PE Course: Classical Control

http://www.cs.aaue.dk/contribution/courses/fall2009/DE5/CC/Course.html
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Solutions to MM2-Essentials for Feedback Control

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1 Exercise

1. Consider a pendulum system which up-end of the pendulum is pivoted on a fixed surface. Through the first-principle modeling, a mathematical model can be obtained as

$$T_c - mglsin\theta = I\ddot{\theta}, \tag{1}$$

where T_c is the applied external torque, θ is the pendulum's deviated angle from the vertical position, I is the moment of initial of the pendulum, and it can be estimated as $I = ml^2$.

- Is this ODE model (7) linear or nonlinear? and why?

  Answer: It is not linear due to the sinusoid function.

- Suppose the motion is small enough that we can let $sin\theta \approx \theta$, get a linear ODE model from (7) based on this approximation.

  Answer: The linearized model is:

$$T_c - mgl\theta = I\ddot{\theta}. \tag{2}$$

- Suppose there is no any external force acting on the system, i.e., $T_c = 0$, but the pendulum has an initial angle position $\theta(0) = 0.5rad$ and initial angular velocity $\dot{\theta}(0) = 0$, can you image the dynamic behavior of the pendulum?

  Answer: The pendulum will have a periodical oscillation with an equal maximal amplitude.

- Based on above assumption, get a quantitative description of the pendulum performance. (Hint: derive the solution of an ODE)

  Answer: Based on the linearized system (2), take the Laplace transform on both sides. There is

$$Is^2\Theta(s) - s\theta(0) - \dot{\theta}(0) + mgl\Theta(s) = 0, \tag{3}$$

  which leads to the following w.r.t. the initial condition $\dot{\theta}(0) = 0$:

$$\Theta(s) = \frac{s\theta(0)}{Is^2 + mgl}, \tag{4}$$

  By taking the inverse Laplace transform, there is

$$\theta(s) = \theta(0)cos(\sqrt{\frac{g}{l}}t) = 0.5cos(\sqrt{\frac{g}{l}}t). \tag{5}$$

- Write a piece of M-file using the ode23 or ode45 solver in Matlab to simulate the considered system behavior. Compare the simulated result with the theoretical solution you obtained from last question.

  Answer:
  The main file pendulummain.m:

```
clear
global g l
g=9.8; l=0.5;
```

```
t0=0; tf=10;
x0=[0.5; 0];
[t,x]=ode23('pendulum',[t0 tf],x0);
subplot(1,2,1);
plot(t,x(:,1)); grid
xlabel('time'); ylabel('Amplitude');
subplot(1,2,2);
plot(abs(fft(x(:,1))));
axis([0 100 0 50]); grid;
xlabel('frequency'); ylabel('Spectrum Amplitude');
```

The own-defined function: pendulum.m:

```
function xdot=pendulum(t,x)
global g l
xdot=zeros(2,1);
xdot(1)=x(2);
xdot(2)=-g/l * x(1);
```

The simulation result is shown in the following figure 1.

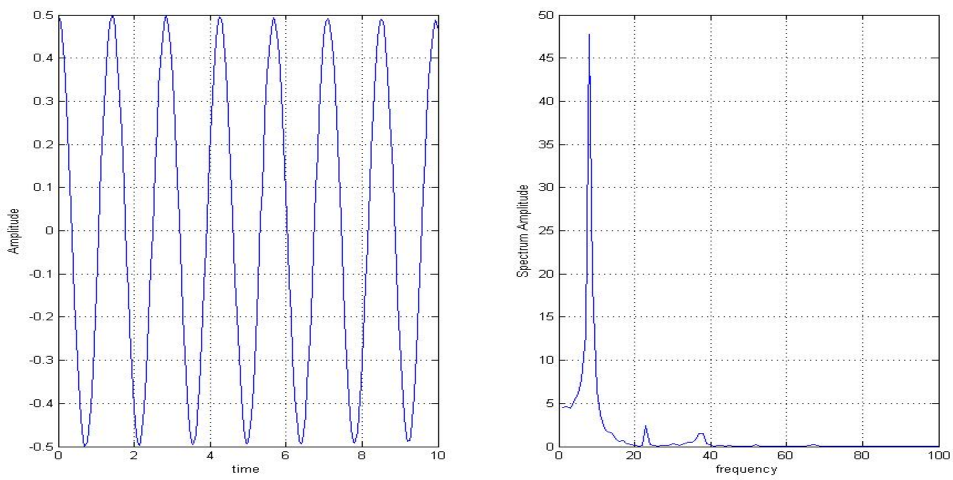

Figure 1: Simulation and spectrum of the free response of ideal pendulum system

- Regard T_c as the system input and θ as the system output, derive the transfer function of the considered linear system.

  Answer: The transfer function is:

$$G(s) = \frac{\Theta(s)}{T_c(S)} = \frac{1}{I^2 s + mgl}. \tag{6}$$

- What're the system poles and zeros? what kind of information we can gain by this pole-zero analysis?

  Answer: the system poles are: $\pm\sqrt{\frac{g}{l}}$. There is no zero of the considered system. There is a pair of purely complex poles, thereby we suspect the system has some periodic behavior with the period equalling to $\sqrt{\frac{g}{l}}$.

- Can you manage to get a Simulink model of this considered system? and simulate it under the condition that $\theta(0) = 0.5rad$, $\dot{\theta}(0) = 0$ and $T_c = 0$ as well.

  Answer: See the following simulink model and simulation result.

- Suppose the system is exposed to some wind disturbance, which can be regarded as an external torque, denoted as T_d, acting on the pendulum at the opposite direction of T_c. Can you extend this consideration into model (7)? so that what kind of system (SISO, SIMO, MISO, MIMO) you obtain?

  Answer: The system model (7) will be extended as

$$T_c - mglsin\theta - T_d = I\ddot{\theta}, \tag{7}$$

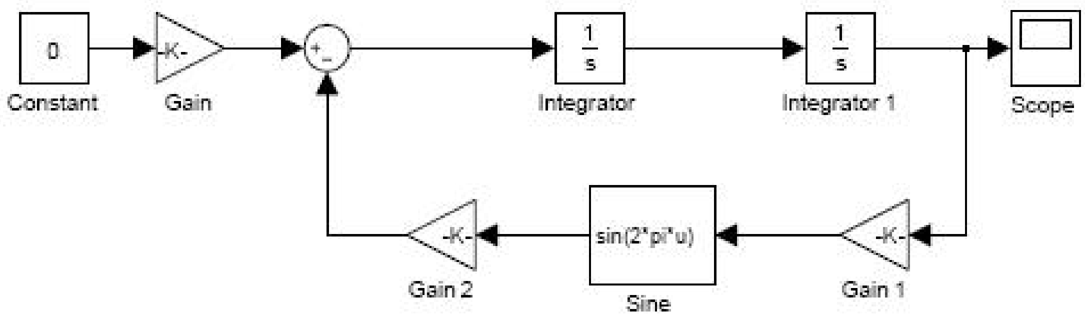

Figure 2: Simulink model of the nonlinear pendulum system

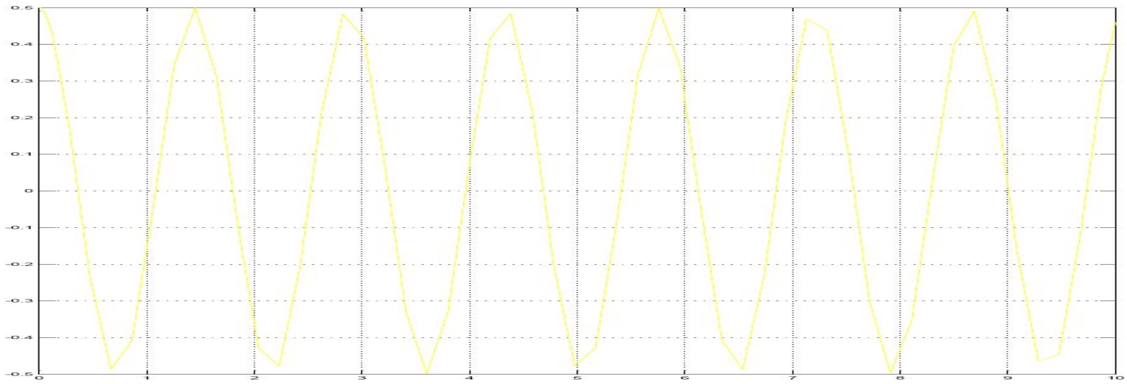

Figure 3: Simulation result of the free response of nonlinear pendulum system

then we get a MISO system, i.e., 2 inputs (T_c and T_d) and 1 output θ.

- Suppose the wind disturbance can be modeled as $T_d = K_f\dot{\theta}$, where K_f is some constant value, can you update the pendulum model and simulate the new system? What a kind of different result do you get, comparing with the situation where T_f is neglected?

  Answer: the system transfer function becomes

  $$G(s) = \frac{\Theta(s)}{T_c(S)} = \frac{1}{I^2 s + k_f s + mgl}. \tag{8}$$

  A new function is defined as pendulumfriction.m:

  ```
  function xdot=pendulumfriction(t,x)
  global g l
  xdot=zeros(2,1);
  xdot(1)=x(2);
  xdot(2)=-g/l * x(1)- 0.2/(0.2*0.5^2) * x(2);
  ```

  A simulation result with $k_f = 0.2$ is shown in the following figure 4.
  A simulink model can be updated as shown in Fig.5. Clearly, the friction disturbance damps the original oscillation. You can try to use different k_f value to check different damping behaviors (e.g., underdamped, overdamped, critically damped).

2. Check the modeling of a crane system (or inverted pendulum) studied in Section 2.1 from the CC textbook. A linearized model can be obtained as

$$\begin{aligned}(I + m_p l^2)\ddot{\theta} + m_p g l \theta &= -m_p l \ddot{x} \\ (m_t + m_p)\ddot{x} + b\dot{x} + m_p l \ddot{\theta} &= u\end{aligned} \tag{9}$$

- By regrading u as system input and θ as system output, derive the transfer function of above system.
- Try to implement a simulink model of the original nonlinear system, i.e.,

$$\begin{aligned}(I + m_p l^2)\ddot{\theta} + m_p g l sin\theta &= -m_p l \ddot{x} cos\theta \\ (m_t + m_p)\ddot{x} + b\dot{x} + m_p l \ddot{\theta} cos\theta - m_p l \dot{\theta}^2 sin\theta &= u.\end{aligned} \tag{10}$$

Manage these parts by your group, and the content should be included in your project report.

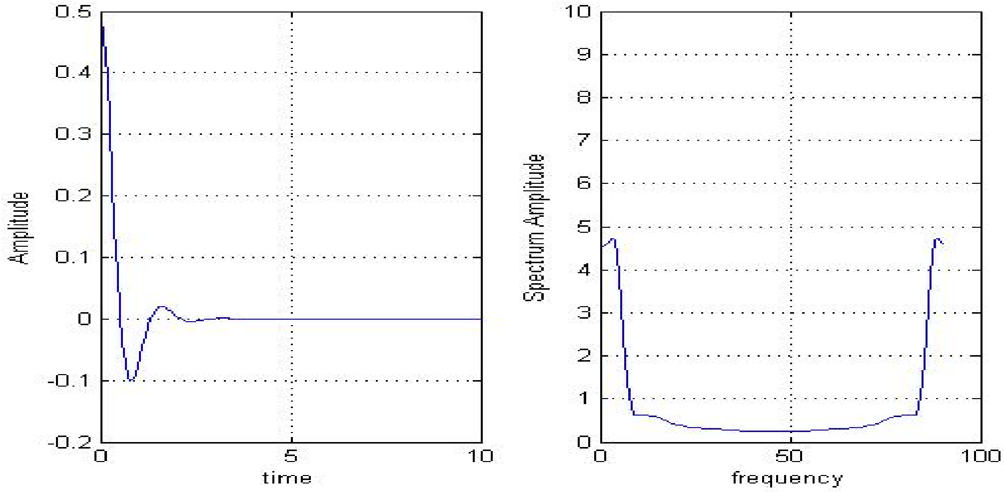

Figure 4: Simulation and spectrum of the response of the damped pendulum system

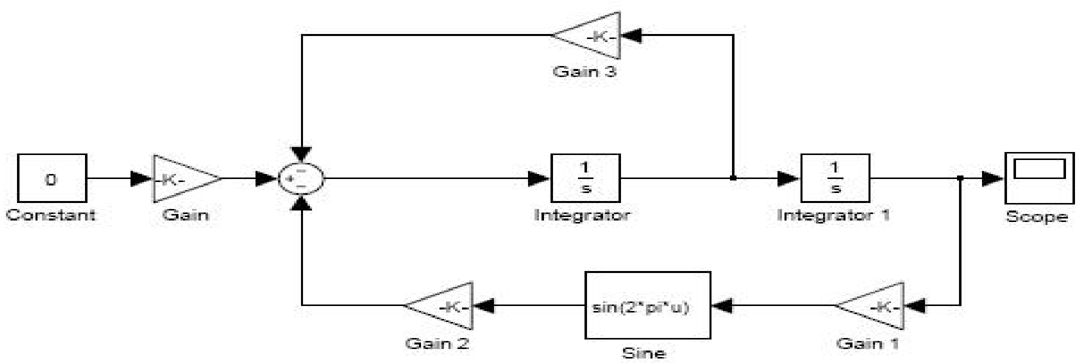

Figure 5: Simulink model of the nonlinear damped pendulum system