Digital Control

Lecture 3

Outline

1. Digital Controller Design
2. Lead Compensator for Antenna - Design Example

What have we talked about in MM2?

- Sampling rate selection
- Equivalents between continuous & digital Systems

Sampling theorem

Theoretical lower limit

$$\frac{\omega_s}{\omega_b} > 2$$

ω_s: sampling frequency
ω_b: required closed-loop bandwidth

Smooth response

Practical limits

$$20 < \frac{\omega_s}{\omega_b} < 40$$

ω_s: sampling frequency
ω_b: required closed-loop bandwidth

Requirements to prefilter

Breakpoint of prefilter, ω_c

$$\omega_b < \omega_c < \frac{\omega_s}{2}$$

ω_s: sampling frequency
ω_b: required closed-loop bandwidth
ω_c: breakpoint of filter

Prefilter should filter lowest noise frequencies while not disturbing highest system frequencies!

Accommodating for sampling effect delay

Approximation of delay

$$e^{-sT/2} \approx \frac{2/T}{s + 2/T}$$

Accommodation

Investigate effect of half-sample delay on continuous system before digitizing:

- Root locus wrt. T
- Analyze using frequency based methods
  - Effect on phase margin/damping

Digital equivalents of continuous-time specifications

Transient response

- Overshoot/damping, M_p, ζ
- Rise time, ω_n
- Settling time, ω_n, ζ, σ

Steady-state response

- Steady-state: $z \rightarrow 1$, $(\lim_{k\rightarrow\infty} f(k) = \lim_{z\rightarrow 1}(z-1)F(z))$
- System type: number of pure integrators in open-loop, $(z=1)$
- System input
  - Step, $\frac{z}{z-1}$
  - Ramp, $\frac{Tz}{(z-1)^2}$
  - Parabola, $\frac{T^2}{2}\frac{z(z+1)}{(z-1)^3}$

Outline

1. Digital Controller Design
   - Emulation Method for Digital Control

2. Lead Compensator for Antenna - Design Example
   - Effect of Sample Times
   - Accommodation for Sampling Delay
   - Effect of Sampling Method

Digital Controller Design

Digital controller can be obtained using:

- Emulation, which finds the discrete equivalent of a continuous controller
- Direct discrete design (next lecture)

Frequency Issues

Continuous Systems

For a minimum-phase transfer function, the phase is uniquely determined by the magnitude curve:

$$\angle G(j\omega) \approx n \times 90^\circ$$

where n is the slope of $G(j\omega)$ in units of decade of amplitude

Discrete Systems

The amplitude and phase relationship is lost!
The prediction of stability from the amplitude curve alone for minimum-phase systems is lost
It is typically necessary to determine both magnitude and phase for discrete systems

Emulation Method

1. A *continuous* controller is designed
2. Sample time is selected
3. Discrete equivalent is computed
4. Evaluation of design

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Case Study: Antenna Control

General System Model:

$$J\ddot{\theta} + B\dot{\theta} = T_c + T_d$$

Discarding the disturbances T_d gives the transfer function:

$$\frac{\Theta(s)}{U(s)} = \frac{1}{s\left(\frac{s}{a} + 1\right)}$$

where $a = \frac{B}{J} = 0.1$ and $u(t) = \frac{T_c(t)}{B}$.

Design Specifications:

- Overshoot to a step input less than 16% (PM $\approx$ 55)
- Settling time to 1% in less than 10s
- Tracking error to ramp of slope $0.01\frac{\text{rad}}{\text{sec}}$ less than 0.01rad
- Sampling time to give at least 10 samples in a rise-time

Lead Compensator Design for Antenna (FC pp. 375)

Step 1

Design the low frequency gain K with respect to the steady-state error specification

Antenna system case: $K = 1$

Step 2

Determine the needed phase lead

```
sys=tf(1,[10 1 0]);
margin(sys)
```

PM=18 at $\omega = 0.308$

Lead Compensator Design for Antenna (FC pp. 375)

Step 3

Using lead contribution of $\phi_{max} = 45$ should result in PM=63 which is 8 more than needed.

Step 4

Determine:

$$\alpha = \frac{1 - \sin \phi_{max}}{1 + \sin \phi_{max}} = \frac{1 - \sin 45}{1 + \sin 45} = 0.1716$$

Step 5

$$T = \frac{1}{\omega_{max}\sqrt{\alpha}} = \frac{1}{\frac{\omega_n}{2}\sqrt{\alpha}} = \frac{2}{0.92\sqrt{\alpha}} = 5.248$$

Giving a zero in $s = -\frac{1}{T} = -0.19$ and a pole in $s = -\frac{1}{\alpha T} = -1.11$.

Lead Compensator Design for Antenna (FC pp. 375)

Step 6

Draw the compensated frequency response, check PM

Using the formulation:

$$D(s) = \frac{Ts + 1}{\alpha Ts + 1}$$

we use:

```
sysD=tf([5.3 1],[0.9 1])
sysC=sys*sysD
margin(sysC)
step(feedback(sysC,1))
```

Lead Compensator Design for Antenna (FC pp. 375)

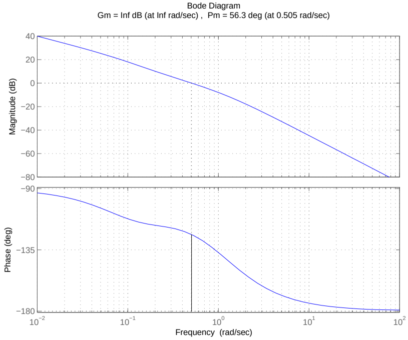


Figure: Frequency response

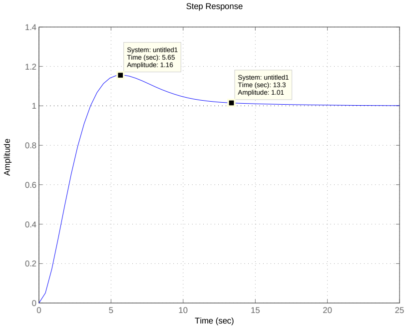


Figure: Step response

Lead Compensator Design for Antenna (FC pp. 375)

Step 7

Step 7: Iterate on the design until all specifications are met

```
sysD=tf([10 1],[1 1])
sysC=sys*sysD
margin(sysC)
sysCL=feedback(sysC,1)
step(sysCL)
```

Lead Compensator Design for Antenna (FC pp. 375)

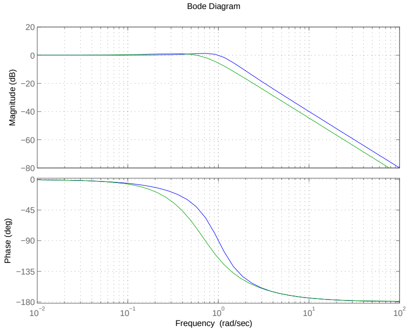


Figure: Frequency response

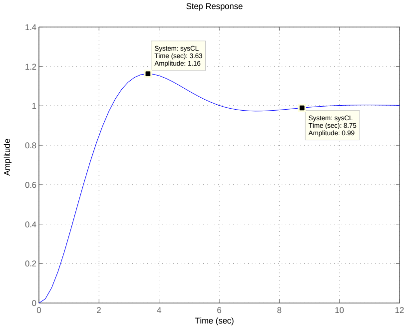


Figure: Step response

Digital Lead Compensator for Antenna - Fast Sampling

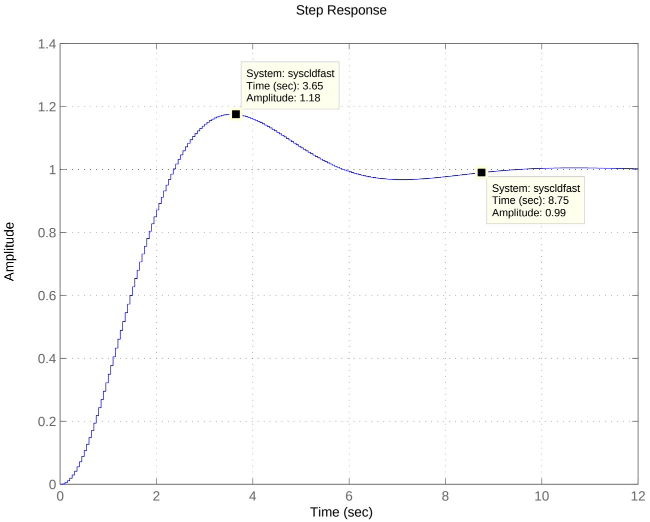
Continuous lead controller

$$D(s) = \frac{10s + 1}{s + 1}$$

Digitization - Fast Sample Rate

```
sysc=tf(1,[10 1 0]);
lead=tf([10 1],[1 1]);
syslead=sysc*lead;
Ts=1/20;
leadd1=c2d(lead,Ts,'zoh');
sysd=c2d(sysc,Ts,'zoh');
syscld=feedback(sysd*leadd1,1);
step(syscld)
```

Digital Lead Compensator for Antenna - Fast Sampling



Digital Lead Compensator for Antenna - Slow Sampling

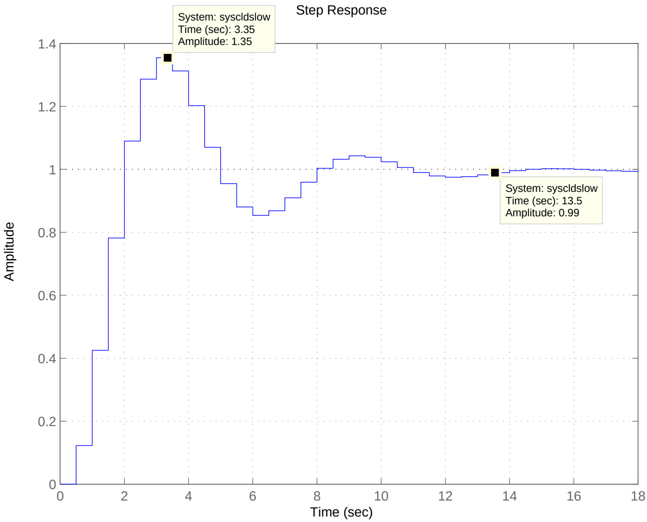
Continuous lead controller

$$D(s) = \frac{10s + 1}{s + 1}$$

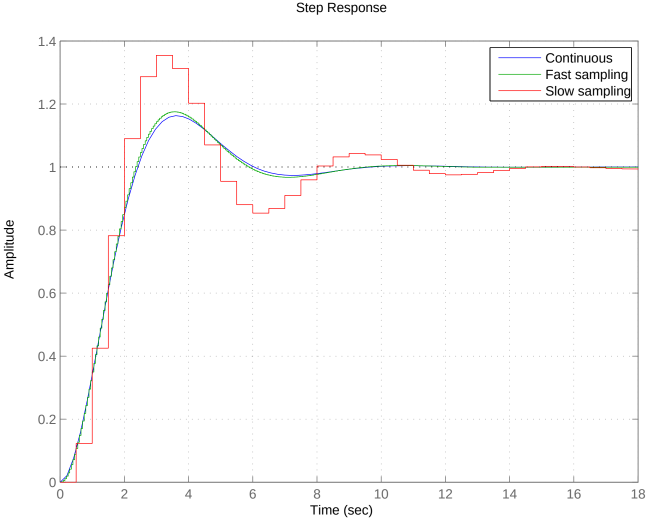
Digitization - Slow Sample Rate

```
sysc=tf(1,[10 1 0]);
lead=tf([10 1],[1 1]);
syslead=sysc*lead;
Ts=1/2;
leadd1=c2d(lead,Ts,'zoh');
sysd=c2d(sysc,Ts,'zoh');
syscld=feedback(sysd*leadd1,1);
step(syscld)
```

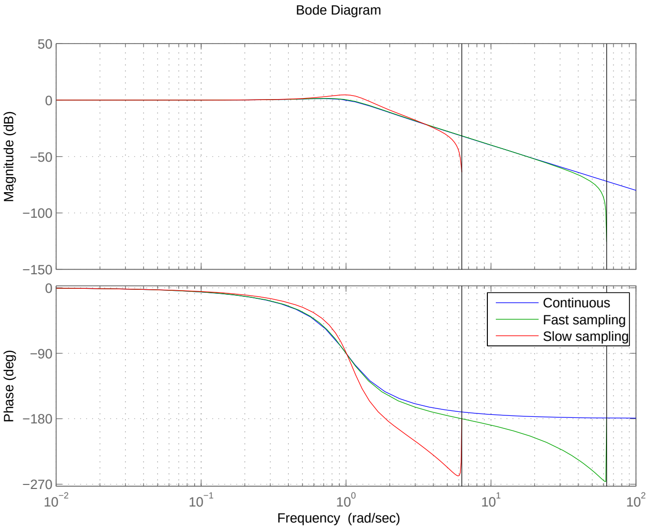
Digital Lead Compensator for Antenna - Slow Sampling



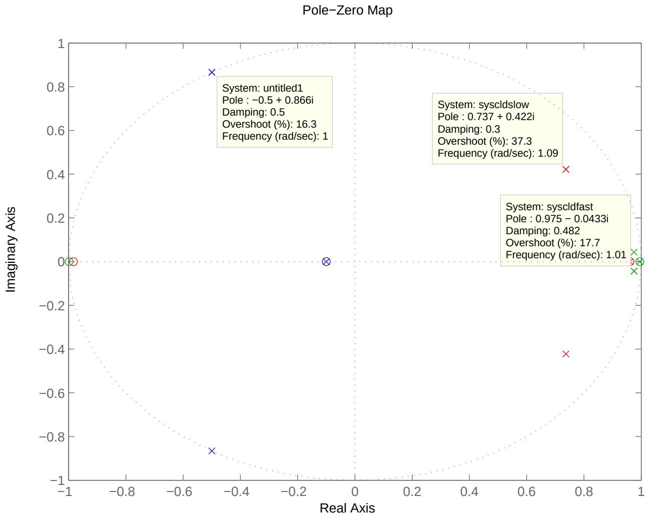
Effect of Sample Time on Step Response



Effect of Sample Time on Frequency Response



Effect of Sample Time on Pole Locations



Incorporating Sampling Delay in System

Continuous System

$$G(s) = \frac{1}{s(10s+1)}$$

Continuous System with Delay

$$G_d(s) = \frac{2/T}{s+2/T}\frac{1}{s(10s+1)}$$

Lead Compensator for System using Slow Sampling Rate

Inserting $T = 1/2$

$$G_d(s) = \frac{2/T}{s + 2/T} \frac{1}{s(10s + 1)}$$
$$= \frac{4}{s(s + 4)(10s + 1)}$$

Lead Compensator Design for Antenna

Step 1

Design the low frequency gain K with respect to the steady-state error specification

Steady-state unchanged from original system: $K = 1$

Step 2

Determine the needed phase lead

```
sys=tf(1,[10 41 4 0]);
margin(sys)
```

PM=14 at $\omega = 0.308$

Lead Compensator Design for Antenna

Step 3

Using lead contribution of $\phi_{max} = 50$ should result in PM=64 which is 9 more than needed.

Step 4

Determine:

$$\alpha = \frac{1 - \sin \phi_{max}}{1 + \sin \phi_{max}} = \frac{1 - \sin 50}{1 + \sin 50} = 0.1325$$

Step 5

$$T = \frac{1}{\omega_{max}\sqrt{\alpha}} = \frac{1}{0.4\sqrt{(0.1325)}} = 6.869$$

Giving a zero in $s = -\frac{1}{T} = -0.1456$ and a pole in $s = -\frac{1}{\alpha T} = -1.099$.

Lead Compensator Design for Antenna

Step 6

Draw the compensated frequency response, check PM

Using the formulation:

$$D(s) = \frac{Ts + 1}{\alpha Ts + 1}$$

we use:

```
sysD=tf([6.9 1],[0.9 1])
sysC=sys*sysD
margin(sysC)
step(feedback(sysC,1))
```

Lead Compensator Design for Antenna

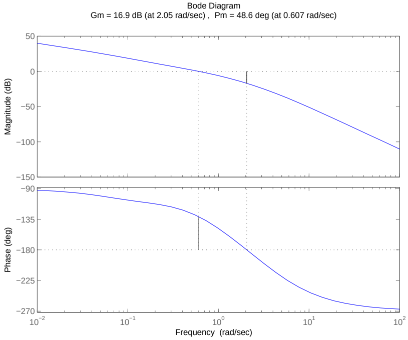


Figure: Frequency response

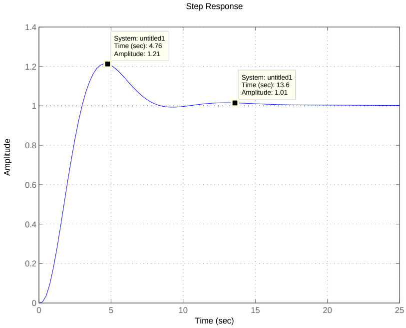


Figure: Step response

Lead Compensator Design for Antenna

Step 7

Step 7: Iterate on the design until all specifications are met

```
sysD=tf([7.5 1],[0.68 1])
sysC=sys*sysD
margin(sysC)
sysCL=feedback(sysC,1)
step(sysCL)
```

Lead Compensator Design for Antenna

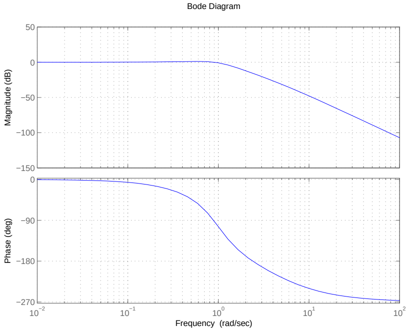


Figure: Frequency response

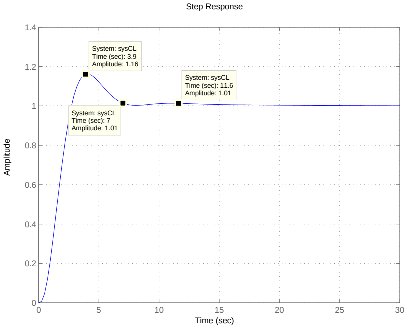


Figure: Step response

Digital Lead Compensator for Antenna - Slow Sampling

Continuous lead controller

$$D(s) = \frac{7.5s + 1}{0.68s + 1}$$

Digitization - Slow Sample Rate

```
sysc=tf(1,[10 1 0]);
lead=tf([7.5 1],[0.68 1]);
syslead=sysc*lead;
Ts=1/2;
leadd1=c2d(lead,Ts,'zoh');
sysd=c2d(sysc,Ts,'zoh');
syscld=feedback(sysd*leadd1,1);
step(syscld)
```

Digital Lead Compensator for Antenna - Comparison

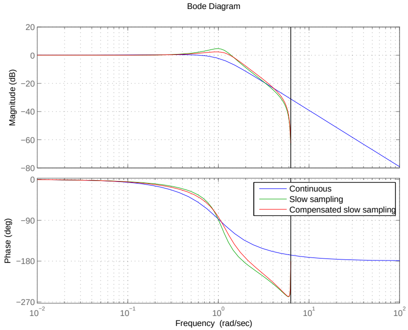


Figure: Frequency response

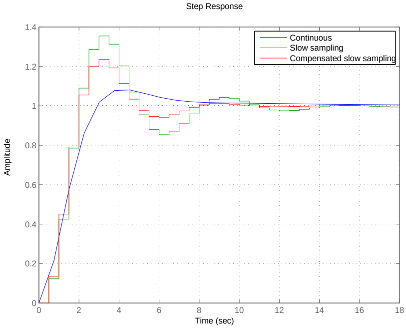


Figure: Step response

Discretization in MATLAB

MATLAB

```
sysd=c2d(sys,Ts,method)
```

method:

- 'zoh': Zero order hold
- 'foh': First order hold (academic)
- 'tustin': Bilinear approximation (trapezoidal)
- 'prewarp': Tustin with a specific frequency used for prewarp
- 'matched': Matching continuous poles with discrete

Discretization of Lead Compensator - Fast Sample Rate

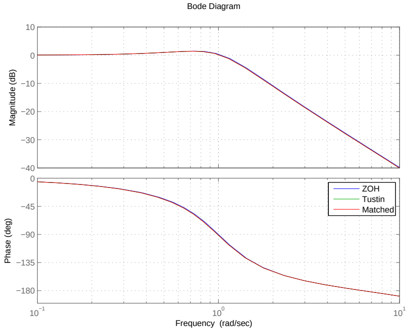


Figure: Frequency response

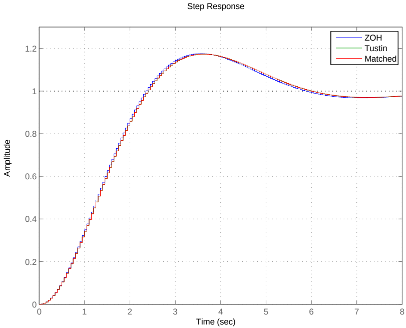
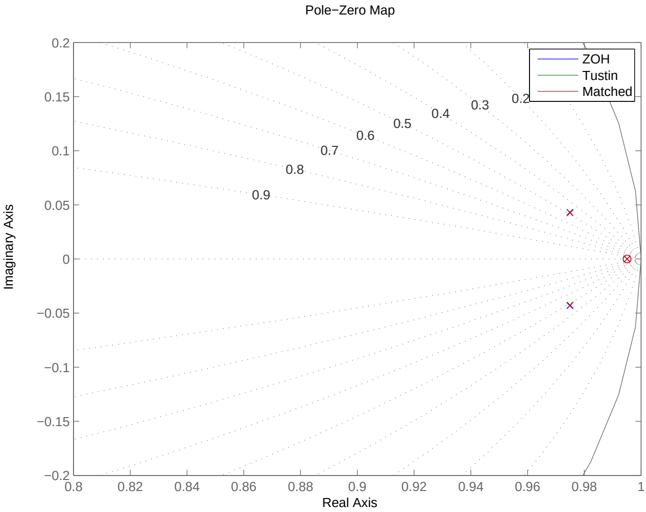


Figure: Step response

Discretization of Lead Compensator - Fast Sample Rate



Discretization of Lead Compensator - Slow Sample Rate

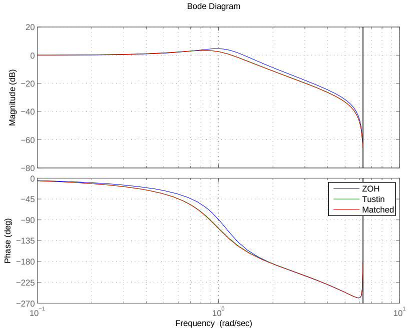


Figure: Frequency response

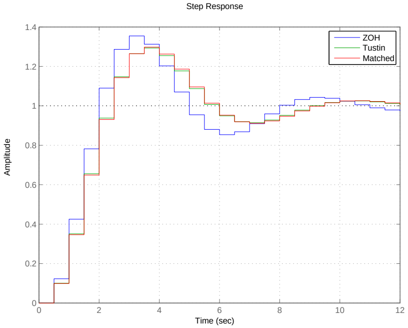
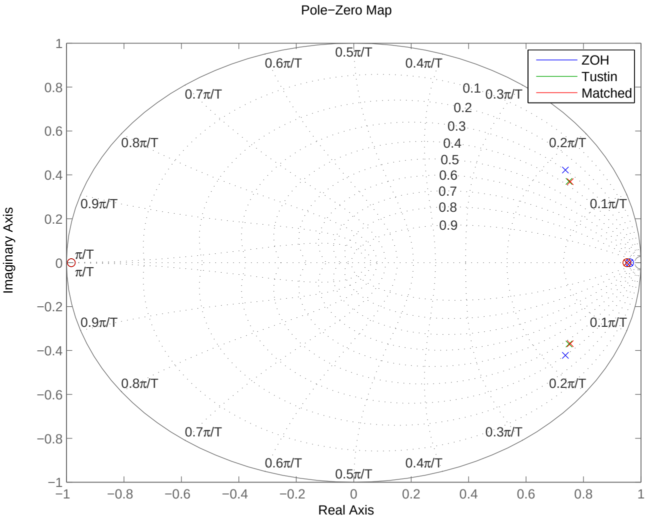


Figure: Step response

Discretization of Lead Compensator - Slow Sample Rate



Some important things to remember

Discretization of compensator

Use the method suited for implementation in the system

Discrete equivalent of plant

- Use method corresponding to implementation (usually ZOH)
- Simulink can combine discrete compensator with continuous plant (digitization of plant not necessary)

Book: Digital Control

- Problem 7.4
- Problem 7.5
- Problem 7.7