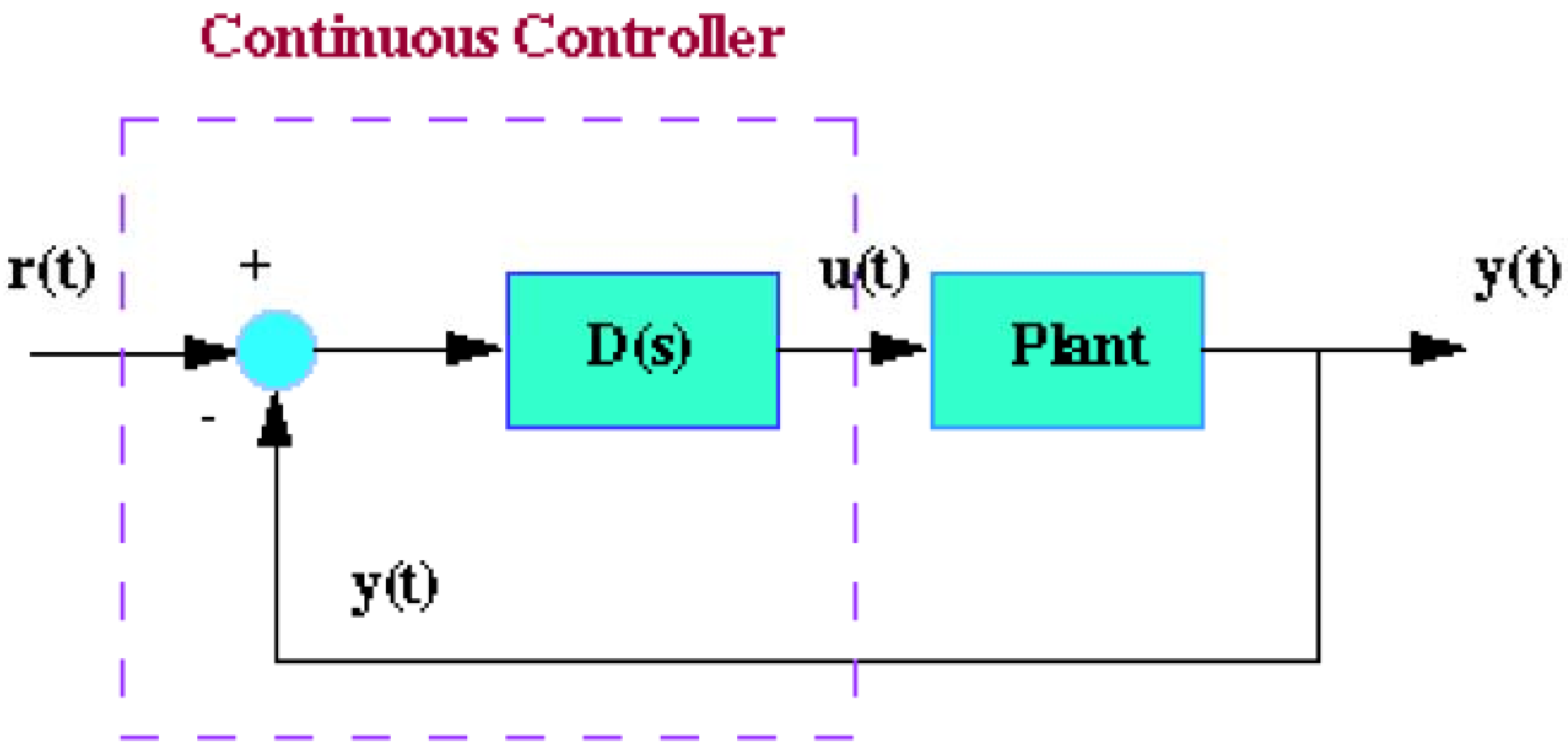
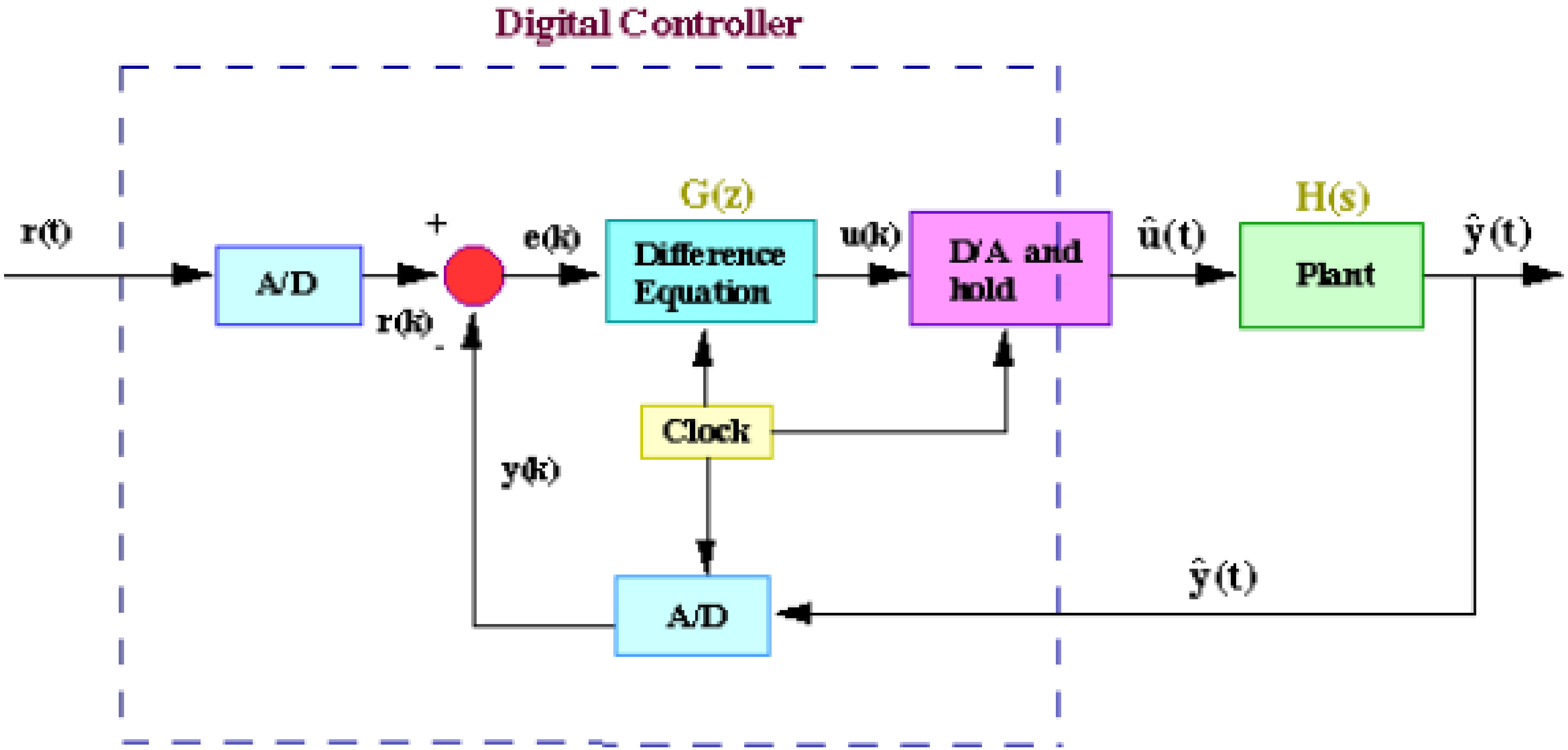
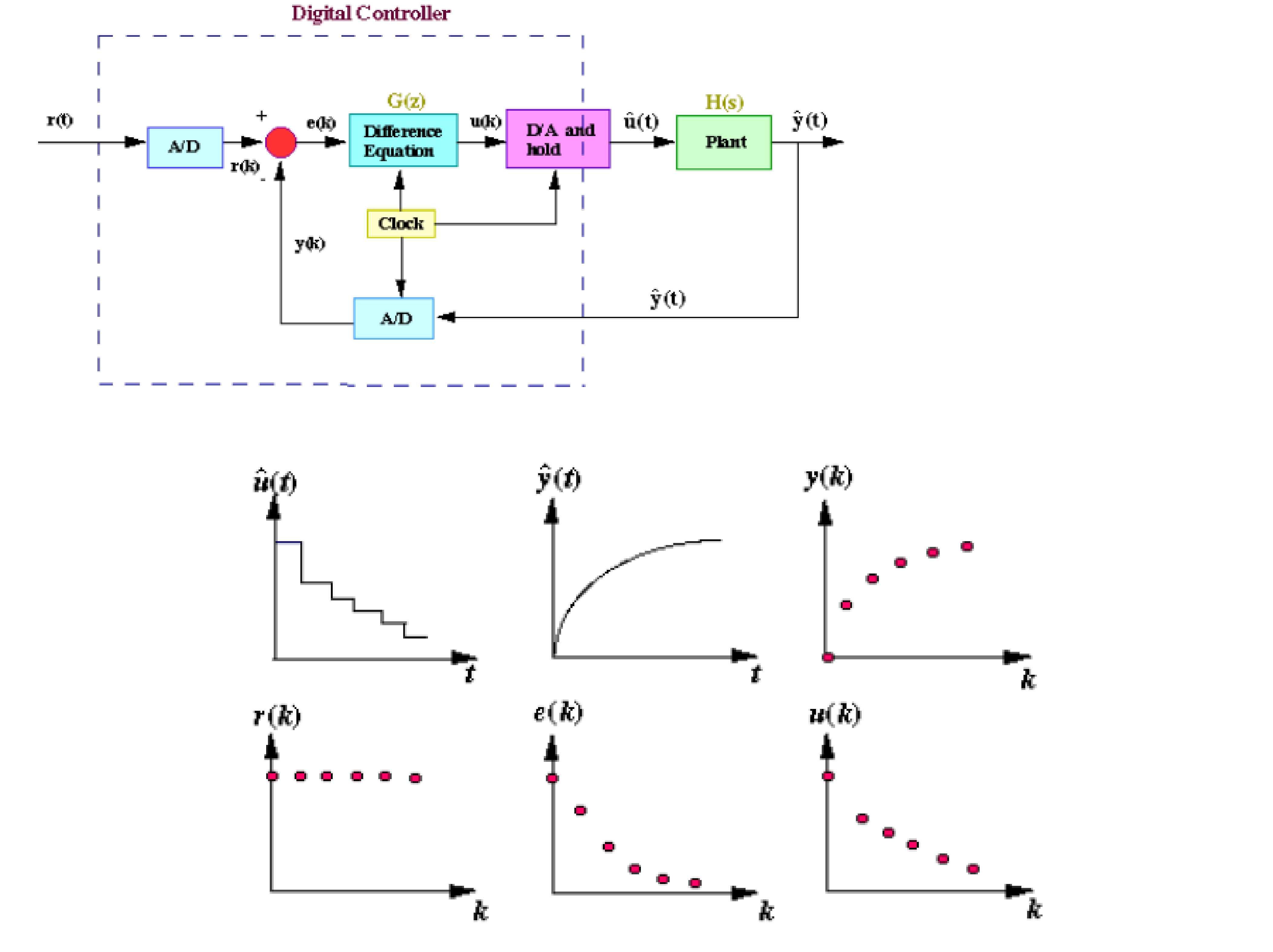
Digital Control

Lecture 1


Continuous Controller
r(t)
+
-
D(s)
u(t)
Plant
y(t)
y(t)


Digital Controller
r(t)
A/D
+
e(k)
G(z)
Difference Equation
u(k)
D/A and hold
û(t)
H(s)
Plant
ŷ(t)
r(k)
-
y(k)
Clock
A/D
ŷ(t)



Digital Controller
r(t)
A/D
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û(t)
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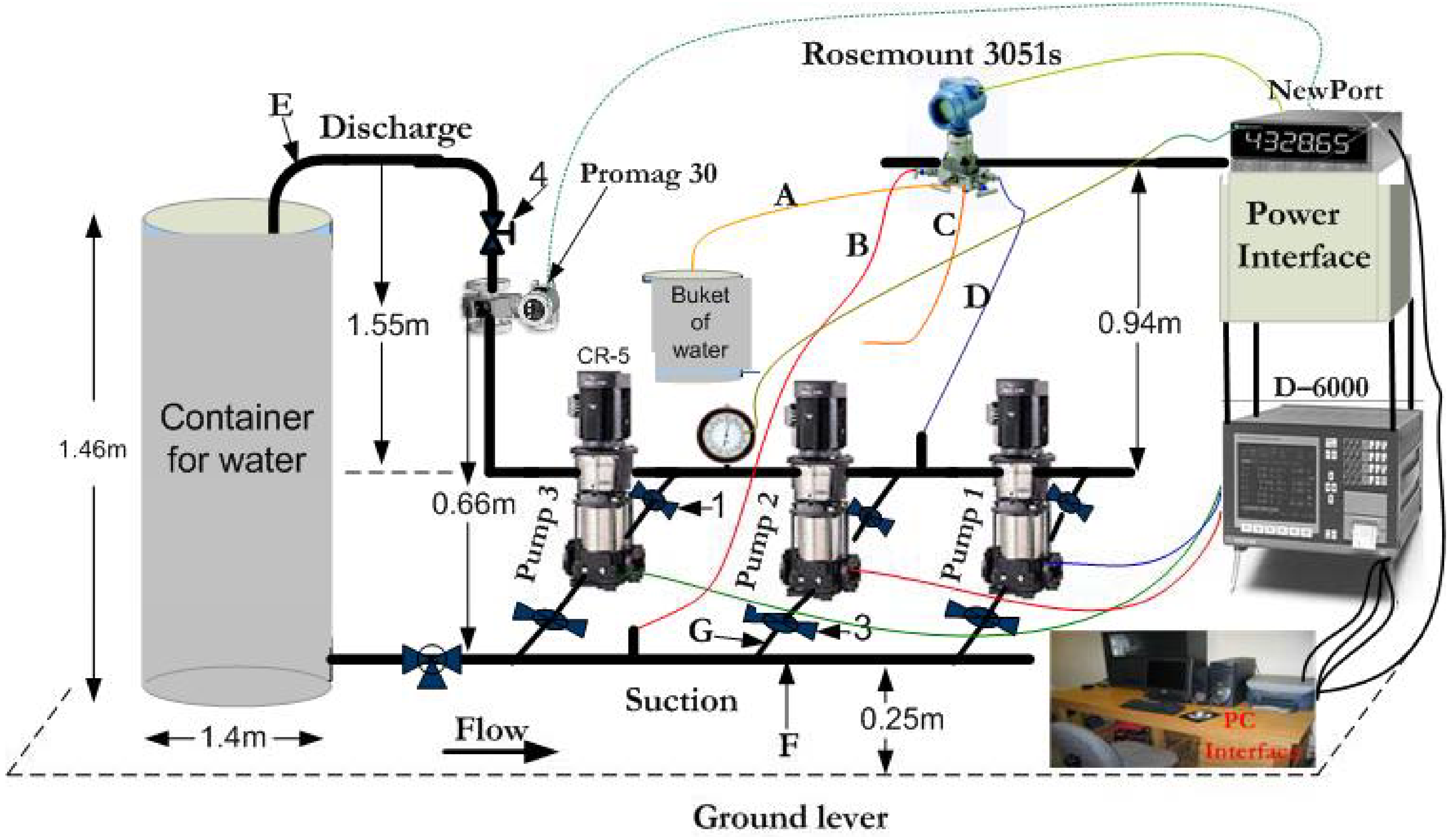
ŷ(t)
û(t)
t
ŷ(t)
t
y(k)
k
r(k)
k
e(k)
k
u(k)
k


Rosemount 3051s
NewPort
432865
E
Discharge
4
Promag 30
A
B
C
D
Power
Interface
1.46m
1.55m
Buket
of
water
0.94m
CR-5
D–6000
Container
for water
0.66m
Pump 3
1
Pump 2
Pump 1
G
3
Suction
F
0.25m
PC
Interface
1.4m
Flow
Ground lever

Questions

- How to represent digital signals & systems?
- How to obtain an equivalent digital representation of a analog system?
- How to analyze digital signal/system features?
- How to develop a digital controller?
- What needs to be concerned when implementing a digital controller?

Outline

1. Discrete Transfer Functions
2. Discretization

Outline

Definition of z-transform

For a discrete signal $e[k]$ with values $e_0, e_1, \ldots, e_k, \ldots$ the z-transform is given by:

z-Transform

$$E(z) \hat{=} \mathcal{Z}\{e[k]\}$$
$$\hat{=} \sum_{k=-\infty}^{\infty} e_k z^{-k}$$

Discrete signal can be obtained by sampling a continuous signal with a sample time T

Description of a structured signal

Given a structural relationship between sequences a z-domain equivalent can be obtained:

Sequences

$$\sum_{k=-\infty}^{\infty} e_k z^{-k} = 1 + a_1 \sum_{k=-\infty}^{\infty} e_{k-1} z^{-k}$$

z-domain

$$E(z) = \frac{1}{1 - a_1 z^{-1}}$$

Discrete transfer functions

Given input signal $E(z)$ and output signal $U(z)$ a transfer function describing their relationship can be given as:

Transfer function

$$H(z) = \frac{U(z)}{E(z)}$$

Discrete transfer functions - continued

Transfer function expressions

$$
\begin{aligned}
H(z) &= \frac{b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}} \\
&= \frac{b_0 z^n + b_1 z^{n-1} + \cdots + b_m z^{n-m}}{z^n + a_1 z^{n-1} + a_2 z^{n-2} + \cdots + a_n} \qquad n \geq m \\
&= \frac{b(z)}{a(z)}
\end{aligned}
$$

MATLAB

```
sys=tf(num,den,Ts)
```

Discrete System Models

Zeros and poles - General formula

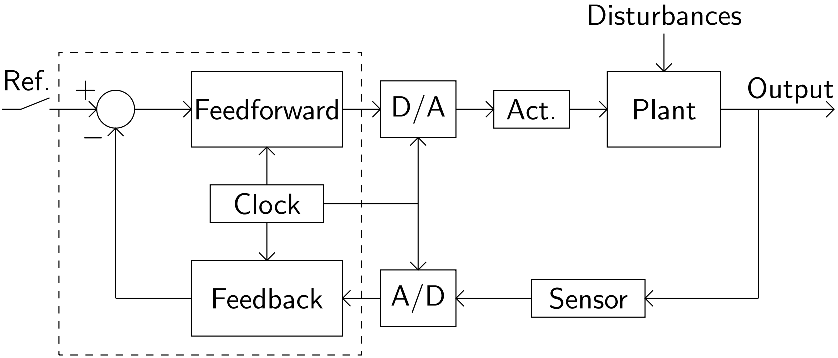
$$H(z) = \frac{U(z)}{E(z)} = K \frac{\prod_{k=1}^{m}(z - z_k)}{\prod_{i=1}^{n}(z - p_i)}$$

MATLAB

```
sys=zpk(z,p,k,Ts)
```

Outline

Basic Digital Control System



Zero Order Hold effect on continuous system

Basic property of ZOH

- Input to ZOH
  - Unit pulse, $\delta(kT)$
- Output from ZOH
  - Square pulse, $1(kT) - 1(kT - T)$

Effect on continuous system $G(s)$ after ZOH

$$\begin{aligned} Y(s) &= \frac{1}{s}G(s) - e^{-Ts}\frac{1}{s}G(s) \\ &= \left(1 - e^{-Ts}\right)\frac{G(s)}{s} \end{aligned}$$

Compensating for ZOH effect in z-transform

Incorporating ZOH in z-transform

$$G(z) = \mathcal{Z}\left\{Y(kT)\right\} = \mathcal{Z}\left\{\left(1 - e^{-Ts}\right)\frac{G(s)}{s}\right\}$$

Converting to z-domain

$$G(z) = \left(1 - z^{-1}\right)\mathcal{Z}\left\{\frac{G(s)}{s}\right\}$$

ZOH discretization example

Continuous system

$$G(s) = \frac{a}{s+a}$$

ZOH equivalent

$$G(z) = \frac{1-e^{-aT}}{z-e^{-aT}}$$

Numerical integration of differential equations

Continuous system

$$G(s) = \frac{U(s)}{E(s)} = \frac{a}{s+a}$$

$$\dot{u}(t) + au(t) = ae(t)$$

Solving differential equation

$$u(t) = u(kT - T) + \int_{kT-T}^{kT} -au(\tau) + ae(\tau)d\tau$$

Approximations of s for different rules

Forward rectangular rule

$$s \approx \frac{z-1}{T}$$

Backward rectangular rule

$$s \approx \frac{z-1}{Tz}$$

Trapezoidal rule

$$s \approx \frac{2}{T}\frac{z-1}{z+1}$$

Direct mapping of zeros/poles

Pole mapping

$$s_p = -a + jb \qquad \Rightarrow z_p = e^{-aT} \angle bT$$

Zero mapping (finite)

$$s_z = -a + jb \qquad \Rightarrow z_z = e^{-aT} \angle bT$$

Direct mapping of zeros/poles - continued

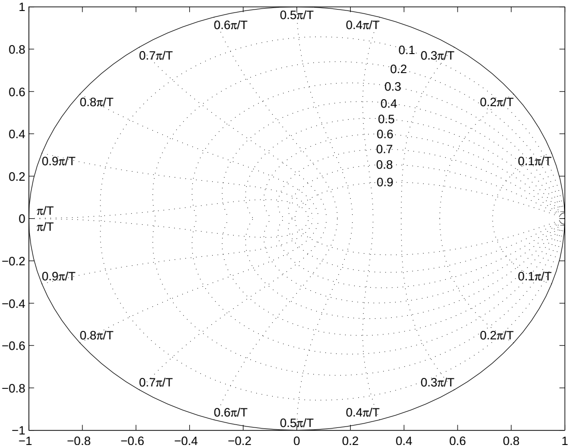
Zero mapping (infinite, no delay)

$$s_z = \infty \qquad \Rightarrow z_z = -1$$

Zero mapping (infinite, with n sample delay)

$$s_z = \infty \qquad \Rightarrow z_z = \infty \text{ (n zeros)}$$
$$s_z = \infty \qquad \Rightarrow z_z = -1 \text{ (remaining zeros)}$$

The Discrete z-plane



Direct mapping of zeros/poles - continued

Gain matching

$$H(s)|_{s=\omega_0} = H(z)|_{z=e^{T\omega_0}}$$

ω_0 is usually 0 in order to match steady-state gains

Discretization in MATLAB

MATLAB

```
sysd=c2d(sys,Ts,method)
```

method:

- 'zoh': Zero order hold
- 'foh': First order hold (academic)
- 'tustin': Bilinear approximation (trapezoidal)
- 'prewarp': Tustin with a specific frequency used for prewarp
- 'matched': Matching continuous poles with discrete

Stability - Impulse Response

Continuous systems

The system is BIBO stable if and only if the impulse response $h(t)$ is absolutely integrable

Discrete systems

The system is BIBO stable if and only if the impulse response $h[n]$ is absolutely summable

Stability - Characteristic Roots

Asymptotic internal stability

Continuous systems

All poles of the system are strictly in the LHP of the s-plane

Discrete systems

All poles of the system are strictly inside the unit circle of the z-plane

Book: Digital Control

- Problem 6.3 a.i-a.vi+b (use MATLAB when possible)
- Problem 6.4 a.i-a.vi+b (use MATLAB when possible)