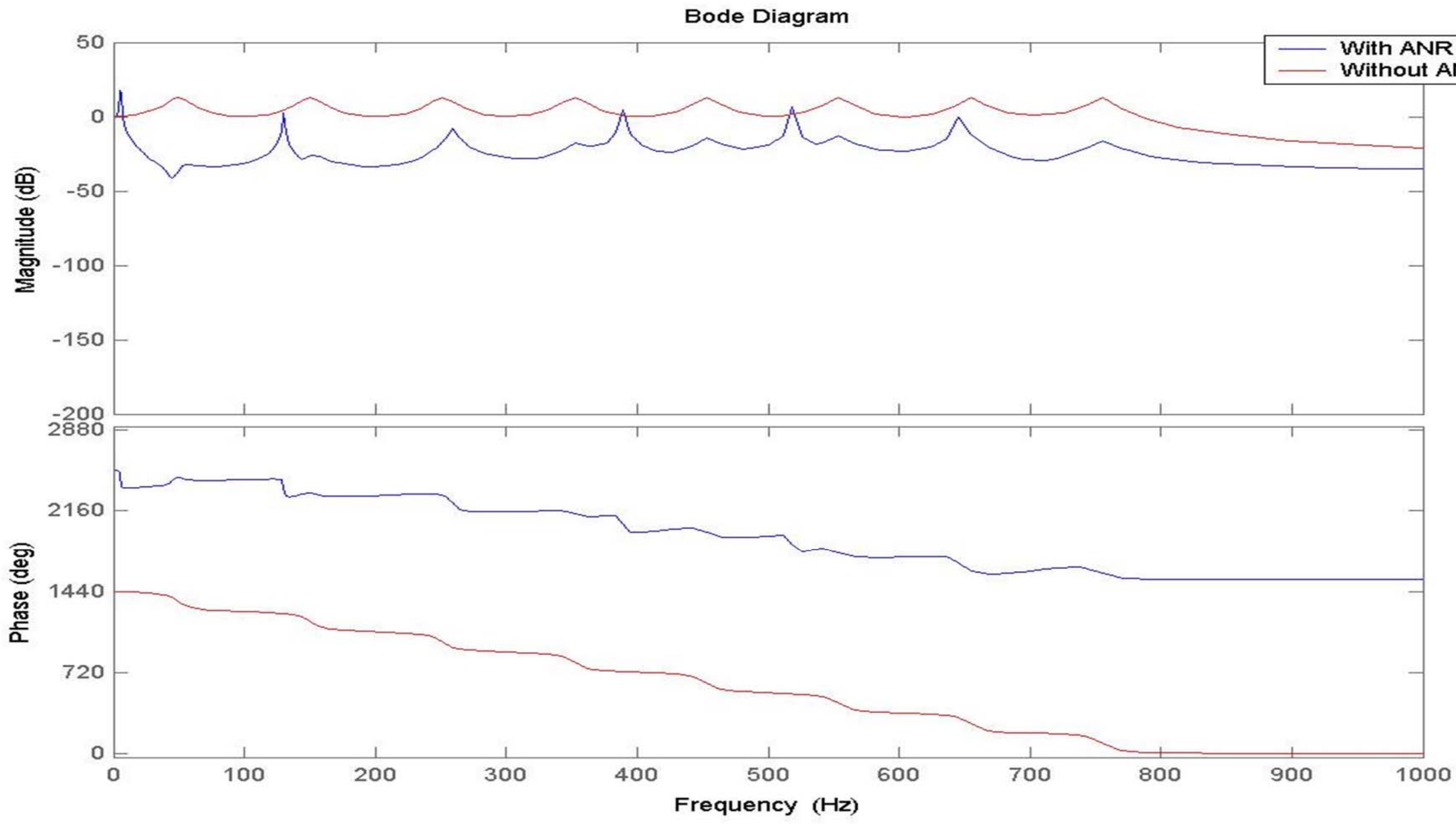
# **Modeling** - Acoustic Duct

**Impedance Verification:**



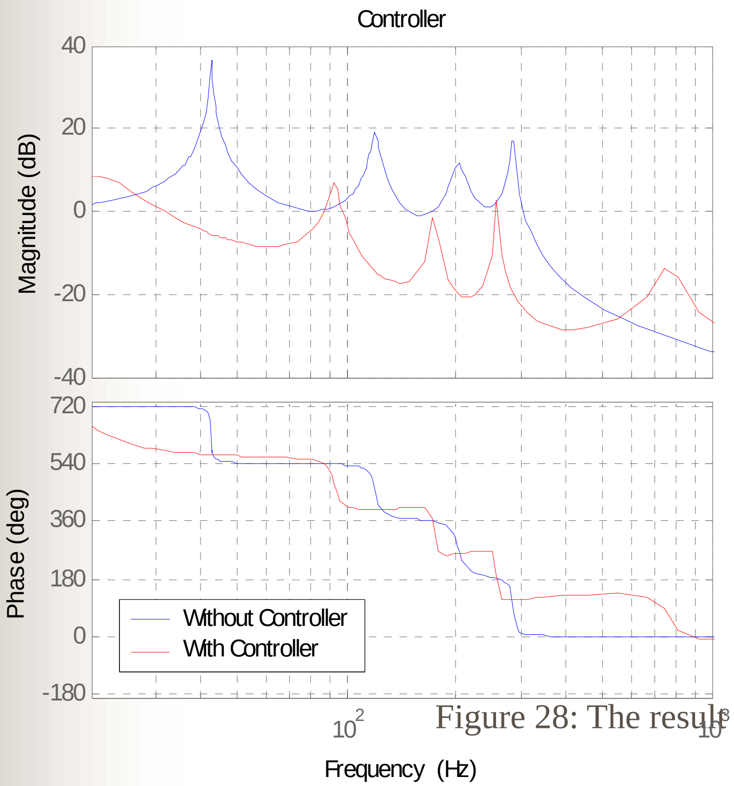

Frequency response of estimated and mathematical model

# Controller Design – Simulation



# Another Designed Controller

$$D(s) = 120 \cdot \frac{s+251}{s+106} \cdot \frac{s+1571}{s+6283}$$



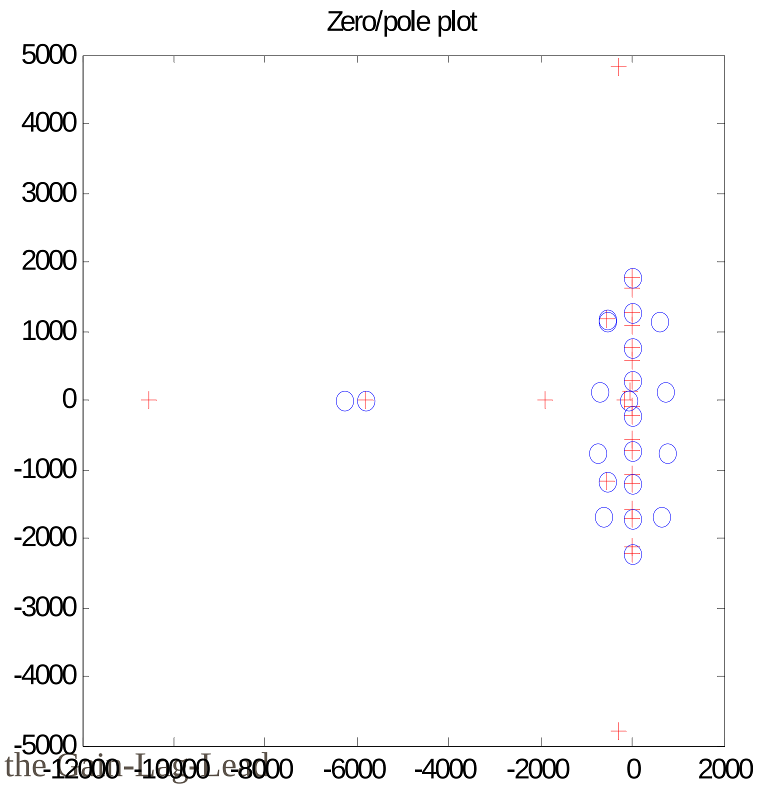
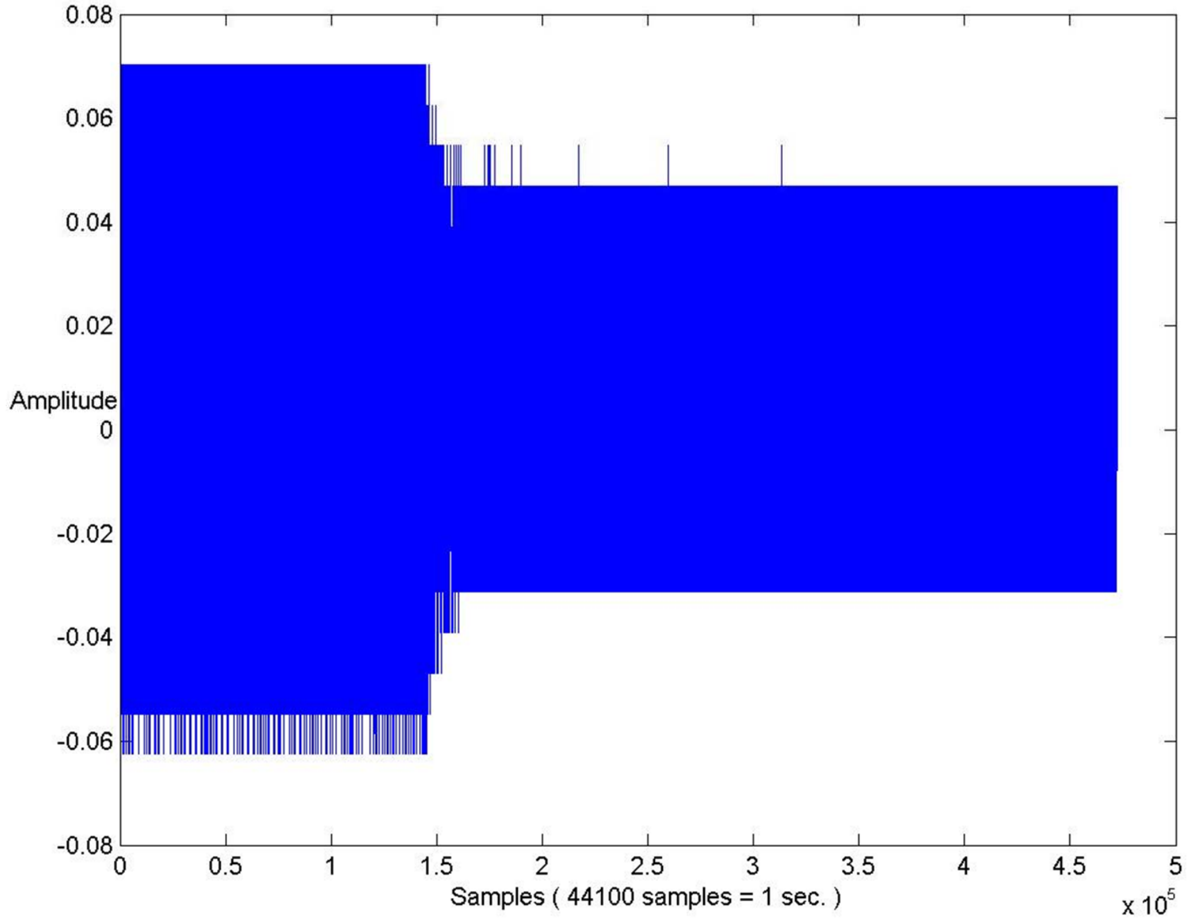




Figure 28: The results of tuning the Gain-Lag-Lead

# Controller for CD-ROM Noise



# Summary of MM 8-9

- How to use bode plot to
  - Analyze the stability (GM,PM)
  - Determine the bandwidth
  - Determine the transient response
  - Determine the system types and steady-state errors
- How to use nyquist diagram
  - Determine the stability
  - Determine the GM,PM