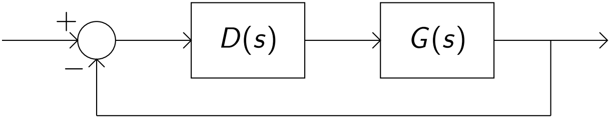
Classical Control

Lecture 5

Outline

Outline

Properties of a General System



Transfer Functions

- Open loop: $D(s)G(s)$
- Closed loop: $\frac{D(s)G(s)}{1+D(s)G(s)}$
- Sensitivity: $\frac{1}{1+D(s)G(s)}$

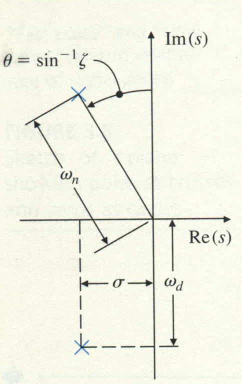
The Goal of Root Locus

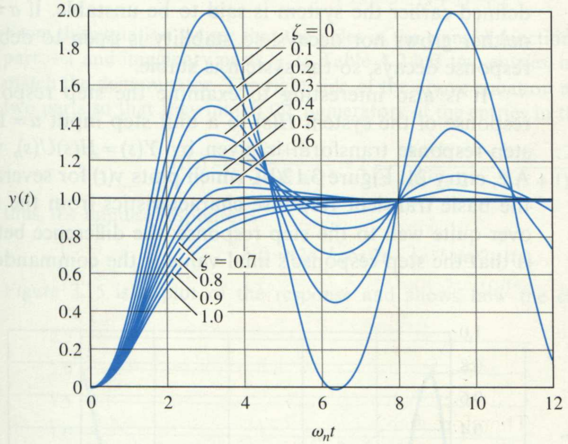
What do we want?

- Show how changes in a systems feedback characteristics and other parameters influence the pole locations
- Study the effects of additional poles and zeros when designing dynamic compensation

Effects of Additional Poles and Zeros

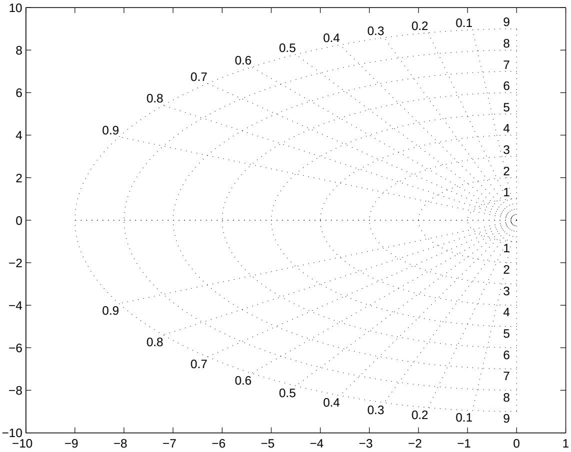
FIGURE 3.11
s-plane plot for a pair of complex poles





S-plane Design in MATLAB

```
sgrid(linspace(0,1,11),linspace(0,9,10),'new')
```



Root Locus

- **Root Locus** is the set of values for s for which the following equation holds for some real value of K

$$1 + KL(s) = 0$$

which is the characteristic equation for the closed loop system

$$\frac{D(s)G(s)}{1 + D(s)G(s)} = \frac{KL(s)}{1 + KL(s)}$$

The roots of the characteristic equation is thus the poles of the closed loop system

Root Locus

- **Generally** the root locus can be formulated as:

$$
\begin{aligned}
1 + KL(s) &= 0 \\
1 + K\frac{b(s)}{a(s)} &= 0 \\
a(s) + Kb(s) &= 0 \\
L(s) &= -\frac{1}{K}
\end{aligned}
$$

Root Locus: Example 1 (FC pp. 234)

System

$$G(s) = \frac{1}{s(s+1)}$$

$$D(s) = K$$

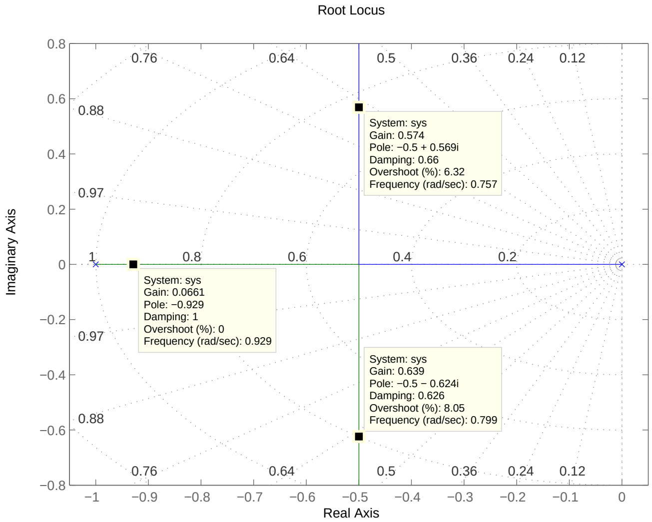
$$1 + D(s)G(s) = 1 + K\frac{1}{s(s+1)} = 1 + KL(s)$$

$$L(s) = \frac{1}{s(s+1)}$$

MATLAB

```
sys=tf(1,[1 1 0]);
rlocus(sys)
axis([-1.05 0.05 -0.8 0.8])
```

Root Locus: Example 1



Root Locus: Example 2 (FC pp. 235)

Plant

$$G(s) = \frac{1}{s(s+c)}$$

$$D(s) = 1$$

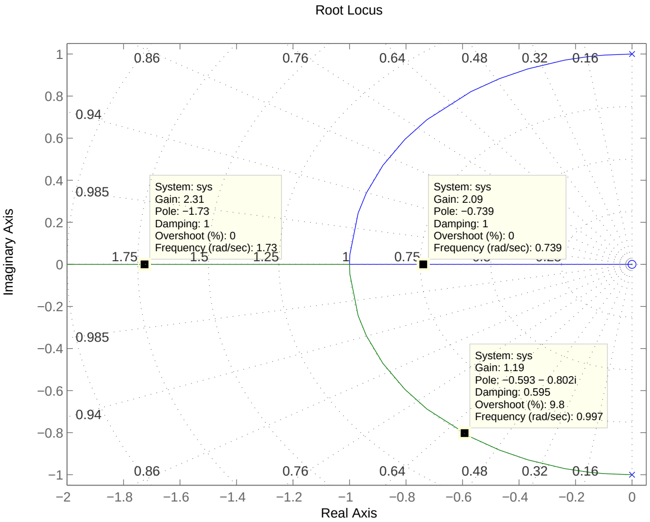
$$1 + D(s)G(s) = 1 + \frac{1}{s(s+c)} = 0 = s^2 + cs + 1 = a(s) + cb(s)$$

$$L(s) = \frac{b(s)}{a(s)} = \frac{s}{s^2+1}$$

MATLAB

```
sys=tf([1 0],[1 0 1]);
rlocus(sys)
axis([-2 0.05 -1.05 1.05])
```

Root Locus: Example 2



Sketching a Root Locus (FC 3rd ed. pp. 260)

Summary: Guidelines for Plotting a 180° Root Locus

1. Mark poles with an × and zeros with a ○.
2. Draw the locus on the real axis to the left of an odd number of real poles plus zeros.
3. Draw the asymptotes, centered at α and leaving at angles ϕ_l, where

$$n - m = \text{number of asymptotes}$$

$$\alpha = \frac{\sum p_i - \sum z_i}{n-m} = \frac{-a_1 + b_1}{n-m},$$

$$\phi_l = \frac{180^\circ + 360^\circ(l-1)}{n-m}, \qquad l = 1, 2, \ldots, n-m.$$

4. Compute locus departure angles from the poles and arrival angles at the zeros:

$$q\phi_{\text{dep}} = \sum \psi_i - \sum \phi_i - 180^\circ - 360^\circ l,$$
$$q\psi_{\text{arr}} = \sum \phi_i - \sum \psi_i + 180^\circ + 360^\circ l,$$

where q is the order of the pole or zero and l takes on q integer values so that the angles are between $\pm 180^\circ$.

5. If further refinement is required at the stability boundary: Assume $s_0 = j\omega_0$, and compute the point(s) where the locus crosses the imaginary axis for positive values of K, and/or use Routh's stability criterion. (This step may not be necessary.)
6. Use the results from the study of multiple roots to help in sketching how locus segments come together and break away: Two segments come together at 180° and break away at $\pm 90^\circ$. Three locus segments approach each other at relative angles of 120° and depart at angles rotated by 60°.
7. Complete the locus, using the facts developed in the previous steps and making reference to the illustrative loci for guidance. The locus branches start at poles and end at zeros or infinity.

Root Locus: Example 3

Plant ($D(s) = K$)

$$G(s) = \frac{s+1}{s^2(s+4)} = L(s)$$

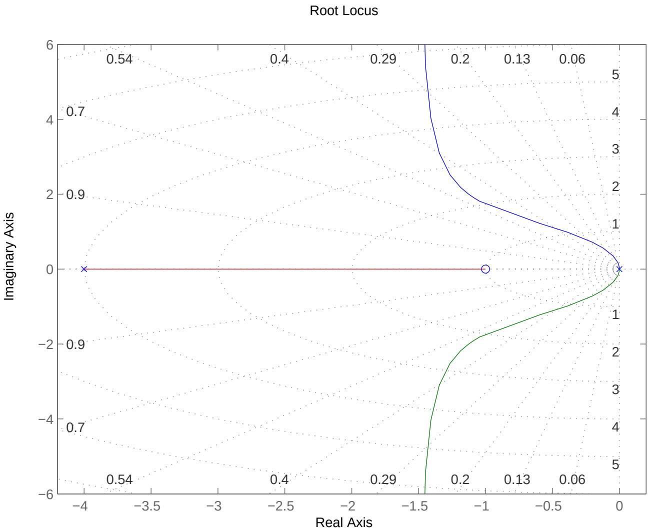
$$a(s) = s^3 + 4s^2 \qquad b(s) = s + 1$$

$$b\frac{da}{ds} - a\frac{db}{ds} = 0 \qquad s = 0, -1.75 \pm 0.9682j$$

MATLAB

```
sys=tf([1 1],[1 4 0 0]);
rlocus(sys)
axis([-4.2 0.2 -6 6])
```

Root Locus: Example 3



Root Locus: Example 4

Plant ($D(s) = K$)

$$G(s) = \frac{s+1}{s^2(s+9)} = L(s)$$

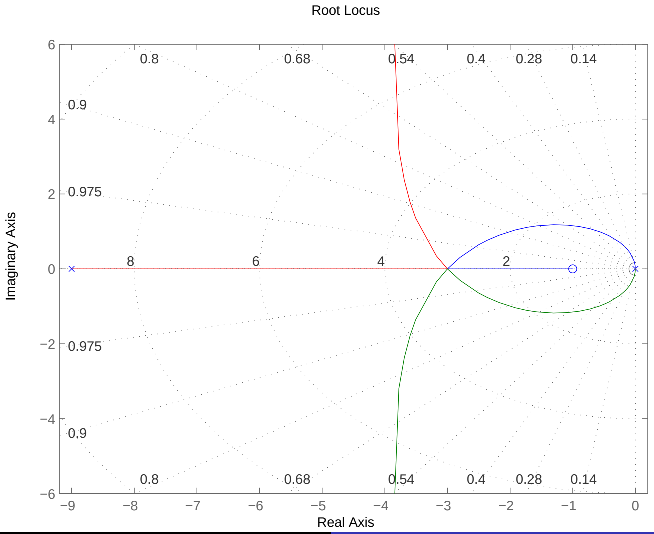
$$a(s) = s^3 + 9s^2 \qquad b(s) = s + 1$$

$$b\frac{da}{ds} - a\frac{db}{ds} = 0 \qquad s = 0, -3, -3$$

MATLAB

```
sys=tf([1 1],[1 9 0 0]);
rlocus(sys)
axis([-9.2 0.2 -6 6])
```

Root Locus: Example 4



Root Locus: Example 5

Plant ($D(s) = K$)

$$G(s) = \frac{s+1}{s^2(s+12)} = L(s)$$

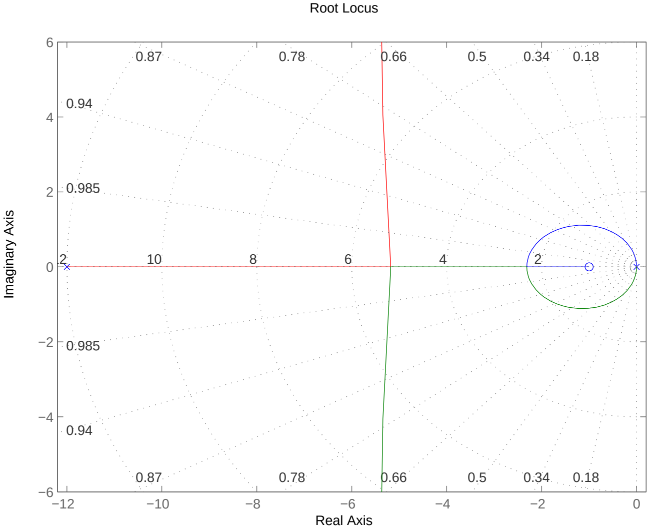
$$a(s) = s^3 + 12s^2 \qquad b(s) = s + 1$$

$$b\frac{da}{ds} - a\frac{db}{ds} = 0 \qquad s = 0, -2.3, -5.2$$

MATLAB

```
sys=tf([1 1],[1 12 0 0]);
rlocus(sys)
axis([-12.2 0.2 -6 6])
```

Root Locus: Example 5



Root Locus: Example 6

Plant ($D(s) = K$)

$$G(s) = \frac{1}{s(s+2)[(s+1)^2+4]} = L(s)$$

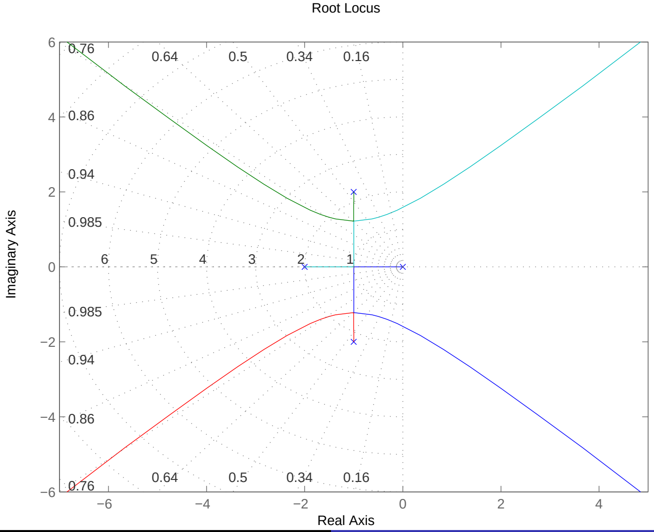
$$a(s) = s^4 + 4s^3 + 9s^2 + 10s \qquad b(s) = 1$$

$$b\frac{da}{ds} - a\frac{db}{ds} = 0 \qquad s = -1, -1 \pm 1.2j$$

MATLAB

```
sys=tf(1,[1 4 9 10 0]);
rlocus(sys)
axis([-7 5 -6 6])
```

Root Locus: Example 6



Outline

1. What is Root Locus
   - Why use Root Locus?
   - Root Locus
   - Sketching a Root Locus
2. Root Locus Design
   - Gain Controller
   - Lead Compensation
   - Lag Compensation

Root Locus Design

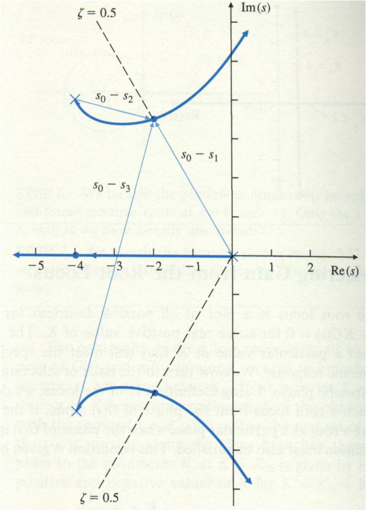
- **Objective**: Select a particular value of K that will meet the specifications for static and dynamic response

$$1 + KG(s) = 0$$

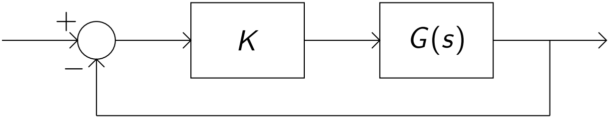
- **Magnitude Condition**:

$$K = \frac{1}{|G(s)|}$$

Root Locus Design



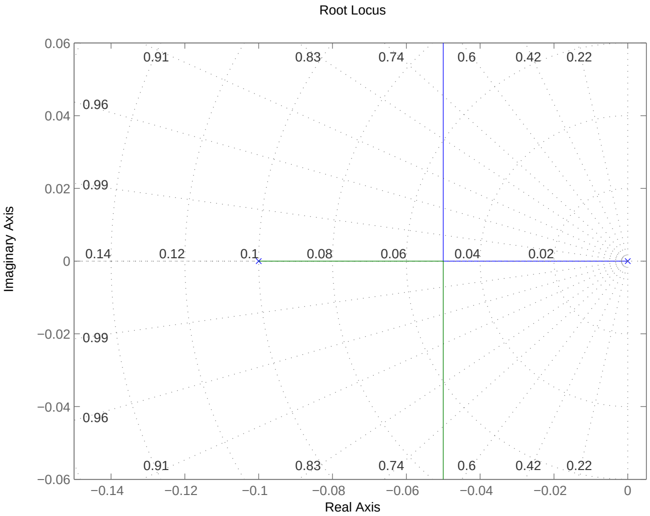
Gain Controller?



MATLAB

```
sys=tf(1,[10 1 0]);
rlocus(sys)
axis([-0.15 0.005 -0.06 0.06])
grid
```

Gain Controller?

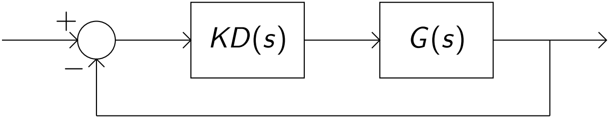


Dynamic Compensation

- **Objective**: If satisfactory process dynamics cannot be obtained by a gain adjustment alone then some modification or compensation of the dynamics is needed

$$1 + KD(s)G(s) = 0$$

- **Lead and Lag Compensation**:
  - **Lead Compensation** acts mainly to lower rise time and decrease the transient overshoot: $D(s) = \frac{s+z}{s+p}, z < p$
  - **Lag Compensation** acts mainly to improve the steady-state accuracy: $D(s) = \frac{s+z}{s+p}, z > p$



Lead Compensation

- Approximates PD control
- Moves the locus to the *left* and typically improves the system *damping*
- Zero and pole selection:
  - The zero is placed in the neighborhood of the closed-loop ω_n as determined by rise-time or settling-time requirements
  - The pole is located at a distance *3 to 20* times the value of the zero location

Lead Compensation: Example

System

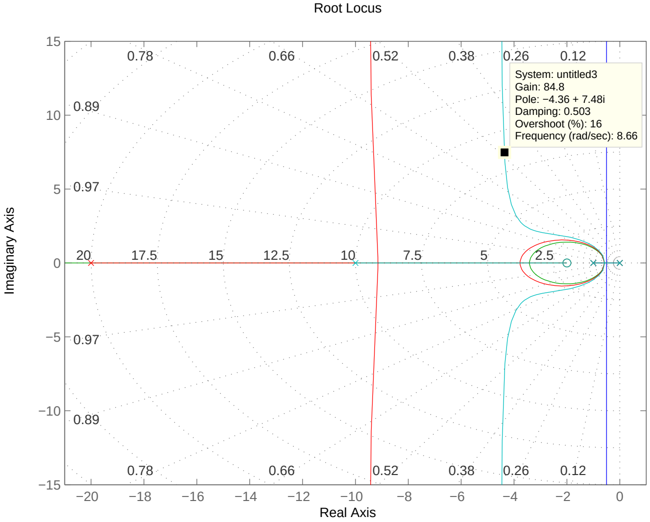
$$G(s) = \frac{K}{s(s+1)}$$

$$D_1(s) = s+2 \qquad D_2(s) = \frac{s+2}{s+20} \qquad D_3(s) = \frac{s+2}{s+10}$$

MATLAB

```
sys=tf(1,[1 1 0]);
D1=tf([1 2],[1]);
D2=tf([1 2],[1 20]);
D3=tf([1 2],[1 10]);
rlocus(sys,D1*sys,D2*sys,D3*sys)
axis([-21 1 -15 15])
```

Lead Compensation: Example



Lead Compensation: Design Example

Design the closed loop response for the following system to have a damping ratio $\zeta > 0.5$ and a natural frequency $\omega_n > 7\frac{rad}{sec}$.

System

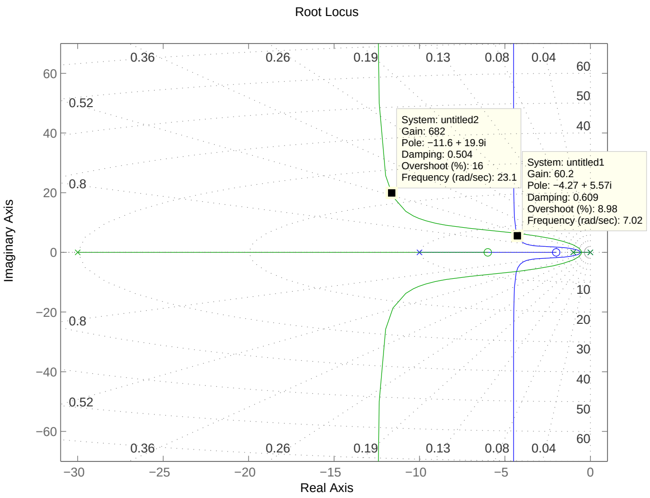
$$G(s) = \frac{K}{s(s+1)}$$

$$D_1(s) = \frac{s+2}{s+10} \qquad D_2(s) = \frac{s+6}{s+30}$$

MATLAB

```
sys=tf(1,[1 1 0]);
D1=tf([1 2],[1 10]);
D2=tf([1 6],[1 30]);
rlocus(D1*sys,D2*sys)
axis([-31 1 -70 70])
```

Lead Compensation: Design Example



Lead Compensation: Antenna

Step 1

Select the lead compensator as $D(s) = \frac{s+0.9}{s+10}$

```
sysDG=tf(1,[10 1 0])*tf([1 0.9],[1 10]);
```

Step 2

Draw the root locus to determine K

```
rlocus(sysDG)
```

Lead Compensation: Antenna

Step 3

Construct the closed-loop system and check whether the requirements are satisfied or not

```
step(feedback(K*sysDG,1))
```

Step 4

If the requirements are not satisfied, iterate the design procedure

Lag Compensation

- Approximates PI control
- Creates a zero and a pole:
  - Effect of the zeros are to move the locus to the left
  - Effect of the poles are to press the locus towards the right
- Zero and pole selection:
  - Select a pole near $s = 0$
  - Select the zero nearby such that the *dipole* does not significantly interfere with the response
- The lag compensation usually degrades stability

Lag Compensation: Example

System

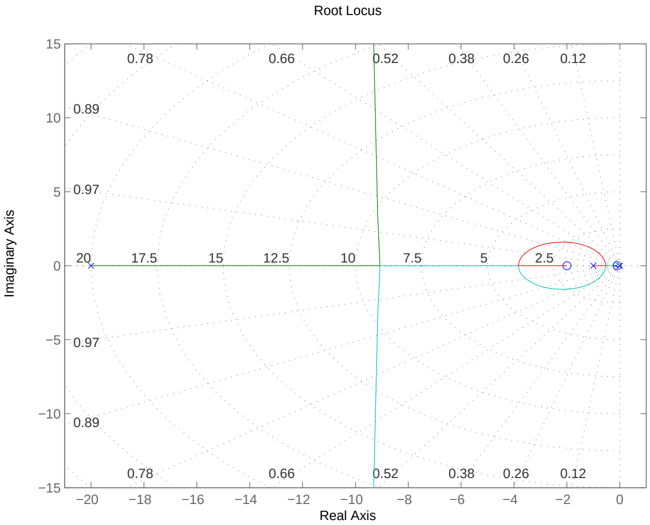
$$G(s) = \frac{K}{s(s+1)}$$

$$D_1(s) = \frac{s+2}{s+20} \qquad D_2(s) = \frac{s+0.1}{s+0.01}$$

MATLAB

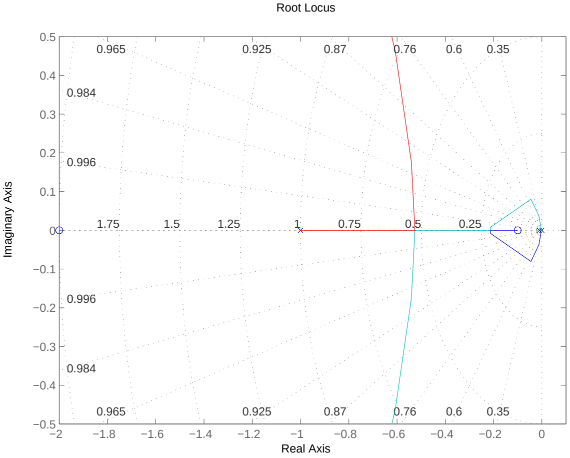
```
sys=tf(1,[1 1 0]);
D1=tf([1 2],[1 20]);
D2=tf([1 0.1],[1 0.01]);
rlocus(D1*D2*sys)
axis([-21 1 -15 15])
```

Lag Compensation: Example



Lag Compensation: Example

```
axis([-2 0.1 -0.5 0.5])
```



Book: Feedback Control

1. Exercise 5.2 in FC pp 296
2. Consider a DC motor control using a PI controller, Where the motor and PI controller are modeled as

$$G(s) = \frac{K}{(\tau s + 1)}, \ D(s) = \frac{K_p(T_i s + 1)}{T_i s}$$

   with parameters $K = 30, \ \tau = 0.35, \ T_i = 0.041$. Use the root locus method to determine the largest vaule of K_p such that $\zeta = 0.45$.
3. Try to use the root locus method to design a lead compensator for the examplified antenna system.