Classical Control
Lecture 7

Outline

Introduction to MIMO Systems
- What is a MIMO System?
- Description of a MIMO System

Analysis and Design for MIMO Systems
- MIMO System Analysis and Design
- Decoupling and Pole Assignment

MIMO System Example: Flight Control System

MIMO Plant
- 6 Degrees of Freedom
- Eighth order ODE
- Longitudinal Motion
- Lateral Motion
- Measured Signals
- Control Surfaces

MIMO Controller

Characteristics of MIMO System
- Multiple Input Channels and/or Multiple Output Channels
- Coupling Phenomena Among Channels

Coupling Phenomena for Lateral Motion

\[
\begin{align*}
\delta r &: \text{rudder} \\
\delta a &: \text{ailerons} \\
r &: \text{yaw rate} \\
p &: \text{roll rate}
\end{align*}
\]
Description of MIMO Systems

Transfer function matrices

\[ G(s) = \begin{bmatrix} G_{11}(s) & \cdots & G_{1m}(s) \\ \vdots & \ddots & \vdots \\ G_{p1}(s) & \cdots & G_{pm}(s) \end{bmatrix} \]

\( G_{ij}(s) \): Transfer function from \( j \text{th input to } i \text{th output} \)

Using \textit{tf} in MATLAB

Specify numerator and denominator of each SISO entry:

```matlab
nums={[0.1134 0.924]; [0.3378 -0.318]};
dens={{1.78 4.48 1; 2.07 1} [0.361 1.09 1; 2.93 1]};
sys=tf(nums,dens,'inputname',{'u1' , 'u2'},'outputname',{'y1' , 'y2'});
```

See \textit{mm7mimodescrip.m},

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Description of MIMO Systems - Example

**MATLAB**

```matlab
nums={[0.1134 0.924]; [0.3378 -0.318]};
dens={{1.78 4.48 1; 2.07 1} [0.361 1.09 1; 2.93 1]};
sys=tf(nums,dens,'inputname',{'u1' , 'u2'},'outputname',{'y1' , 'y2'});
```

**Bode Diagram**

- From: \( u_1 \) To: \( y_1 \)
- From: \( u_2 \) To: \( y_1 \)
- From: \( u_1 \) To: \( y_2 \)
- From: \( u_2 \) To: \( y_2 \)

**Impulse Response**

- From: \( u_1 \) To: \( y_1 \)
- From: \( u_2 \) To: \( y_2 \)

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Description of MIMO Systems - Example

Step Response

MIMO System Connections

Serial connection:
\[ G_1(s)G_2(s) \neq G_2(s)G_1(s) \]

Feedback connection:
\[ G(s) = G_1(s)[I + G_1(s)G_2(s)]^{-1} = [I + G_2(s)G_1(s)]^{-1}G_1(s) \]

Basic Analysis of MIMO Systems

- Frequency analysis: bode, nyquist
- Time domain analysis: impulse, step, poles, tzero
- Root locus: Does not work (only applicable to SISO systems)
- Useful tool: ltiview

Control Design for MIMO Systems

- Frequency response design:
  - Decoupling technique: Diagonal dominance matrix,...
  - Inverse Nyquist array method
  - Characteristic locus method
  - Quantitative Feedback Technique (QFT)
  - ...

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Decoupling Design Technique

**Motivation:** Directly use SISO techniques for MIMO system design

**Idea:** Decouple multiple input and output channels into several independent single input and output channels

**Example**

Transfer function

\[ G(s) = \begin{bmatrix} 0.1134 & 0.924 \\ 1.78s^2 + 4.48s + 1 & 2.07s + 1 \\ 0.361s^2 + 1.09s + 1 & 2.93s + 1 \end{bmatrix} \]

refactorized into \( u_1 \to y_2 \) and \( u_2 \to y_1 \)

\[ G_1(s) = \frac{0.3378}{0.361s^2 + 1.09s + 1}, \quad G_2(s) = \frac{0.924}{2.07s + 1} \]

**Decoupling Design Technique - Example**

*Bode Diagram*

*Step Response*