PE Course: Classical Control

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MM2: Essentials for Feedback Control

1 Reading

- Section 2.1 (models of mechanical systems, p.20-33);
- Section 3.1 (Laplace transform, p.86-110)
- Section 3.2 (Block diagram, p.111-118)

2 Content

- Essentials in using ODE model
- Refresh of Laplace transform
- Block diagram transformation
- Matlab/Simulink issues

3 Exercise

1. Consider a pendulum system which up-end of the pendulum is pivoted on a fixed surface. Through the first-principle modeling, a mathematical model can be obtained as

$$T_c - mglsin\theta = I\ddot{\theta}, \tag{1}$$

where T_c is the applied external torque, θ is the pendulum's deviated angle from the vertical position, I is the moment of initial of the pendulum, and it can be estimated as $I = ml^2$.

   - Is this ODE model (1) linear or nonlinear? and why?
   - Suppose the motion is small enough that we can let $sin\theta \approx \theta$, get a linear ODE model from (1) based on this approximation.
   - Suppose there is no any external force acting on the system, i.e., $T_c = 0$, but the pendulum has an initial angle position $\theta(0) = 0.5rad$ and initial angular velocity $\dot{\theta}(0) = 0$, can you image the dynamic behavior of the pendulum?
   - Based on above assumption, get a quantitative description of the pendulum performance. (Hint: derive the solution of an ODE)
   - Write a piece of M-file using the ode23 or ode45 solver in Matlab to simulate the considered system behavior. Compare the simulated result with the theoretical solution you obtained from last question.
   - Regard T_c as the system input and θ as the system output, derive the transfer function of the considered linear system.
   - What're the system poles and zeros? what kind of information we can gain by this pole-zero analysis?
   - Can you manage to get a Simulink model of this considered system? and simulate it under the condition that $\theta(0) = 0.5rad$, $\dot{\theta}(0) = 0$ and $T_c = 0$ as well.
   - Suppose the system is exposed to some wind disturbance, which can be regarded as an external torque, denoted as T_d, acting on the pendulum at the opposite direction of T_c. Can you extend this consideration into model (1)? so that what kind of system (SISO, SIMO, MISO, MIMO) you obtain?
   - Suppose the wind disturbance can be modeled as $T_f = K_f\dot{\theta}$, where K_f is some constant value, can you update the pendulum model and simulate the new system? What a kind of different result do you get, comparing with the situation where T_f is neglected?

2. Check the modeling of a crane system (or inverted pendulum) studied in Section 2.1 from the CC textbook. A linearized model can be obtained as

$$\begin{aligned} (I + m_p l^2)\ddot{\theta} + m_p g l\theta &= -m_p l\ddot{x} \\ (m_t + m_p)\ddot{x} + b\dot{x} + m_p l\ddot{\theta} &= u \end{aligned} \tag{2}$$

- By regrading u as system input and θ as system output, derive the transfer function of above system.
- Try to implement a simulink model of the original nonlinear system, i.e.,

$$\begin{aligned} &(I + m_p l^2)\ddot{\theta} + m_p g l sin\theta = -m_p l \ddot{x} cos\theta \\ &(m_t + m_p)\ddot{x} + b\dot{x} + m_p l \ddot{\theta} cos\theta - m_p l \dot{\theta}^2 sin\theta = u. \end{aligned} \tag{3}$$