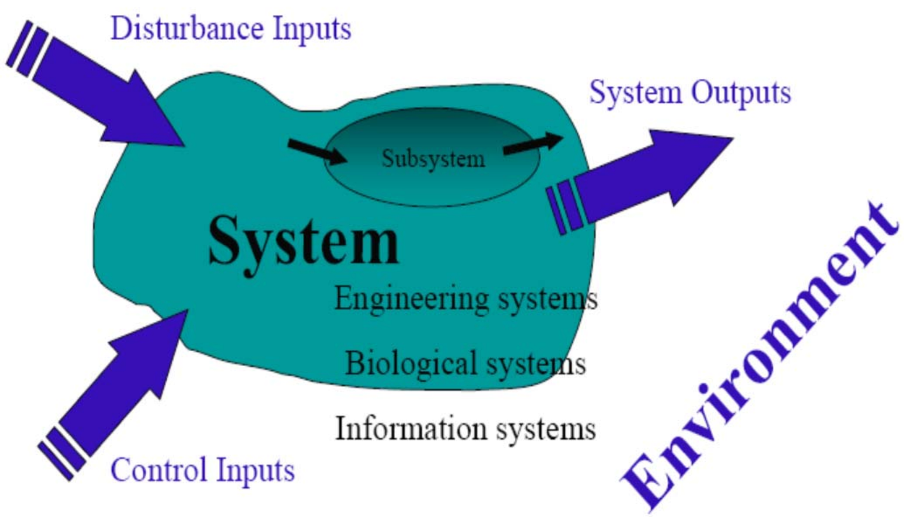
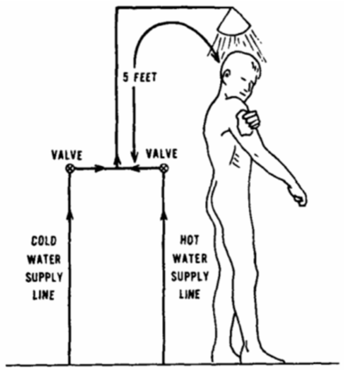
MM1: Basic Concept (I): **System and its Variables**

- A **system** is a collection of components which are coordinated together to perform a function
- Systems interact with their **environment**. The interaction is defined in terms of **variables**
  - System inputs
  - System outputs
  - Environmental disturbances
- **Dynamic system** is a system ehose performance could change according to time

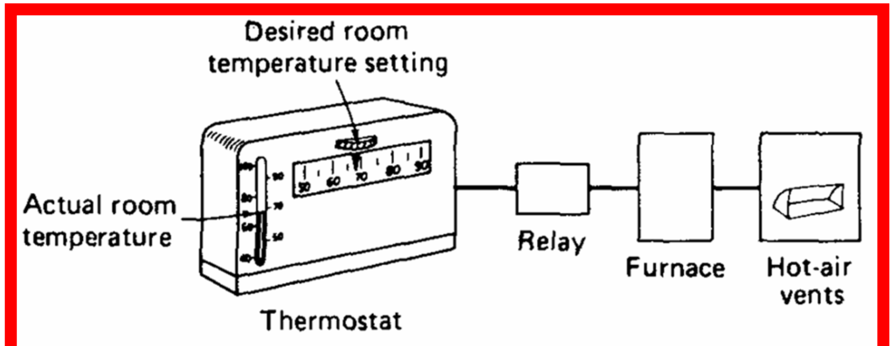


MM1: Basic Concept (II): Control

- **Control** is a process of causing a system (output) variable to conform to some desired status/value
- **Manual Control** is a process where the **control** is handled by human being(s)
- **Automatic Control** is a control process which involves machines only



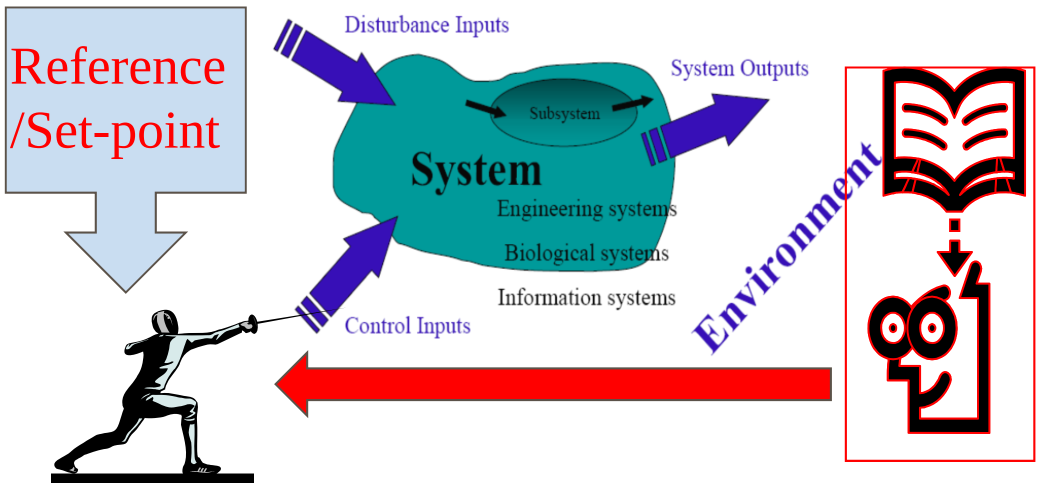
Flow diagram for shower example.



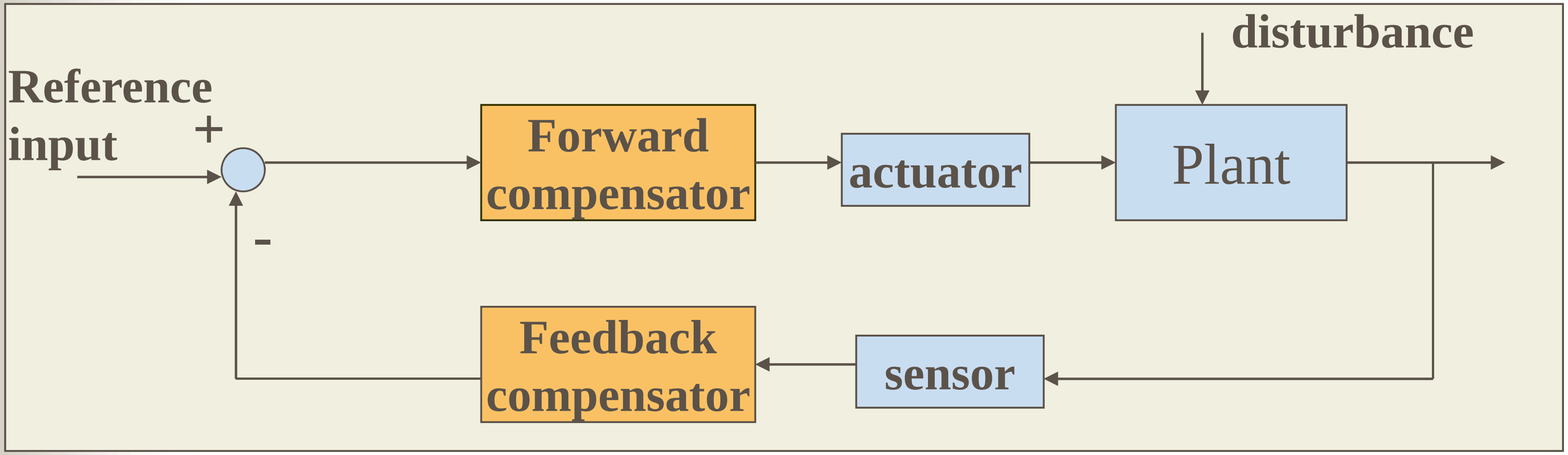
Home heating control system.

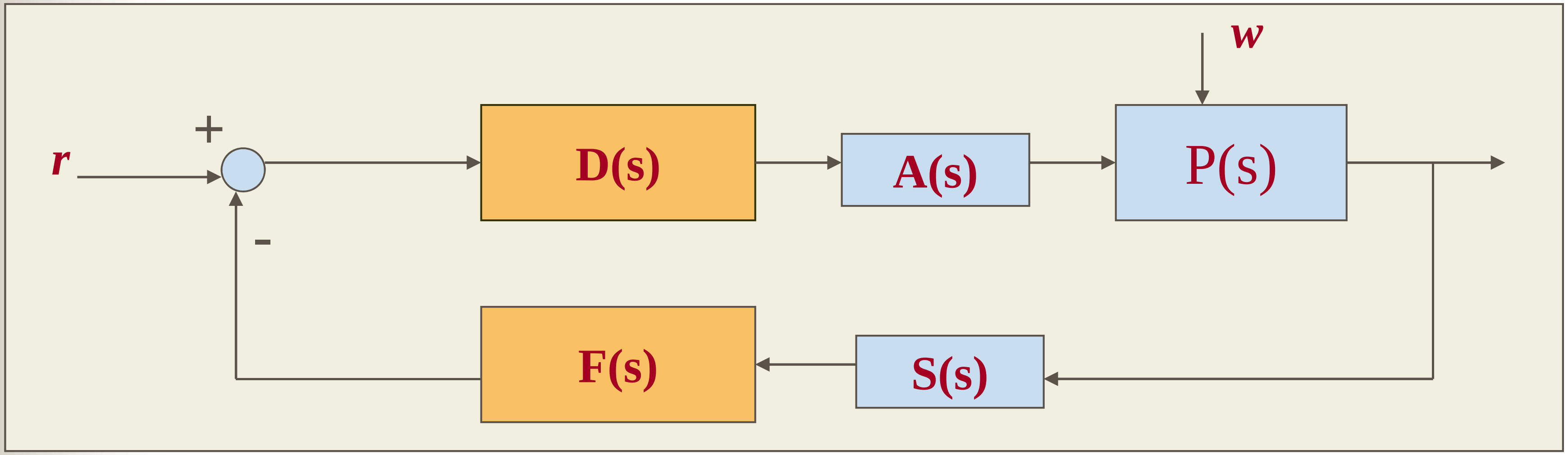
MM1: Control Classification

- **Open-loop Control:** A control process which does not utilize the feedback mechanism, i.e., the output(s) has no effect upon the control input(s)
- **Closed-loop Control:** A control process which utilizes the feedback mechanism, i.e., the output(s) does have effect upon the control input(s)



MM1: Feedback Control – Block Diagrams





The Goals of this lecture (MM2) …

- Essentials in using (ordinary) differential equation model
  - Why use ODE model
  - Linear vs. nonlinear ODE models
  - How to solve an ODE
  - Numerical methods (Matlab)
- Refresh of Laplace transform
  - Key features
  - Transformation from ODE to TF model
- Block diagram transformation
  - Composition /decomposition
  - Signal-flow graph

MM2: ODE Model

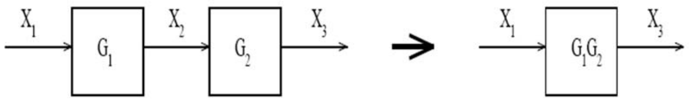
- A general ODE model:

$$a_n \frac{d^n y(t)}{dt^n} + \cdots + a_1 \frac{dy(t)}{dt} + a_0\, y(t) = b_m \frac{d^m u}{dt^m} + \cdots + b_1 \frac{du}{dt} + b_0\, u$$

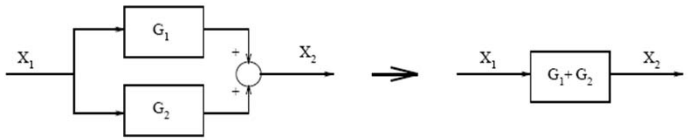
- **SISO, SIMO, MISO, MIMO** models
- Linear system, Time-invance, Linear Time-Invarance (**LTI**)
- Solution of ODE is an explicit description of dynamic behavior
- Conditions for unique solution of an ODE
- Solving an ODE:
  - Time-domain method, e.g., using exponential function
  - Complex-domain method (**Laplace transform**)
  - Numerical solution – CAD methods, e.g., ode23/ode45

MM2: Block diagram Rules

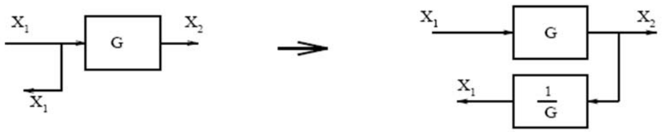
Combining blocks in cascade:



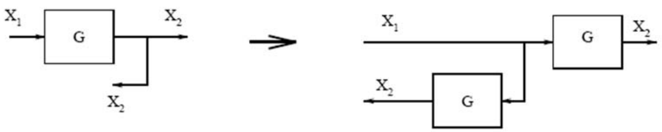
Combining blocks in parallel:



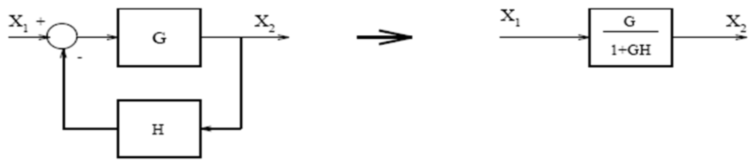
Moving a pickoff point forward:



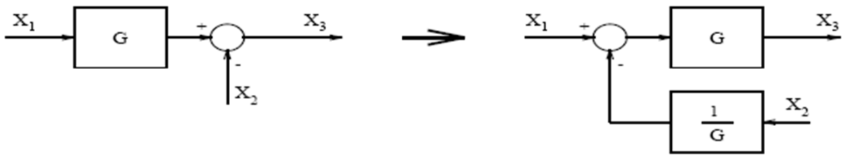
Moving a pickoff point backward:



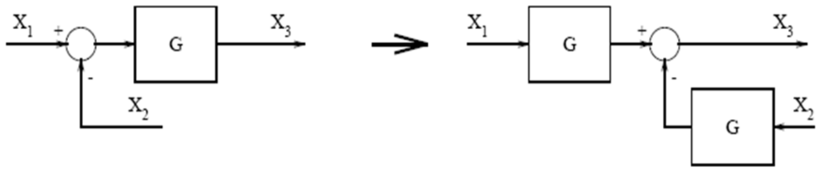
Eliminating a feedback loop:



Moving a summing point backward:



Moving a summing point forward:



MM2: Simulink Block diagram

- System build-up
  - Using TF block
  - Using nonlinear blocks
  - Using math blocks
- Creat subsystems
  - Top-down
  - Bottom-up
- Usage of ode23 & ode45

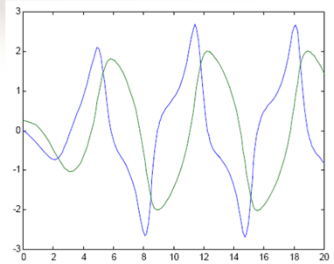
Goals for this lecture (MM3)

- Time response analysis
  - Typical inputs
  - 1st, 2nd and higher order systems

- Performance specification of time response
  - Transient performance
  - Steady-state performance

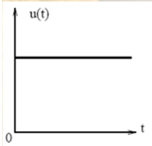
- Numerical simulation of time response

MM3: Time Response Analysis (I)

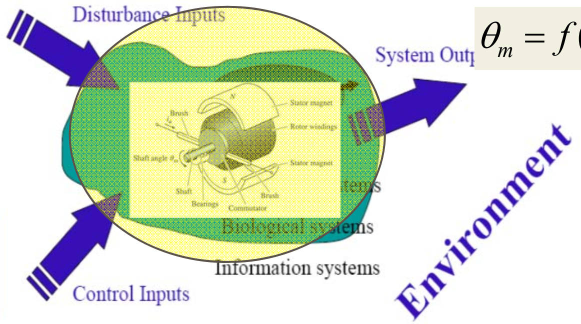


“help ode23” to find out all the other options.

d(t)=0



Typical input u(t)



$$\theta_m = f(\theta_m(0), \dot{\theta}_m(0), v_a, T_e, T_f)$$

Time response y(t)

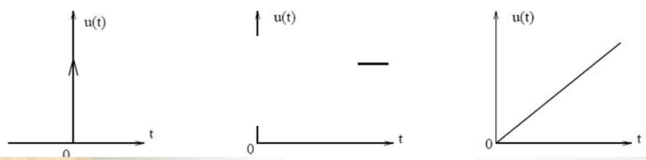
Laplace Trans

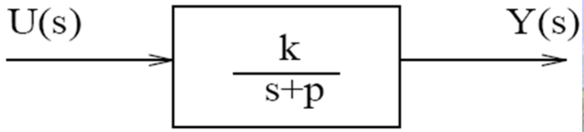
$$\theta_m(0) = \alpha, \quad \dot{\theta}_m(0) = \beta$$

Inv Laplace T.

$$Y(s) = G(s)U(s)$$

$$G(s) = \frac{\sum_{i=0}^{m} b_i s^i}{\sum_{i=0}^{n} a_i s^i}$$





MM3: Time Response Analysis (II)

- Typical inputs: impulse, step and ramp signals
- 1st, 2nd and high-order (LTI) systems

Test signal	$u(t)$	$U(s)$
Impulse	$\delta(t)$	1
Step	1	$1/s$
Ramp	t	$1/s^2$

$G(s) = \dfrac{k}{s+p}$, pole : $-p$, time constant : $\dfrac{1}{p}$

$G(s) = \dfrac{c}{\tau s+1}$, pole : $-\dfrac{1}{\tau}$, time constant : τ

time domain :

$g(t) = ke^{-pt}$ or $g(t) = \dfrac{c}{\tau}e^{-\frac{1}{\tau}t}$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}.$$

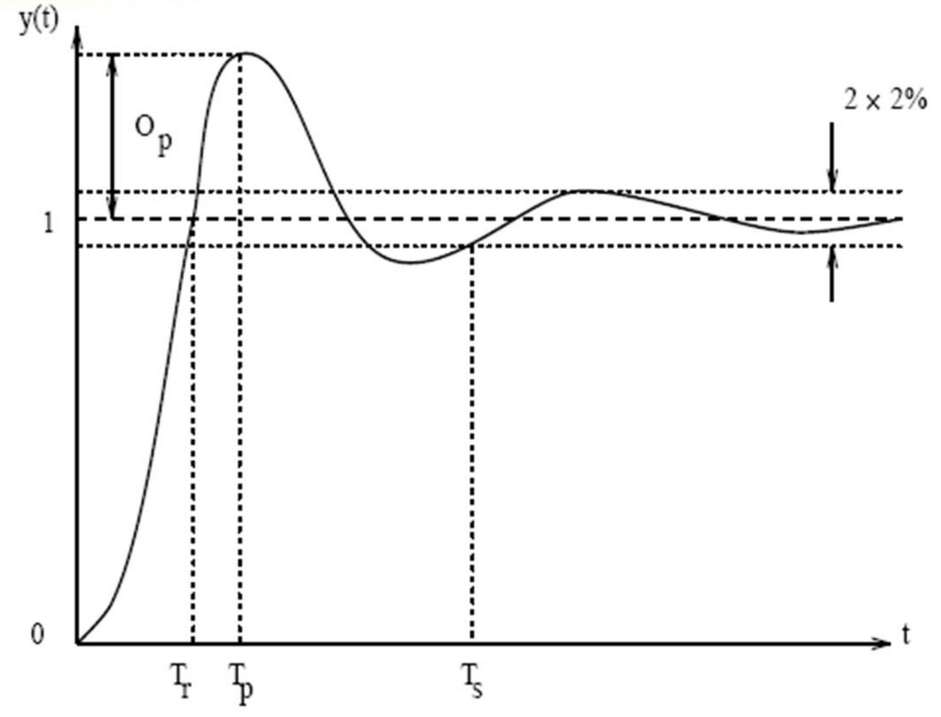
- ξ – *damping ratio*, a dimensionless factor
- ω_n – *natural frequency* with unit rad/s

Time response = excitation response + initial condition response (free response)

$$\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}, \quad \omega_d = \omega\sqrt{1-\xi^2}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

MM3: Performance Specification



$$t_r \cong \frac{1.8}{\omega_n}$$

$$t_s \cong \frac{4.6}{\zeta\omega_n} \cong \frac{4.6}{\sigma}$$

$$M_p \cong \begin{cases} 5\%, & \zeta = 0.7 \\ 16\%, & \zeta = 0.5 \\ 35\%, & \zeta = 0.3 \end{cases}$$

$$t_p \cong \frac{\pi}{\omega_d}, \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

Rise time $T_r = \frac{\pi - \phi}{\omega_d}$

Peak time $T_p = \frac{\pi}{\omega_d}$

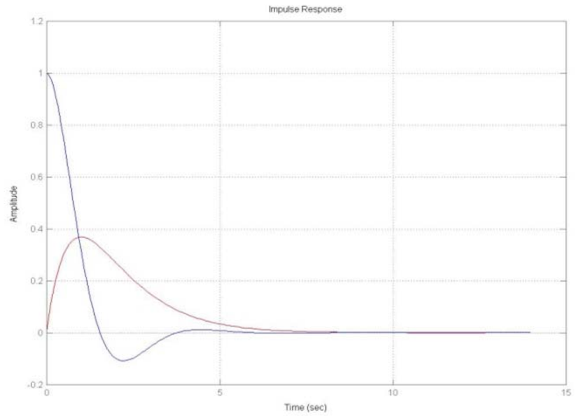
Settling time $T_s \approx \frac{4}{\xi\omega_n}$

Overshoot $O_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$

Steady-state error e_{ss}: difference between input & output as $t \to \infty$

MM3: **Numerical Simulation**

- Impulse response: **impulse(sys)**
- Step response: **step(sys)**
- **ltiview(sys)**
- **Subplot(m,n,1)**

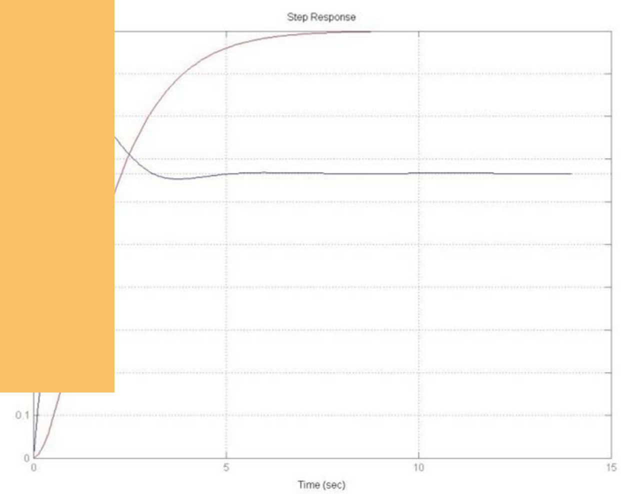


EXAMPLE:

sys1:
num1=[1];
den1=[1 2 1];

Sys2:
num2=[1 2];
den2=[1 2 3];

```
impulse(tf(num1,den1),'r',tf(num2,den2),'b')
step(tf(num1,den1),'r',tf(num2,den2),'b')
```



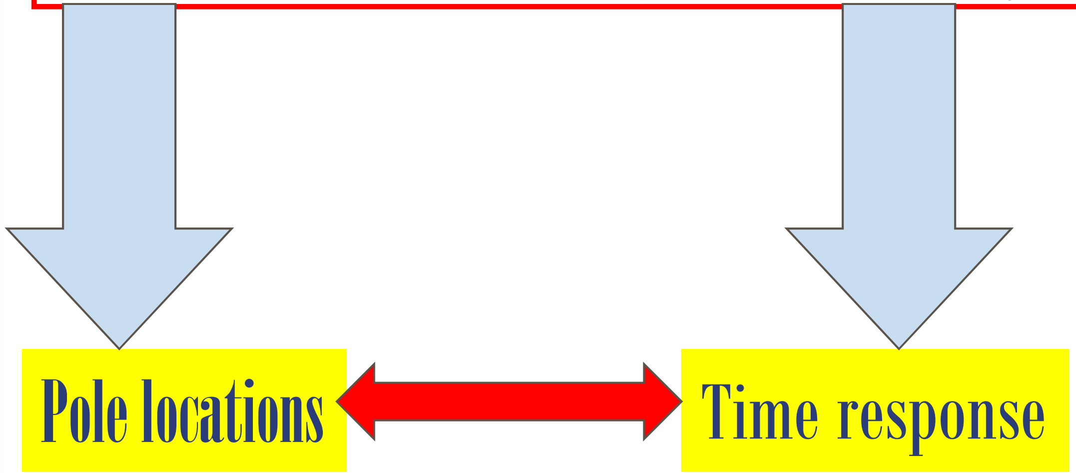
Goals for this lecture (MM4)

- **System poles vs. time responses**
  - **Poles and zeros**
  - **Time responses vs. Pole locations**
- Feedback characteristics
  - Characteristics
  - A simple feedback design
- Block diagram decomposition (simulink)

MM4 : Poles vs Performance

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}.$$

- ξ – *damping ratio*, a dimensionless factor
- ω_n – *natural frequency* with unit rad/s



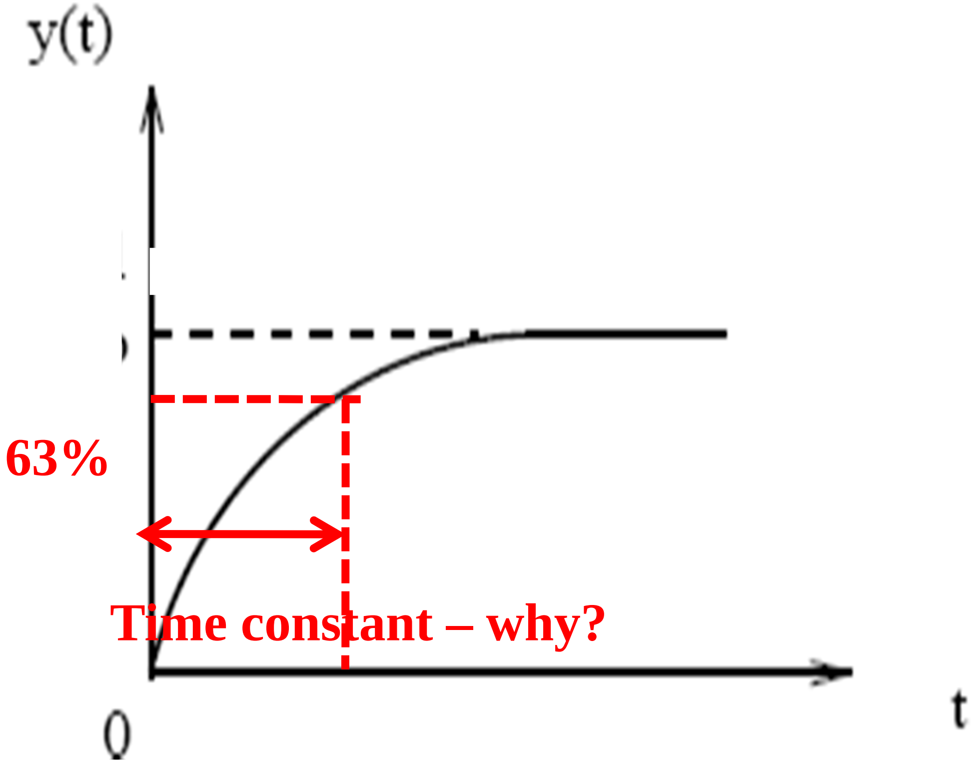
MM4: First-order System

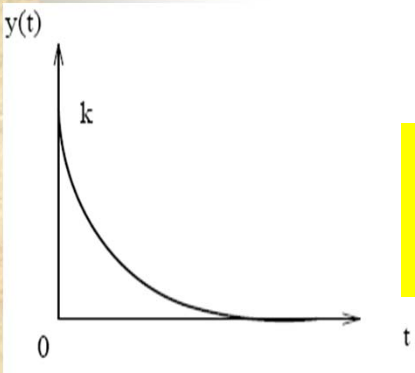
$$G(s)=\frac{1}{\tau s+1},\qquad \text{assume}\quad \tau>0$$

$$\text{pole:}\quad -\frac{1}{\tau},\quad \text{time constant: } \tau,$$

$$\text{Impulse response:}\quad y(t)=L\left(\frac{1}{\tau s+1}\right)=e^{-\frac{1}{\tau}t}$$

$$\text{Step response:}\quad y(t)=L\left(\frac{1}{s(\tau s+1)}\right)=1-e^{-\frac{1}{\tau}t}$$





Time response is determined by the time constant

System pole is the negative of inverse time constant

MM4: Second-Order System

$$G(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n+\omega_n^2},\quad \text{assume } \omega_n>0,\quad \xi\ge 0$$

poles: $p_{1,2}=-\xi\omega_n\pm\omega_n\sqrt{\xi^2-1}$

real (different) poles: $p_{1,2}=-\xi\omega_n\pm\omega_n\sqrt{\xi^2-1},\quad$ if $\xi>1$

real (identical) poles: $p_{1,2}=-\xi\omega_n,\quad$ if $\xi=1$

complex poles: $p_{1,2}=-\xi\omega_n\pm j\omega_n\sqrt{1-\xi^2},\quad$ if $0<\xi<1$

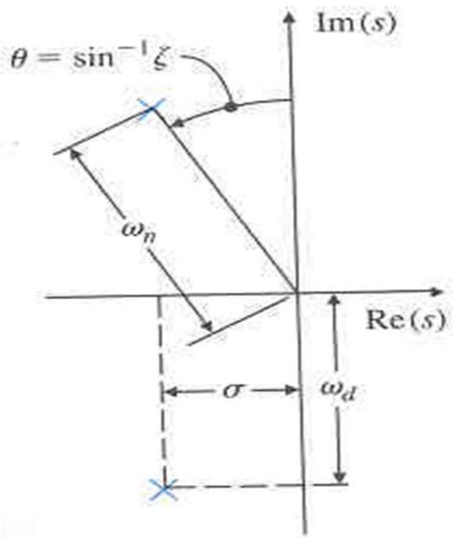
complex poles: $p_{1,2}=\pm j\omega_n,\quad$ if $\xi=0$

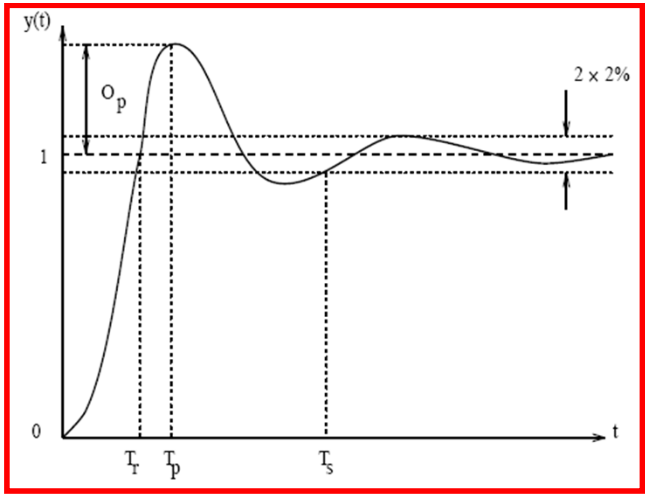
$$t_r\cong\frac{1.8}{\omega_n}$$

$$t_s\cong\frac{4.6}{\zeta\omega_n}\cong\frac{4.6}{\sigma}$$

$$M_p\cong\begin{cases}5\%, & \zeta=0.7\\ 16\%, & \zeta=0.5\\ 35\%, & \zeta=0.3\end{cases}$$

$$t_p\cong\frac{\pi}{\omega_d},\quad \omega_d=\omega_n\sqrt{1-\zeta^2}$$





Rise time $T_r=\dfrac{\pi-\phi}{\omega_d}$

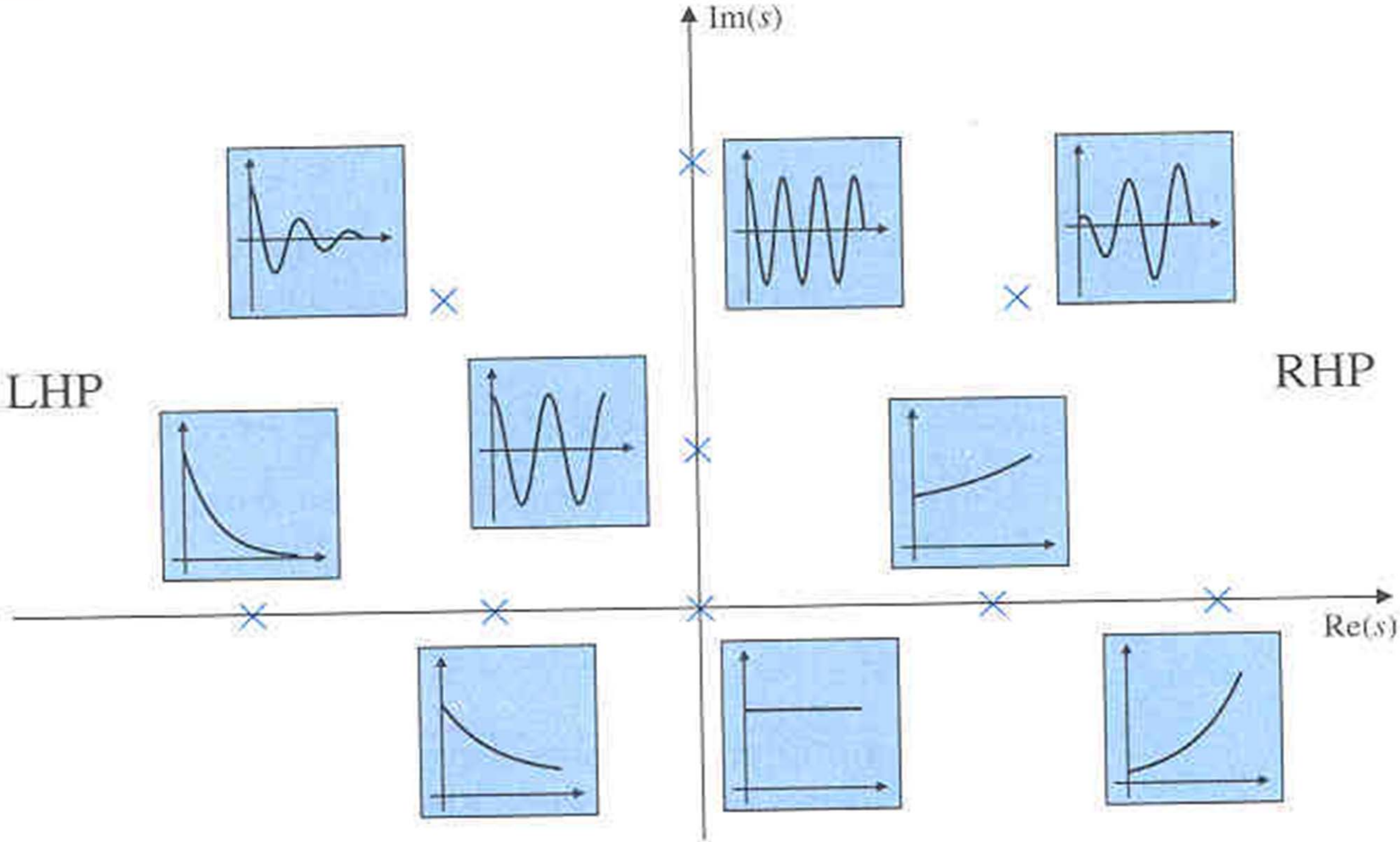
Peak time $T_p=\dfrac{\pi}{\omega_d}$

Settling time $T_s\approx\dfrac{4}{\xi\omega_n}$

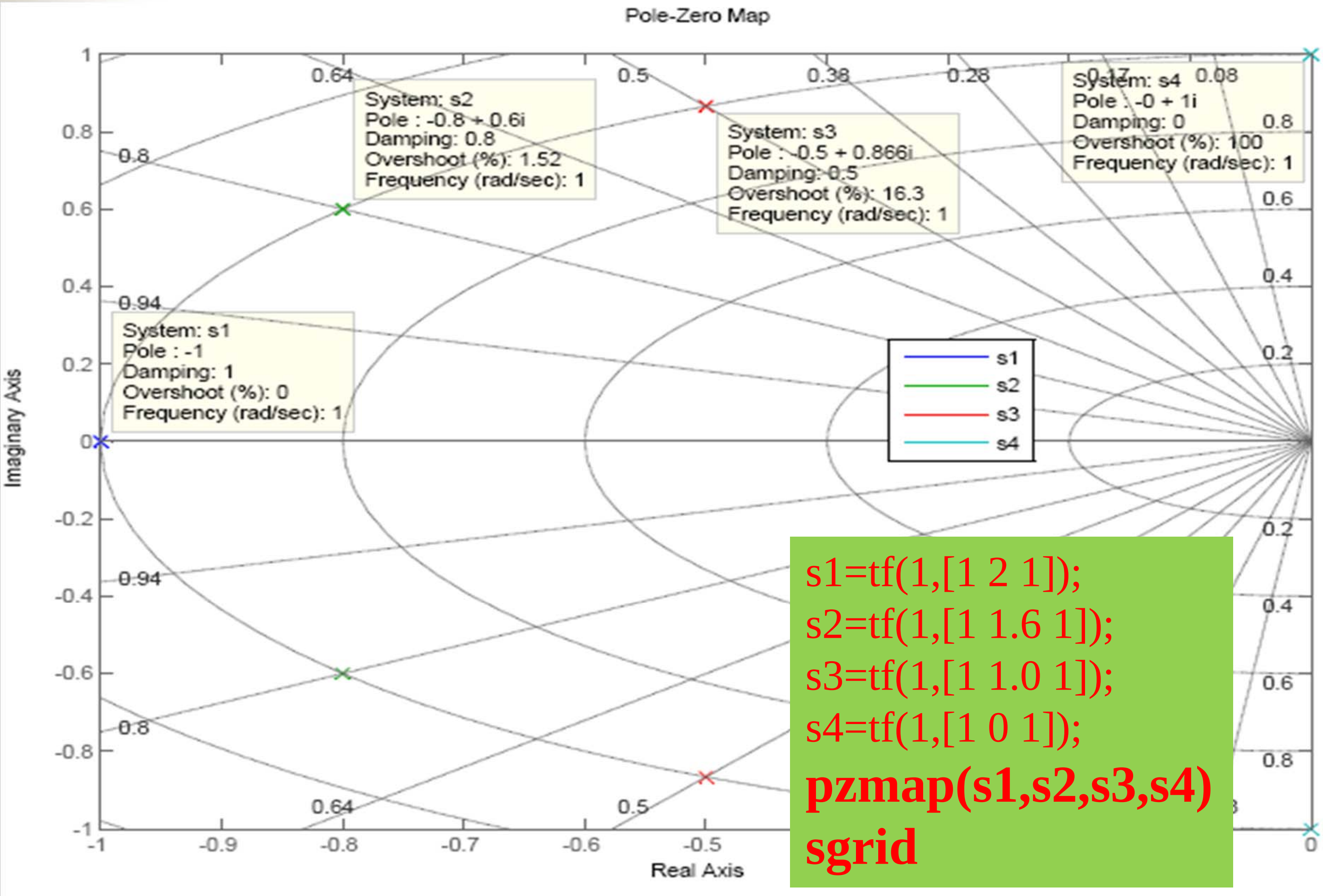
Overshoot $O_p=e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$

$$\phi=\tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi},\quad \omega_d=\omega\sqrt{1-\xi^2}$$

MM4: Summary of Pole vs Performance



MM4: Plot of Pole Locations




```
s1=tf(1,[1 2 1]);
s2=tf(1,[1 1.6 1]);
s3=tf(1,[1 1.0 1]);
s4=tf(1,[1 0 1]);
```

pzmap(s1,s2,s3,s4)

sgrid

Goals for this lecture (MM5)

- **Stability analysis**
  - **Definition of BIBO**
  - **Pole locations**
  - **Routh criteron**
- Steady-state errors
  - Final Theorem
  - DC-Gain
  - Stead-state errors
- Effects of zeros and additional poles

MM5 : BIBO Stability

- A system is said to have **bounded input-bounded output (BIBO) stability** if every bounded input results in a bounded output (regardless of what goes on inside the system)
- The continuous (LTI) system with impuse response **h(t)** is BIBO stable if and only if **h(t)** is absolutely integrallable
- All system poles locate in the left half s-plane - **asymptotic internal stability**
- **Routh Criterion:** For a stable system, there is no changes in sign and no zeros in the first column of the Routh array

MM5 : Steady-State Error

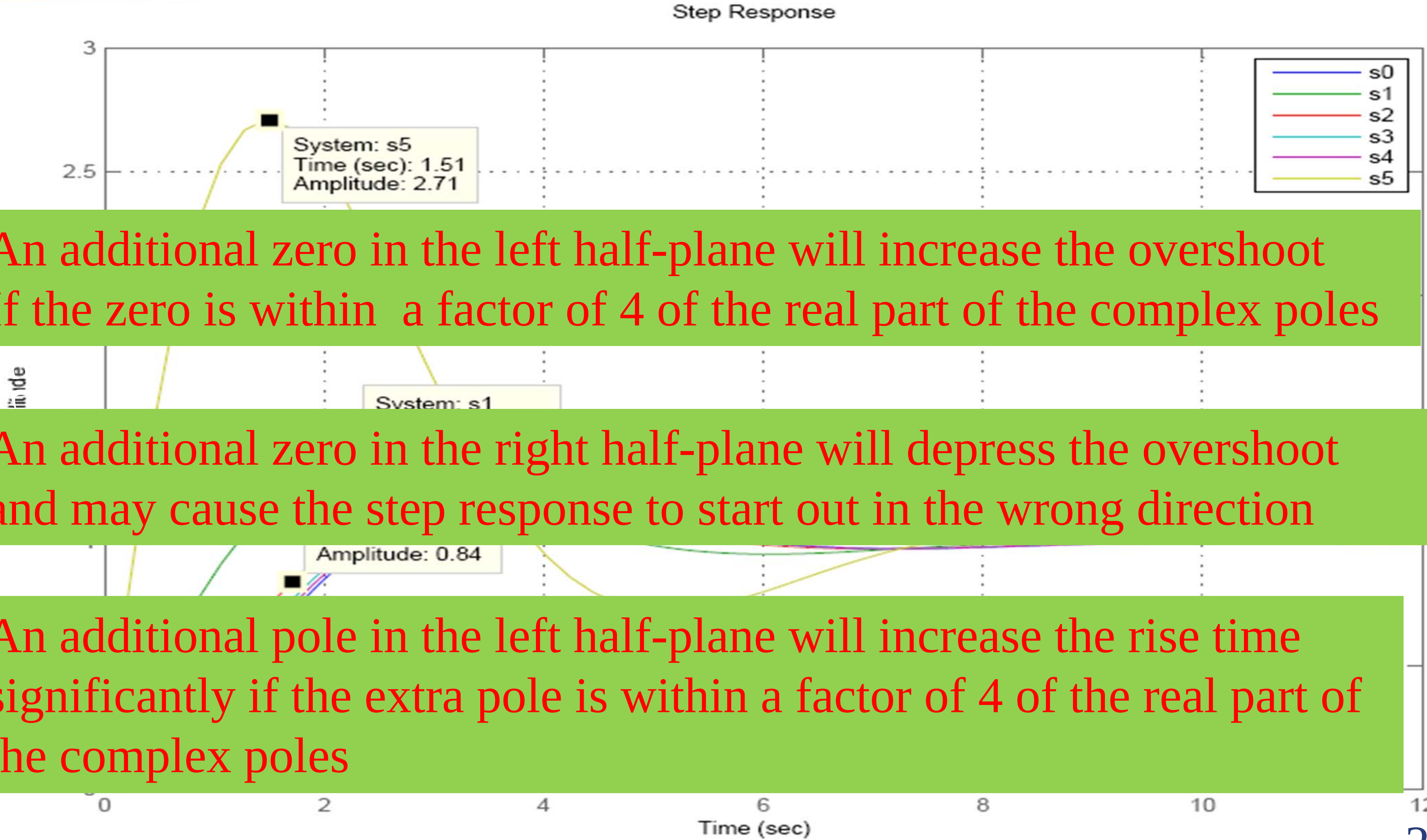
- **Objective:** to know whether or not the response of a system can approach to the reference signal as time increases
- **Assumption:** The considered system is stable
- **Analysis method:** Transfer function + final-value Theorem

$$e(\infty) = \lim_{s\to 0} s(R(s) - Y(s)) = \lim_{s\to 0} s(R(s) - G(s)R(s))$$
$$= \lim_{s\to 0} s(1 - G(s))R(s), \quad R(s) = \frac{1}{s}$$
$$= \lim_{s\to 0} (1 - G(s)) = 1 - G(0)$$

DC-Gain

- **Position-error constant** $K_p = \lim_{s\to 0} G_o(s)$
- **Velocity constant** $K_v = \lim_{s\to 0} sG_o(s)$
- **Acceleration constant** $K_a = \lim_{s\to 0} s^2 G_o(s)$

MM5 : Effect of Additional Zero & Pole

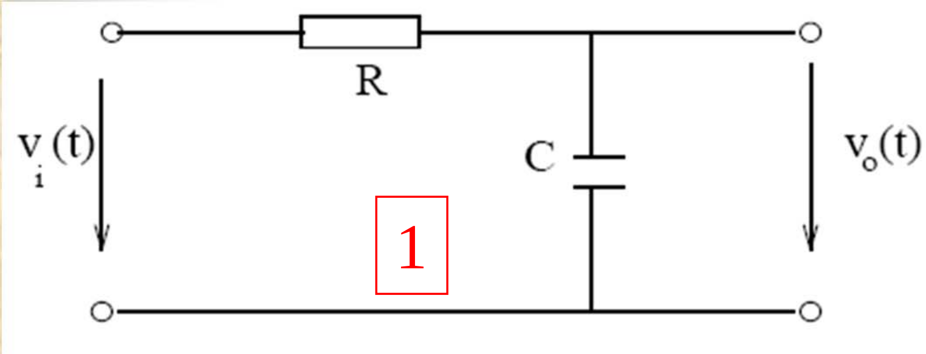


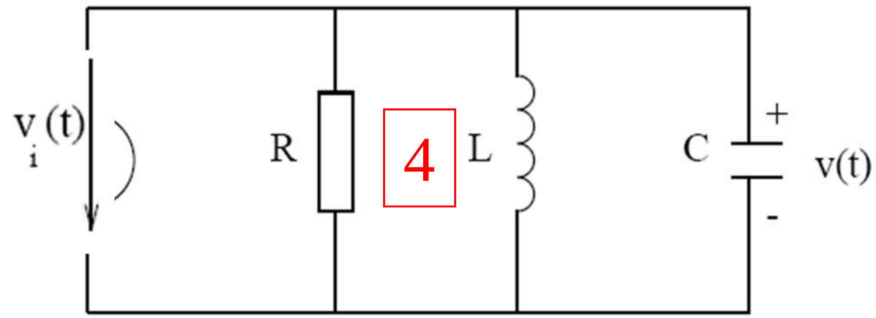
An additional zero in the left half-plane will increase the overshoot If the zero is within a factor of 4 of the real part of the complex poles

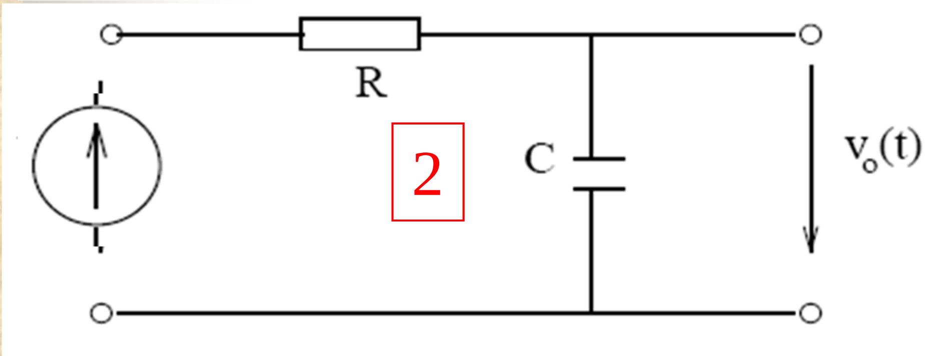
An additional zero in the right half-plane will depress the overshoot and may cause the step response to start out in the wrong direction

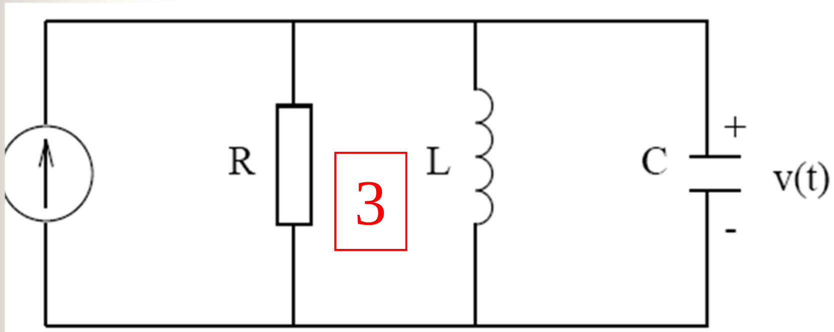
An additional pole in the left half-plane will increase the rise time significantly if the extra pole is within a factor of 4 of the real part of the complex poles

BIBO Stability – **Execise (I)**





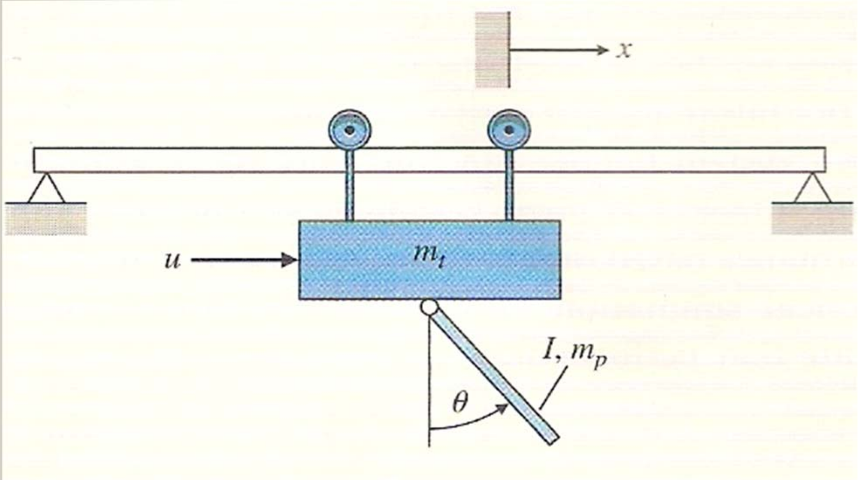




- **Are these systems BIBO stable?**
- Intuitive explanation
- Theoretical analysis

BIBO Stability – **Execise (II)**

- How about the stability of your project systems?

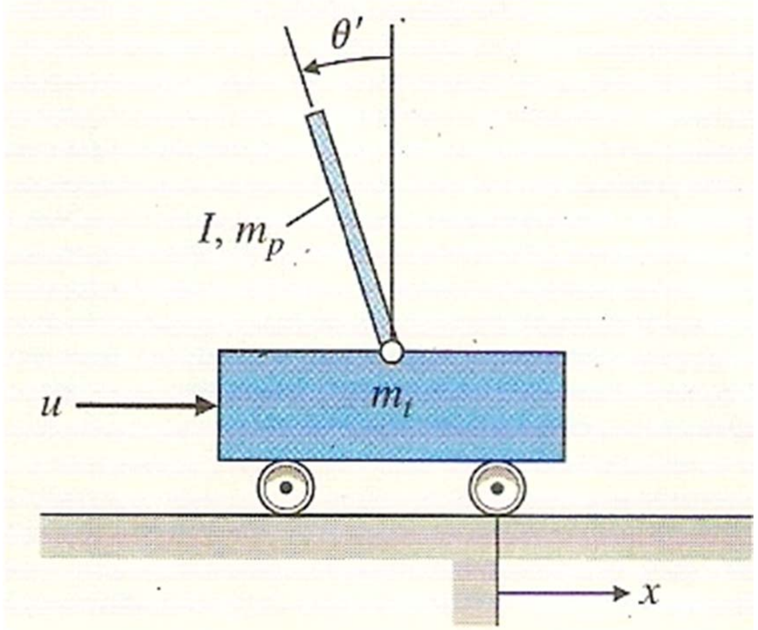


$$(I + m_p l^2)\ddot{\theta} + m_p g l\theta = -m_p l\ddot{x}$$

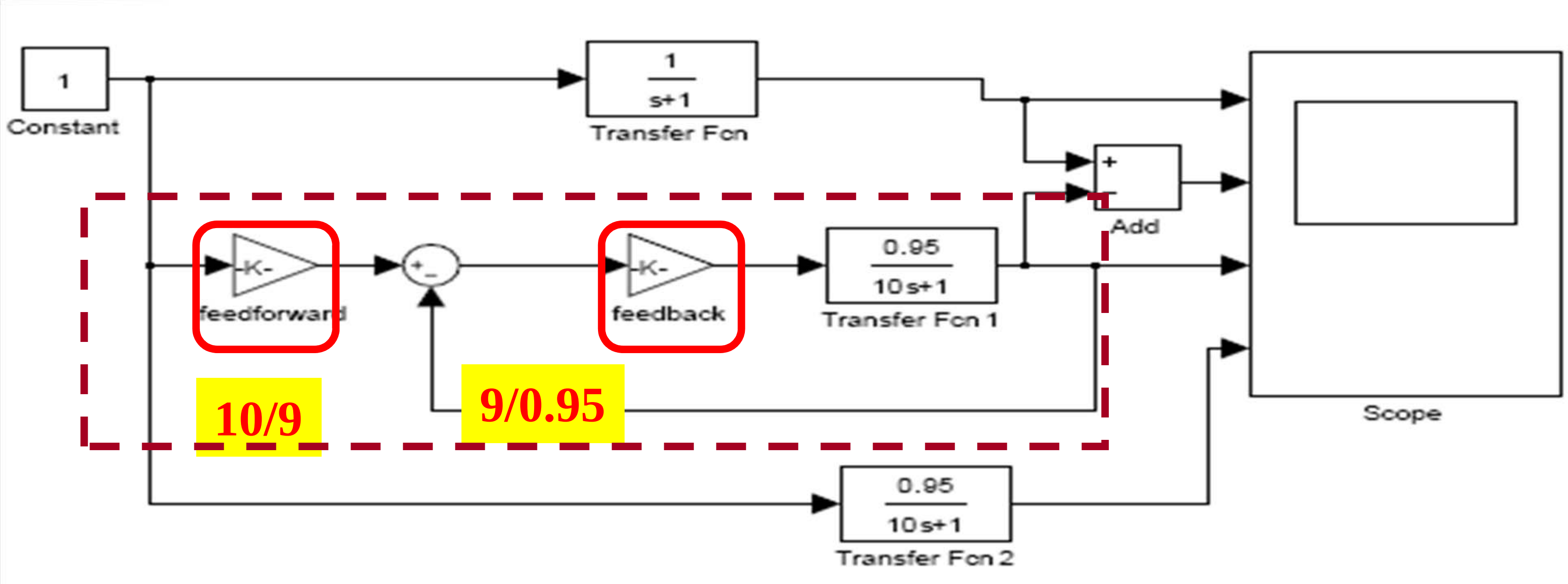
$$(m_t + m_p)\ddot{x} + b\dot{x} + m_p l\ddot{\theta} = u.$$

$$(I + m_p l^2)\ddot{\theta}' - m_p g l\theta' = m_p l\ddot{x}$$

$$(m_t + m_p)\ddot{x} + b\dot{x} - m_p l\ddot{\theta}' = u.$$



Revisit of example: First-order System (II)



- What's the tpye of original system?
- Derive the transfer function of the closed-loop system
- What's the time constant and DC-gain of the CL system?
- What's the feedforward gain so that there is no steady-state error?

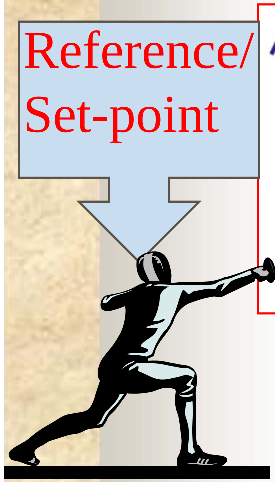
Goals for this lecture (MM6)

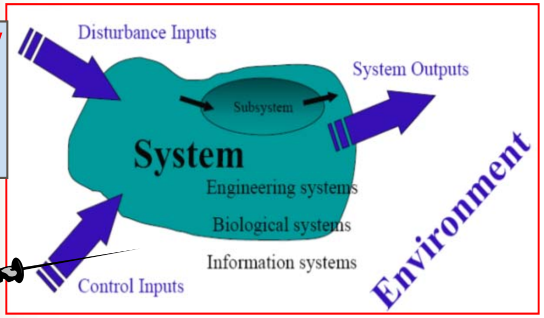
- **Definition characterisitc of PID control**
  - **P- controller**
  - **PI- controller**
  - **PID controller**

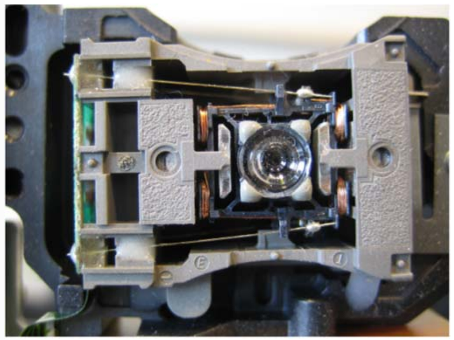
- Ziegler-Nichols tuning methods
  - Quarter decay ratio method
  - Ultimate sensitivity method

Control objectives

Control is a process of causing a system (output) variable to conform to some desired status/value (**MM1**)







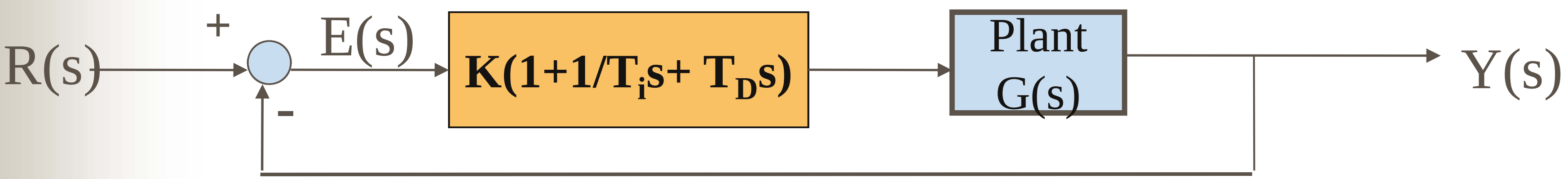
Control Objectives

- **Stable (MM5)**
- **Quick responding (MM3, 4)**
- **Adequate disturbance rejection**
- **Insensitive to model & measurement errors**
- **Avoids excessive control action**
- **Suitable for a wide range of operating conditions**

(extra readings: Goodwin's lecture)

MM6: Characteristics of PID Controllers

- Proportional gain, K_p larger values typically mean faster response. An excessively large proportional gain will lead to process instability and oscillation.
- Integral gain, K_i larger values imply steady state errors are eliminated more quickly. The trade-off is larger overshoot
- Derivative gain, K_d larger values decrease overshoot, but slows down transient response and may lead to instability due to signal noise amplification in the differentiation of the error.



MM6: PID Tuning Methods- Trial-Error

Rules of thumb:

$$K_p > K_i > K_d,$$
$$K_p \approx (5 \sim 10) K_i,$$
$$K_i \approx (5 \sim 10) K_d$$

Procedure:

Step 1: Set $K_i = 0$ & $K_d = 0$. Increase K_p from zero;

Step 2: Fix K_p. Increase K_i from zero;

Step 3: Fix K_p & K_i. Increase K_d from zero.

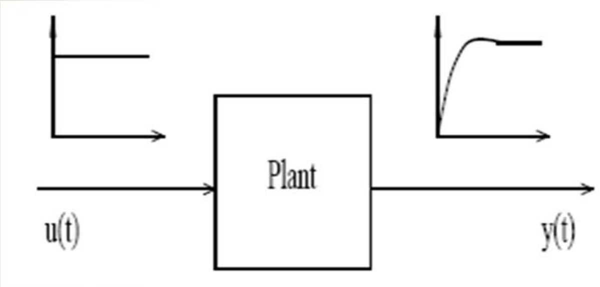
Note: Several iterations of the procedure may be necessary

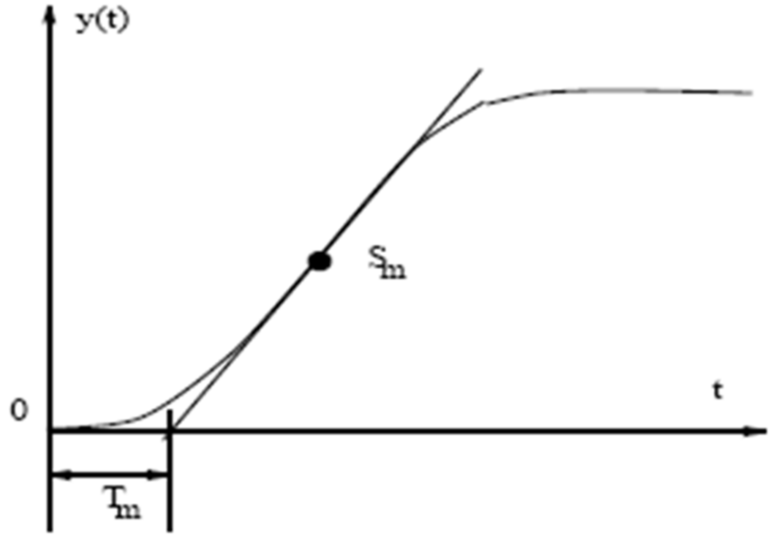
- Adavantages: Simple
- Disadvantages:
  - unsatisfactory performance
  - expensive on-site experiment
  - issues of equipment safety

See Hou Ming's lexture notes

MM6: PID Tuning – Zieglor Niechols (I)

- **Pre-condition**: system has no overshoot of step response



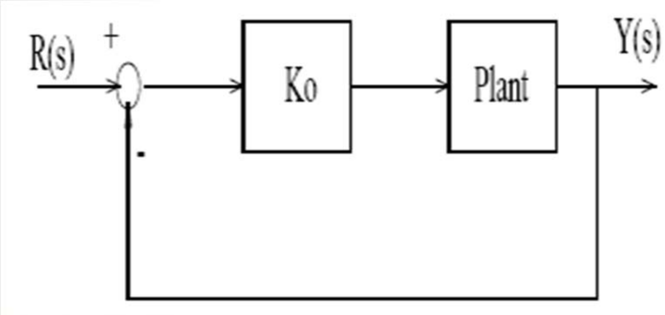


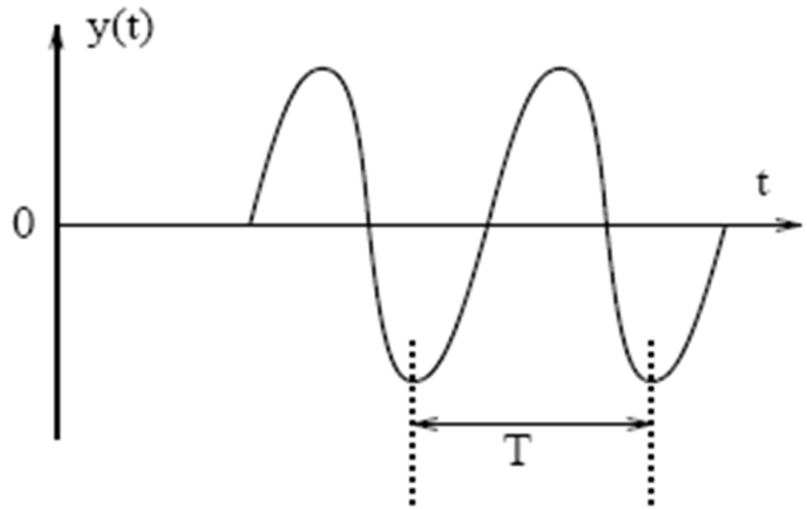
Control	K_p	K_i	K_d
P	$\frac{1}{S_m T_m}$	0	0
PI	$\frac{0.9}{S_m T_m}$	$\frac{3}{S_m}$	0
PID	$\frac{1.2}{S_m T_m}$	$\frac{2.4}{S_m}$	$\frac{0.6}{S_m}$

See Hou Ming’s lexture notes

MM6: PID Tuning – Zieglor Niechols (II)

- **Pre-condition**: system order > 2





Control	K_p	K_i	K_d
P	$\frac{K_o}{2}$	0	0
PI	$\frac{9K_o}{20}$	$\frac{3K_oT_o}{8}$	0
PID	$\frac{3K_o}{5}$	$\frac{3K_oT_o}{10}$	$\frac{3K_oT_o}{40}$

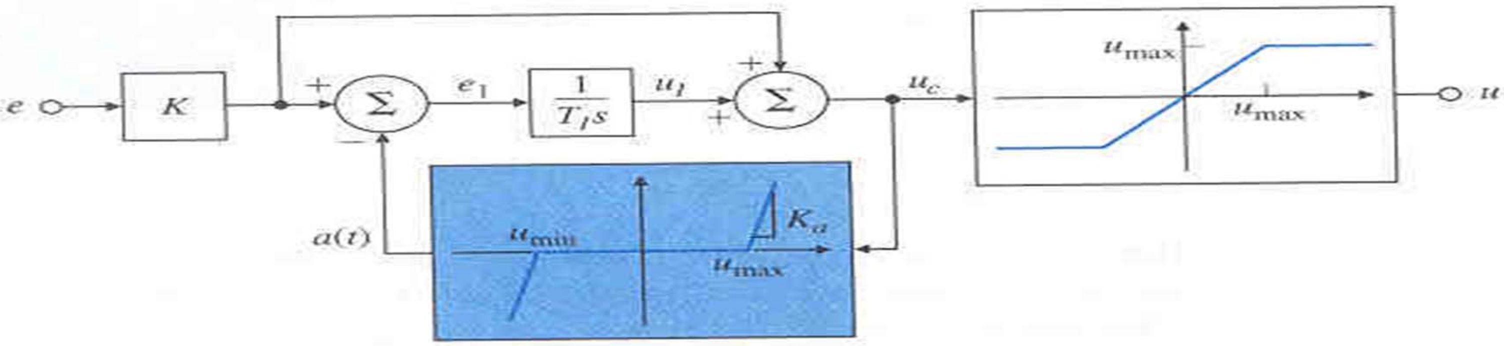
See Hou Ming's lexture notes

Goals for this lecture (MM7)

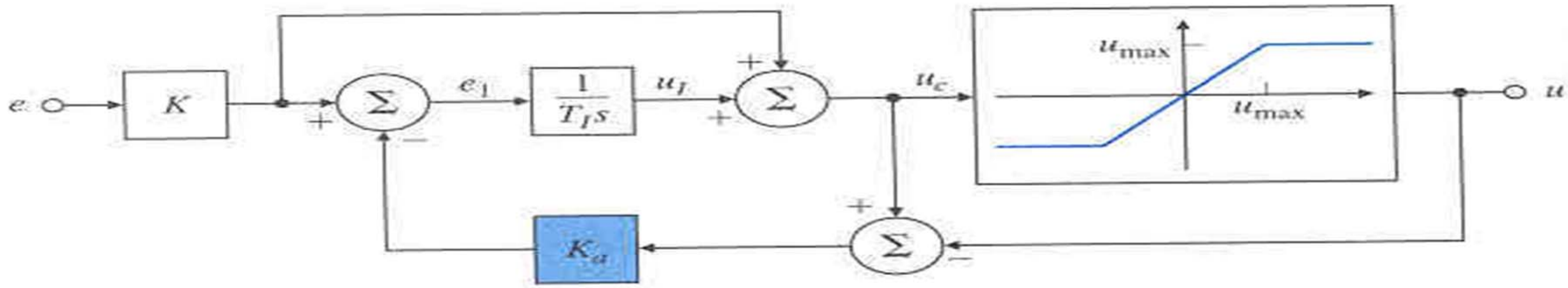
Some **practical issues** when developing a PID controler:

- **Integral windup & Anti-windup methods**
- Derivertive kick
- When to use which controller?
- Operational Amplifier Implementation
- Other tuning methods

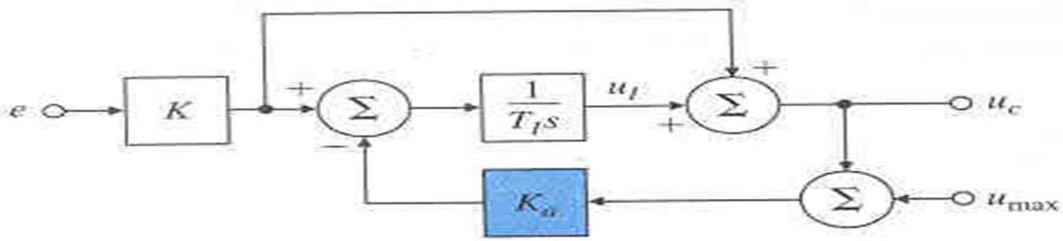
Anti-windup Techniques



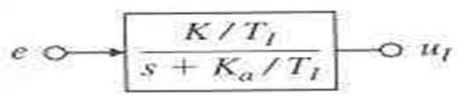
(a)



(b)



(c)



(d)

$$u(t) = K(e(t) + \frac{1}{T_I}\int_{t_0}^{t} e(\tau)d\tau + T_D\dot{y}(t))$$

$$U(s) = K(1 + \frac{1}{T_I s})E(s) + T_D sY(s)$$

Derivative Kick

- **Derivative kick**: if we have a setpoint change, a spike will be caused by D controller, which is called derivative kick.
- Derivative kick can be removed by replacing the derivative term with just output (y), instead of (r_{set}-y)
- Derivative kick can be reduced by introducing a lowpass filter before the set-point enters the system
- The bandwidth of the filter should be much larger than the closed-loop system's bandwidth

$$G(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \qquad \text{(1st order)}$$

Cohen-Coon Tuning Method

- **Pre-condition:** first-order system with some time delay
- **Objective:** ¼ decay ratio & minimum offset

	k_c	T_i	T_D
P	$\frac{1}{k_p}\frac{\tau}{\theta}(1+\frac{\theta}{3\tau})$		
PI	$\frac{1}{k_p}\frac{\tau}{\theta}(\frac{9}{10}+\frac{\theta}{12\tau})$	$\theta\frac{30+3(\theta/\tau)}{9+20(\theta/\tau)}$	
PID	$\frac{1}{k_p}\frac{\tau}{\theta}(\frac{4}{3}+\frac{\theta}{4\tau})$	$\theta\frac{32+6(\theta/\tau)}{13+8(\theta/\tau)}$	$\theta\frac{4}{11+2(\theta/\tau)}$

In the table k_p is the process gain, τ the process time constant and θ the process time delay.

$$G(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \qquad \text{(1st order)}$$

Table 12.3 Controller Design Relations Based on the ITAE Performance Index and a First-Order plus Time-Delay Model

Type of Input	*Type of Controller*	*Mode*	*A*	*B*
Load	PI	P	0.859	−0.977
		I	0.674	−0.680
Load	PID	P	1.357	−0.947
		I	0.842	−0.738
		D	0.381	0.995
Set point	PI	P	0.586	−0.916
		I	1.03[b]	−0.165[b]
Set point	PID	P	0.965	−0.85
		I	0.796[b]	−0.1465[b]
		D	0.308	0.929

[a]Design relation: $Y = A(\theta/\tau)^B$ where $Y = KK_c$ for the proportional mode, τ/τ_I for the integral mode, and τ_D/τ for the derivative mode.
[b]For set-point changes, the design relation for the integral mode is $\tau/\tau_I = A + B(\theta/\tau)$. [8]

Goals for this lecture (MM8)

Essentials for **frequency domain** design methods – **Bode plot**

- **Bode plot analysis**
  - **How to get a Bode plot**
  - **What we can gain from Bode plot**
- How to use bode plot for design purpose
  - Stability margins (Gain margin and phase margin)
  - Transient performance
  - Steady-state performance
- Matlab functions: bode(), margin()

Frequency Response

- The frequency response **$G(j\Omega)$ $(=G(s)|_{s=j\Omega})$** is a representation of the system's response to sinusoidal inputs at varying frequencies

$$G(j\Omega) = |G(j\Omega)|\, e^{\triangleleft G(j\Omega)},$$

- Input $x(n)$ and output $y(n)$ relationship

$$|Y(j\Omega)| = |H(j\Omega)|\, |X(j\Omega)|$$

$$\triangleleft Y(j\Omega) = \triangleleft H(j\Omega) + \triangleleft X(j\Omega)$$

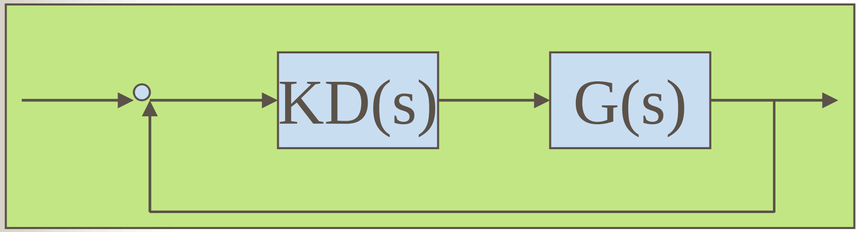
- The frequency response of a system can be viewed
  - via the **Bode plot** (H.W. Bode 1932-1942)
  - via the **Nyquist diagram**

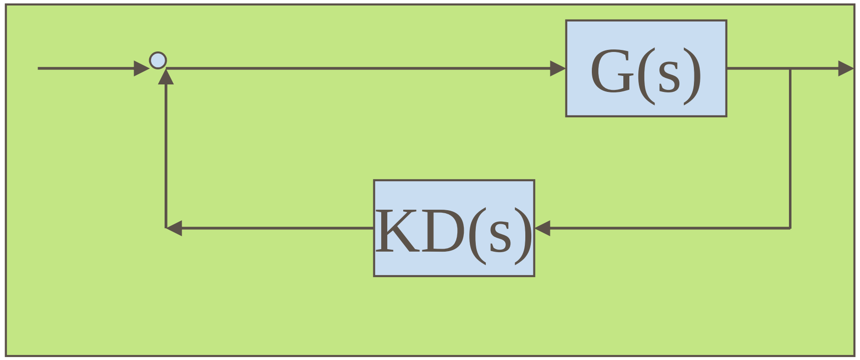
Open-Loop Transfer Function

- **Motivation**

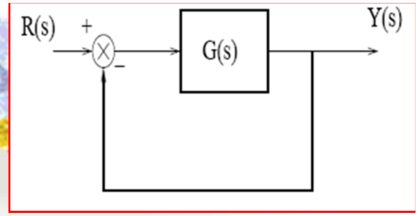
  Predict the closed-loop system's properties using the open-loop system's frequency response

- **Open-loop TF (Loop gain) :** **$L(s)=KD(s)G(s)$**





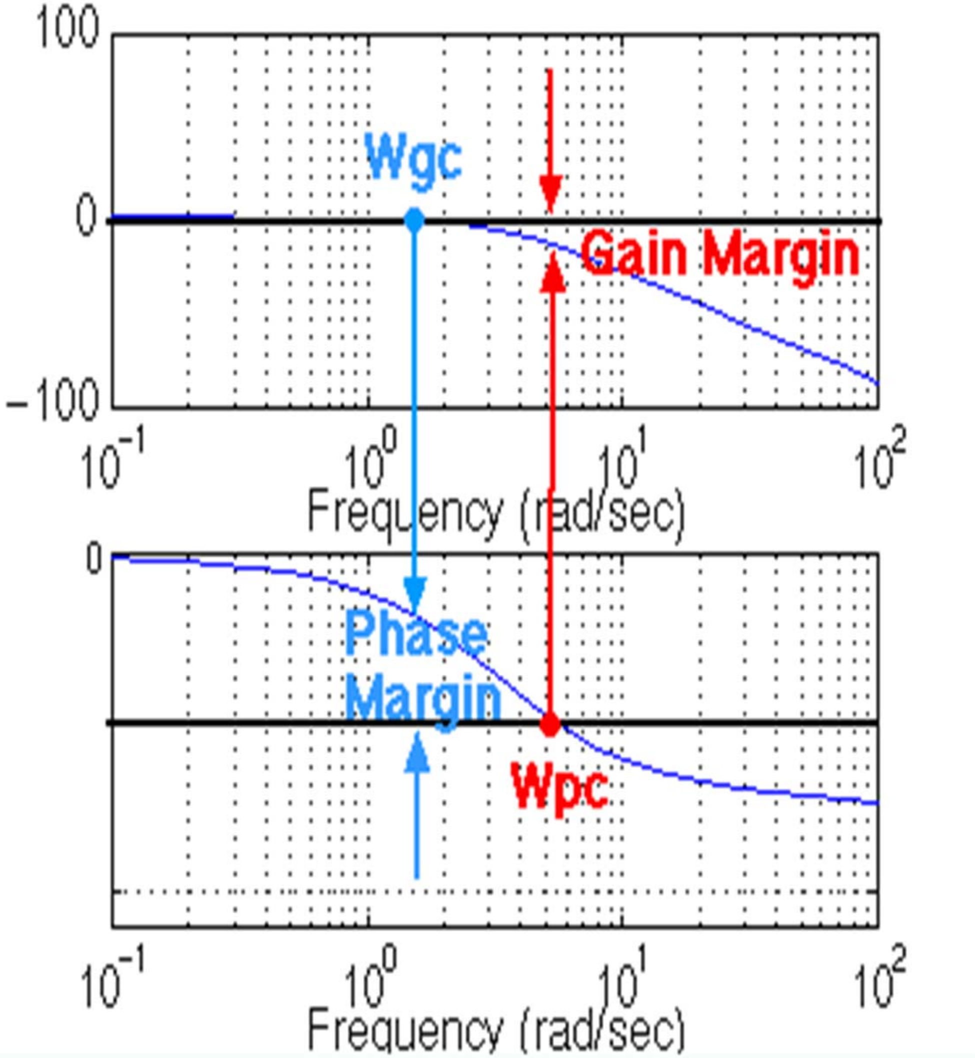
- **Closed-loop:** **$G_{cl}(s)=L(s)/(1+L(s))$, or $G_{cl}(s)=G(s)/(1+L(s))$**



Definition of Phase Margin (PM)

- Bode plot of the open-loop TF

- The **phase margin** is the difference in phase between the phase curve and -180 deg at the point corresponding to the frequency that gives us a gain of 0dB (the **gain cross over frequency**, Wgc).



Remarks of Using Bode Plot

- **Precondition:** The system must be stable in open loop if we are going to design via Bode plots
- **Stability:** If the gain crossover frequency is less than the phase crossover frequency (i.e. Wgc < Wpc), then the closed-loop system will be stable
- **Damping Ratio:** For second-order systems, the closed-loop damping ratio is approximately equal to the phase margin divided by 100 if the phase margin is between 0 and 60 deg
- A very rough estimate that you can use is that the bandwidth is approximately equal to the **natural frequency**

Goals for this lecture (MM9)

- **A design example based on Bode plot**
  - **Open-loop system feature analysis**
  - **Bode plot based design**

- Nyquist Diagram
  - What's Nyquist diagram?
  - What we can gain from Nyquist diagram

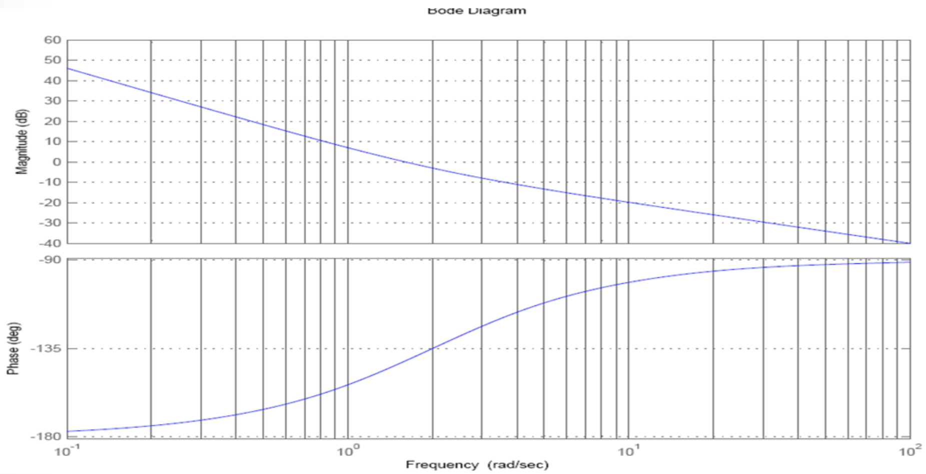
- Matlab functions: nyquist()

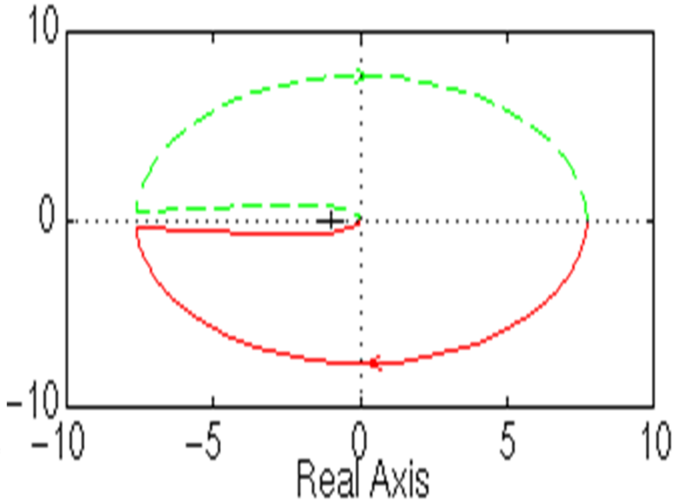
Nyquist Diagram: **Definition**

The Nyquist diagram is a plot of **G(jΩ)** , where **G(s)** is the open-loop transfer function and **Ω** is a vector of frequencies which encloses the entire right-half plane

$$G(j\Omega) = |G(j\Omega)|\, e^{\angle G(j\Omega)},$$

- The Nyquist diagram plots the position its the complex plane , while the Bode plot plots its magnitude and phase separately.





Nyquist Criterion for Stability (MM9)

The Nyquist criterion states that:

- P = the number of open-loop (unstable) poles of G(s)H(s)
- N = the number of times the Nyquist diagram encircles –1
  - clockwise encirclements of -1 count as positive encirclements
  - counter-clockwise (or anti-clockwise) encirclements of -1 count as negative encirclements
- Z = the number of right half-plane (positive, real) poles of the closed-loop system
- The important equation:

$$Z = P + N$$

Goals for this lecture (MM10)

- **An illustrative example**
  - **Frequency response analysis**
  - **Frequency response design**
- Lead and lag compensators
  - What's a lead/lag compensator?
  - Their frequency features
- A systematical procedure for lead compensator design
- A practical design example – Beam and Ball Control

What have we talked in lecture (MM10)?

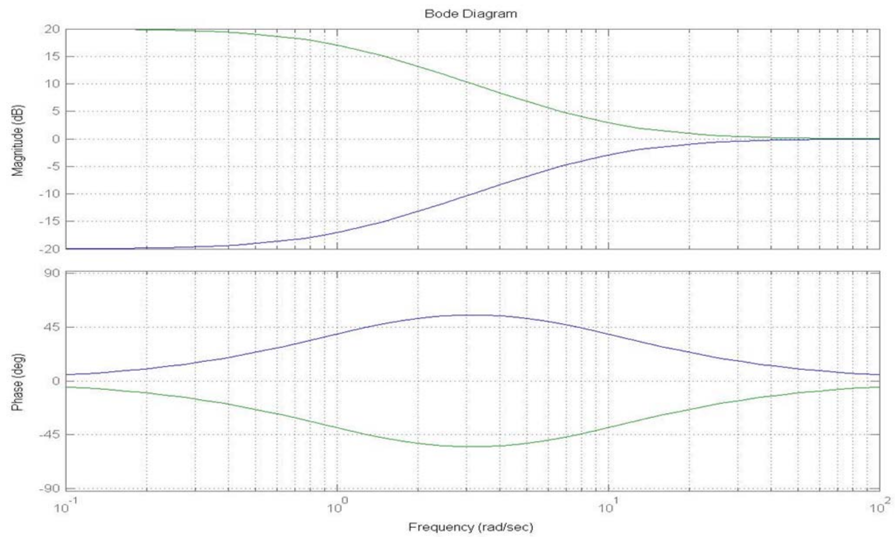
- Lead and lag compensators

  D(s)=(s+z)/(s+p)

  with z < p or z > p

  D(s)=K(Ts+1)/(αTs+1),

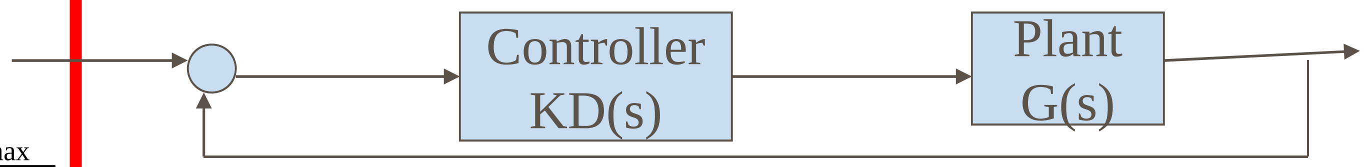
  with α <1 or α >1



- A systematical procedure for lead compensator design

$$\omega_{max} = \frac{1}{T\sqrt{\alpha}}$$

$$\alpha = \frac{1 - \sin \beta_{max}}{1 + \sin \beta_{max}}$$



Exercise

Could you repeat the antenna design using

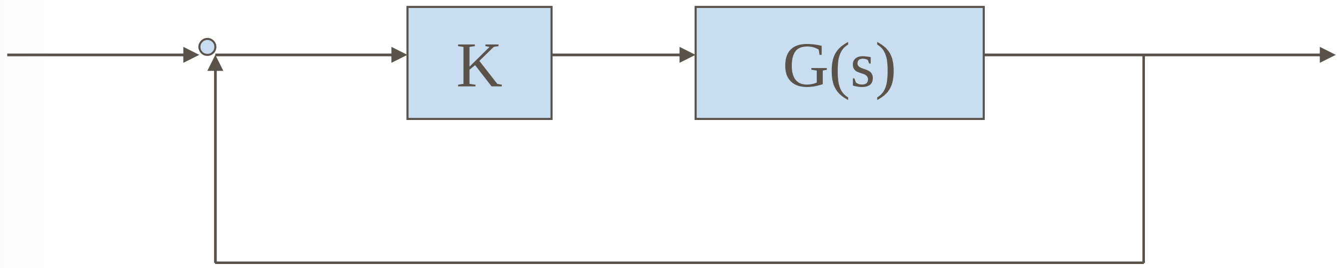
1. Continuous lead compensation;
2. Emulation method for digital control;

Such that the design specifications:

- Overshoot to a step input less than 5%;
- Settling time to 1% to be less than 14 sec.;
- Tracking error to a ramp input of slope 0.01rad/sec to be less than 0.01rad;
- Sampling time to give at at least 10 samples in a rise time.

(Write your analysis and program on a paper!)

1. Introduction - Root Locus



Open-loop trans. Func.: **KG(s)**;
Closed-loop trans. Func.: **KG(s)/(1+KG(s))**
Sensitivity function: **1/(1+KG(s))**

- The root locus of an (open-loop) transfer function **KG(s)** is a plot of the locations (locus) of all possible closed loop poles with proportional gain **K** and unity feedback
- From the root locus we can select a gain such that our closed-loop system will perform the way we want

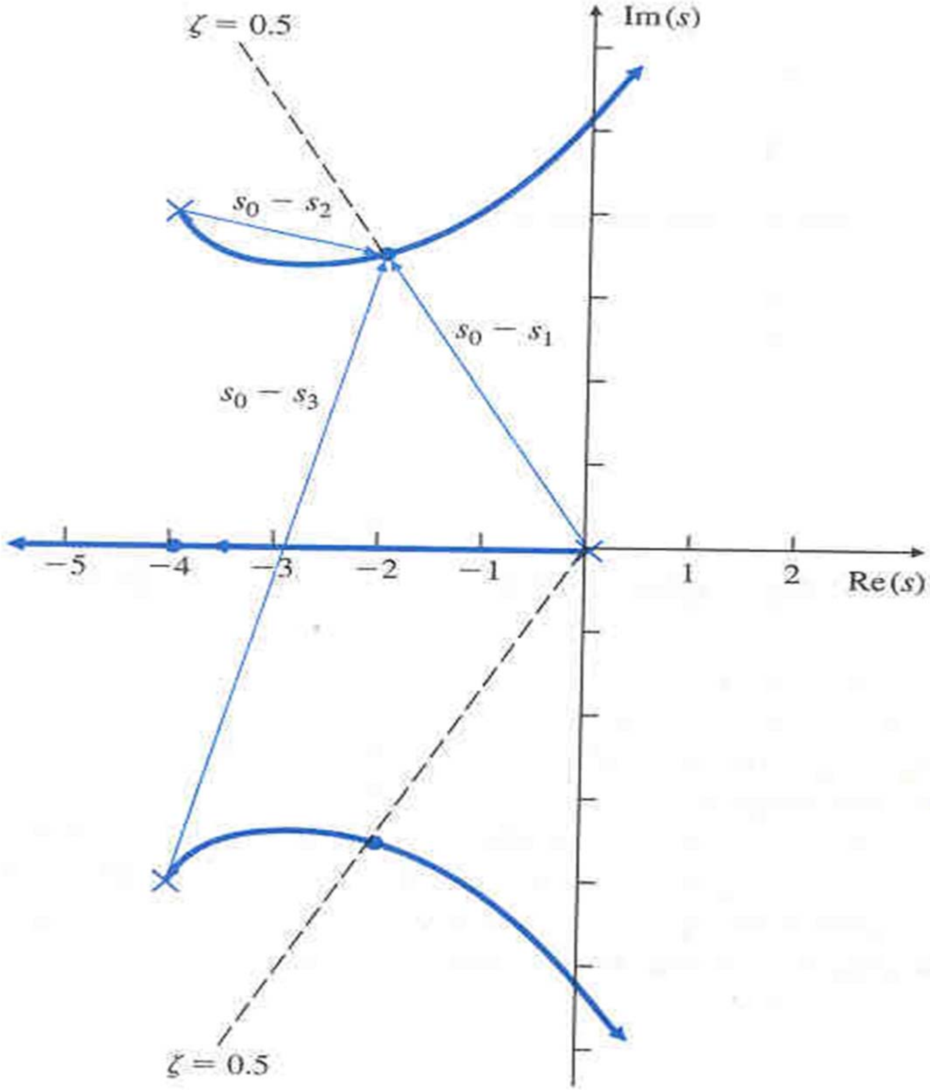
Control Design Using Root Locus (I)

- **Objective:** select a particular value of K that will meet the specifications for static and dynamic

  1+KG(s)=0

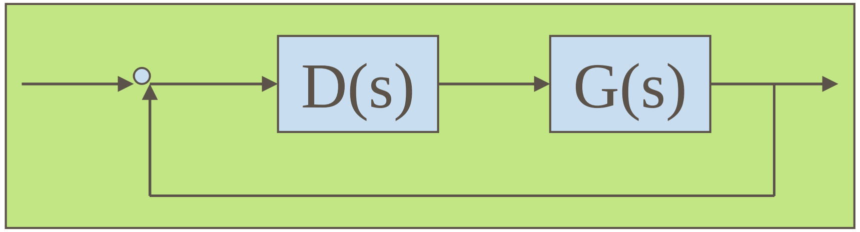
- **Magnitude condition:**

  K=1/|G(s)|



Exercise

- Question 5.2 on FC page.321;
- Consider a DC motor control using a PI controler



Where the motor is modeled as $G(s)=K/(\tau s+1)$ and PI controller is $D(s)=K_p(T_i s+1)/T_i s$, with parameters $K=30$, $\tau=0.35$, $T_i=0.041$. Through the root locus method determine the largest vaule of K_p such that $\xi=0.45$

- Try to use the root locus method to design a lead compensator for the examplifed attenna system.