Classical Control

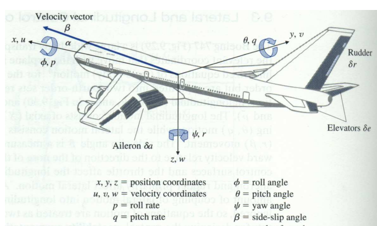
Lecture 7

Outline

MIMO System Example: Flight Control System

MIMO Plant

- 6 Degrees of Freedom
- Eighth order ODE
- Longitudinal Motion
- Lateral Motion
- Measured Signals
- Control Surfaces

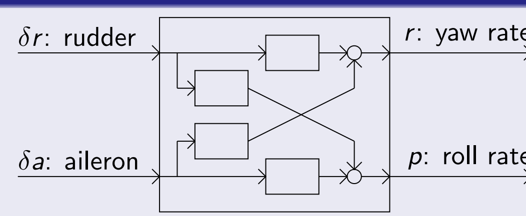


MIMO Controller

Characteristics of MIMO System

- Multiple Input Channels and/or Multiple Output Channels
- Coupling Phenomena Among Channels

Coupling Phenomena for Lateral Motion



Description of MIMO Systems

Transfer function matrices

$$\mathbf{G}(s) = \begin{bmatrix} G_{11}(s) & \cdots & G_{1m}(s) \\ \vdots & \ddots & \vdots \\ G_{p1}(s) & \cdots & G_{pm}(s) \end{bmatrix}$$

$G_{ij}(s)$: Transfer function from jth input to ith output

Using *tf* in MATLAB

Specify numerator and denominator of each SISO entry:

```
nums={num11 num12;num21 num22};
dens={den11 den12;den21 den22};
sys=tf(nums,dens);
```


Description of MIMO Systems - Example

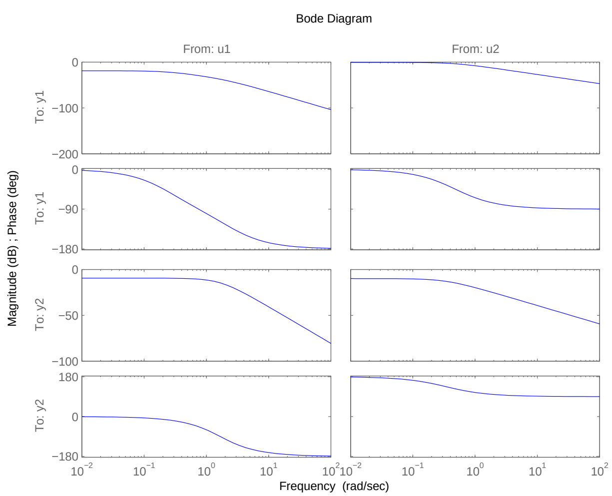
MATLAB

```
nums={[0.1134] [0.924]; [0.3378] [-0.318]};
dens={[1.78 4.48 1] [2.07 1]
      [0.361 1.09 1] [2.93 1]};
sys=tf(nums,dens,'inputname',{'u1' , 'u2'},...
        'outputname',{'y1' , 'y2'})
```

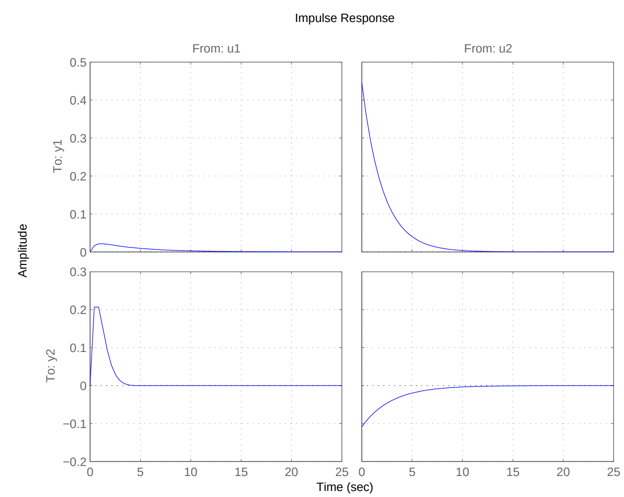
$$sys = \begin{bmatrix} \frac{0.1134}{1.78s^2+4.48s+1} & \frac{0.924}{2.07s+1} \\ \frac{0.3378}{0.361s^2+1.09s+1} & \frac{-0.318}{2.93s+1} \end{bmatrix}$$

See *mm7mimodescrip.m*,

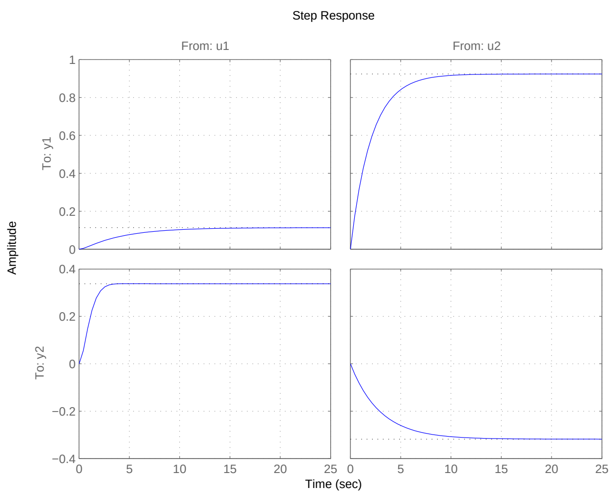
Description of MIMO Systems - Example



Description of MIMO Systems - Example



Description of MIMO Systems - Example



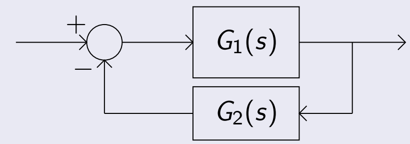
Description of MIMO Systems

MIMO System Connections

Serial connection:

$$\mathbf{G}_1(s)\mathbf{G}_2(s) \neq \mathbf{G}_2(s)\mathbf{G}_1(s)$$

Feedback connection:



$$\begin{aligned}\mathbf{G}(s) &= \mathbf{G}_1(s)[\mathbf{I} + \mathbf{G}_1(s)\mathbf{G}_2(s)]^{-1} \\ &= [\mathbf{I} + \mathbf{G}_2(s)\mathbf{G}_1(s)]^{-1}\mathbf{G}_1(s)\end{aligned}$$

Basic Analysis of MIMO Systems

- **Frequency analysis**: `bode`, `nyquist`
- **Time domain analysis**: `impulse`, `step`, `poles`, `tzero`
- **Root locus**: Does not work (only applicable to SISO systems)
- **Useful tool**: `ltiview`

Control Design for MIMO Systems

- **Frequency response design**:
  - Decoupling technique: Diagonal dominance matrix,...
  - Inverse Nyquist array method
  - Characteristic locus method
  - Quantitative Feedback Technique (QFT)
  - ...

Decoupling Design Technique

- **Motivation**: Directly use SISO techniques for MIMO system design
- **Idea**: Decouple multiple input and output channels into several independent single input and output channels

Example

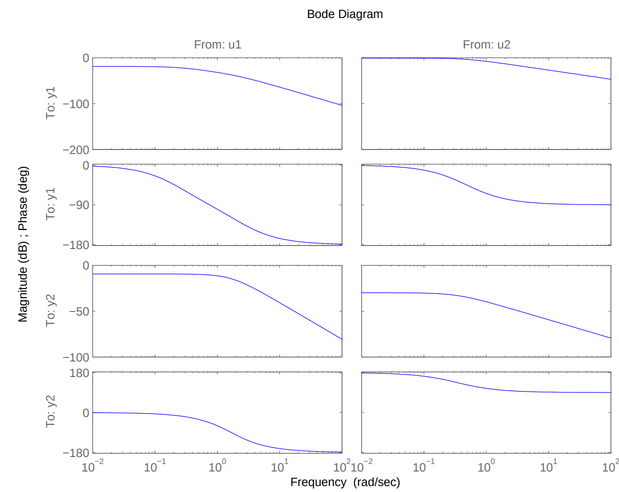
Transfer function

$$G(s) = \begin{bmatrix} \frac{0.1134}{1.78s^2+4.48s+1} & \frac{0.924}{2.07s+1} \\ \frac{0.3378}{0.361s^2+1.09s+1} & \frac{-0.0318}{2.93s+1} \end{bmatrix}$$

refactorized into $u_1 \longrightarrow y_2$ and $u_2 \longrightarrow y_1$

$$G_1(s) = \frac{0.3378}{0.361s^2 + 1.09s + 1} \qquad G_2(s) = \frac{0.924}{2.07s + 1}$$

Decoupling Design Technique - Example



Decoupling Design Technique - Example