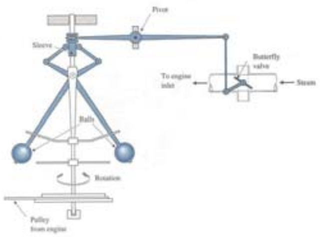


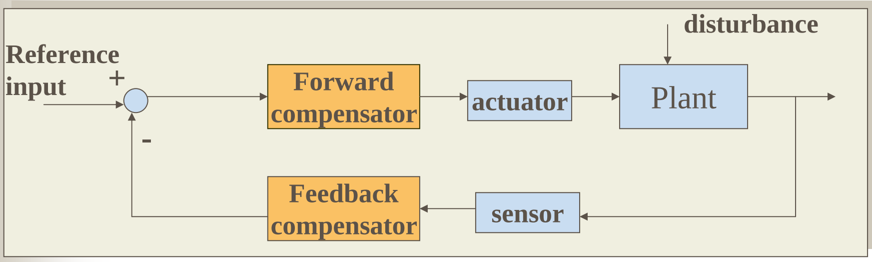
Digital Control - Summary

Quick review – Aanalog control systems

Supporting courses:

- Modeling and simulation
- Classical control

Continuous-time Control



Applications:
- Regulator systems
- Servo or position systems
- Tracking systems
- ...

Targets:
Closed-loop stability
Disturbance attenuation
Good command response
Robustness

Modeling ...

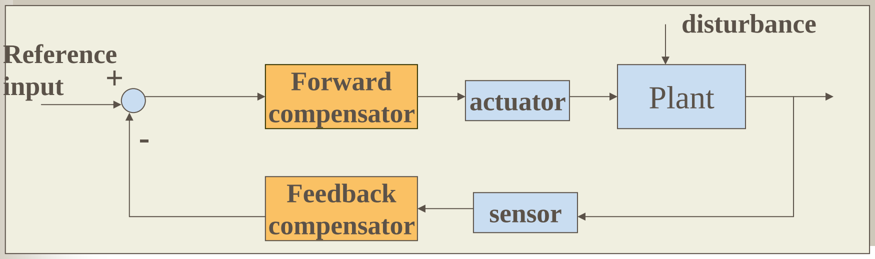
- **Types of models**
- **Physical modeling and experimental modeling...**
- **Differential/difference equations**
- **Transfer functions**
  - **Num-den form**

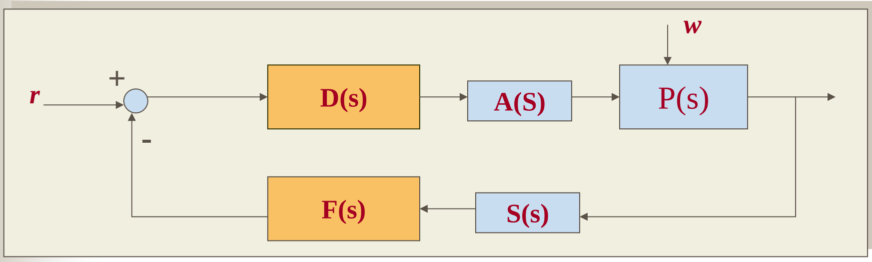
$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_1 s^m + b_2 s^{m-1} + \cdots + b_{m+1}}{a_1 s^n + a_2 s^{n-1} + \cdots + b_{n+1}}, \quad e.g., \quad G(s) = \frac{K}{s^2 + 2\zeta\omega s + \omega^2}$$

$$G(s) = \frac{Y(s)}{U(s)} = K\frac{\prod_{i=1}^{m}(s - z_i)}{\prod_{i=1}^{n}(s - p_i)} \quad e.g., \quad G(s) = \frac{K}{(s - p_1)(s - p_2)}$$

$$p_{1,2} = \zeta\omega \pm \omega\sqrt{1 - \zeta^2}$$

System Connection ...





System Analysis

- Transient response

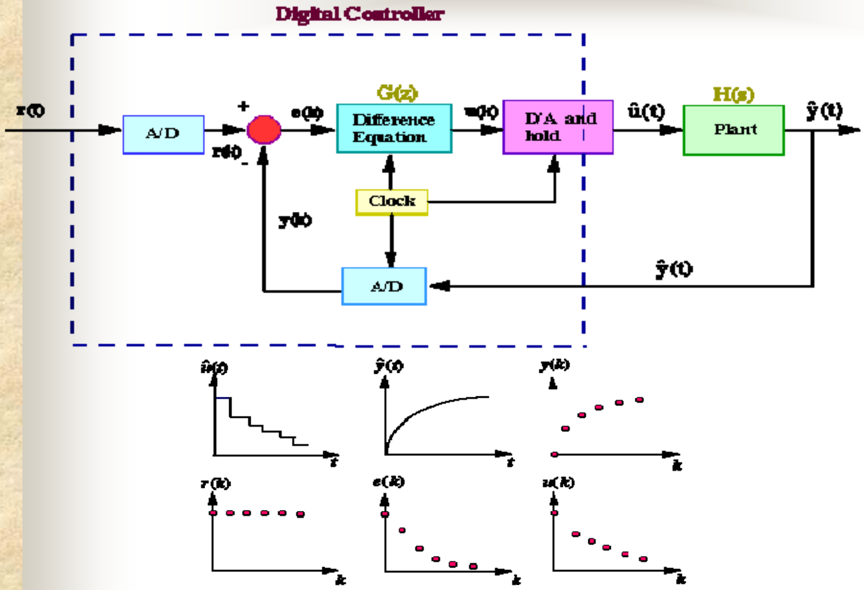
  Impulse response, step response, ...
- Steady-state response (system types: type 0, type I, type II, ..)
- Stability
- System respones vs. Pole locations
- Bode plot
- System stability: gain margin and phase margin

(Analog) Control Design

- Requirement specifications
  - Time-domain: rise time, settling time, overshoot
  - Frequency-domain: bandwidth (3dB frequency)...
- Basic control structure – PID control (with anti-windup)
- Design methods
  - Frequency response design
  - Root locus design
  - Dynamic compensation: lead or lag compensators
  - State-space design

Review of

Digital control systems



Digital signals and systems – descriptions

- Time domain
  - Signals: Sequences
  - systems: difference eqautions
- Frequency domain
  - Z-transform
  - Fourier transform
  - Transfer function

Unique features of digital control comparing with analog control...

- Sampling mechanism
- Discretization
- Quantization effects

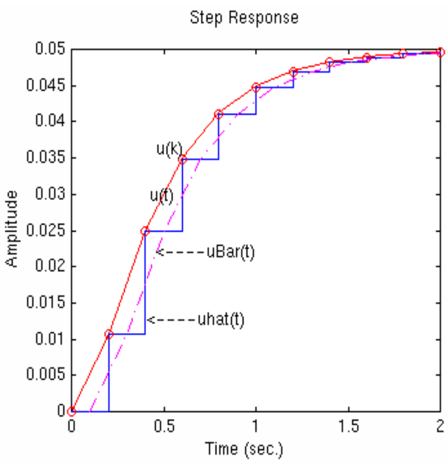
Sampling Theorem

- **Sampling theorem:** the sample rate must be at least twice the required closed-loop bandwidth
- Aliasing problem
- Practically, $20 < \omega_s/\omega_b < 40$
- Side-Effect: A **delay** is caused, which will degrade the stability and damping of the system

Time Delay Caused by Sampling

- A **delay** exists, which will degrade the stability and damping of the system
- A first order TF can be carried out by introducing a delay of **T/2** in the continuous-time analysis

$G_h(s)=(2/T)/(s+2/T)$



Discretization

2 Discretization
- Introducing Zero Order Hold
- Numerical Integration
- Zero-Pole Matching
- Stability

Basic property of ZOH
- Input to ZOH
  - Unit pulse, $\delta(kT)$
- Output from ZOH
  - Square pulse, $1(kT) - 1(kT - T)$

Converting to z-domain

$$G(z) = (1 - z^{-1})\,\mathcal{Z}\left\{\frac{G(s)}{s}\right\}$$

Lecture 1 Digital Cont

Pole mapping

$$s_p = -a + jb \qquad \Rightarrow z_p = e^{-aT}\angle bT$$

Zero mapping (finite)

$$s_z = -a + jb \qquad \Rightarrow z_z = e^{-aT}\angle bT$$

MATLAB

```
sysd=c2d(sys,Ts,method)
```

method:
- 'zoh': Zero order hold
- 'foh': First order hold (academic)
- 'tustin': Bilinear approximation (trapezoidal)
- 'prewarp': Tustin with a specific frequency used for prewarp
- 'matched': Matching continuous poles with discrete

Quantization effects

Why?

– Numbers in computer must be represented by a finite number of bits

Quantization methods

- Truncation
  - Ignore the least significant part(s), e.g., 2^{-l}
- Round-off
  - Check the least significant part with half of full range
  - Maximum error with round-off is half that from trunction

$x = x_q + e$

- Quantization objects:
  - **Variables**
  - Coefficient parameters

- Stcochastic analysis gives a modest results about the deviations propagated of the round-off error through the system
- Worst-case analysis considers the most extreme case
- Steady-state worst-case analysis focuses on the steady-state error
- Round-off signals won't cause changes of system features

Parameter round-off error analysis

- Main concerns:
  - Dynamic response
  - System stability
- Key issue:
  - Different realization structures lead to different results
- Key method:
  - Sensitivity analysis

Digital control systems - **Analysis**

Real-time responses...

Transient response

- Overshoot/damping, M_p, ζ
- Rise time, ω_n
- Settling time, ω_n, ζ, σ

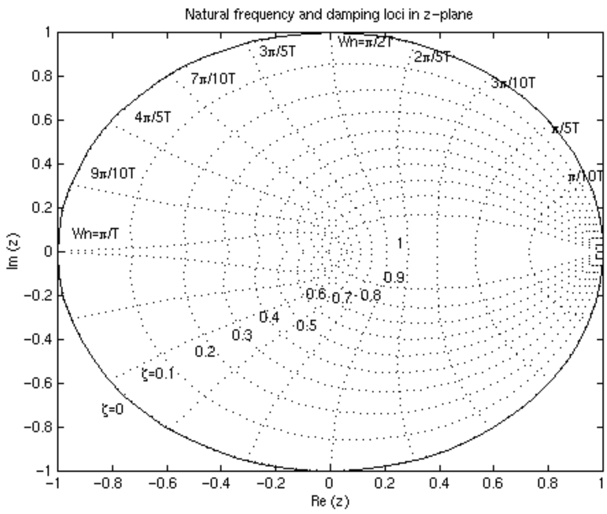
Steady-state response

- Steady-state: $z \longrightarrow 1$, $(\lim_{k\to\infty} f(k) = \lim_{z\to 1}(z-1)F(z))$
- System type: number of pure integrators in open-loop, $(z = 1)$
- System input
  - Step, $\frac{z}{z-1}$
  - Ramp, $\frac{Tz}{(z-1)^2}$
  - Parabola, $\frac{T^2}{2}\frac{z(z+1)}{(z-1)^3}$

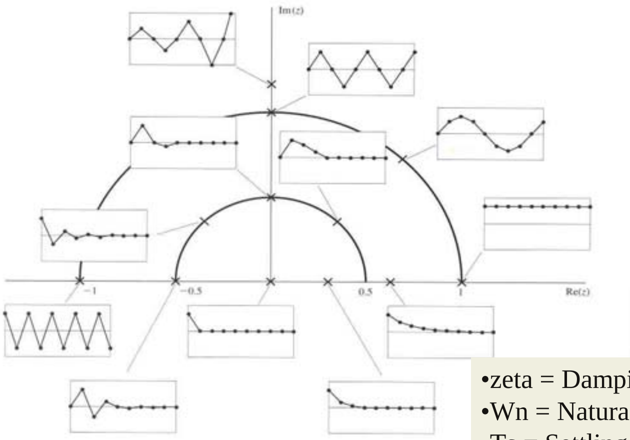
Lecture 2 Digital Control

Z- domain...

- Pole and zero locations
- Stability
- Performance



Effect of Poles for Discrete Case

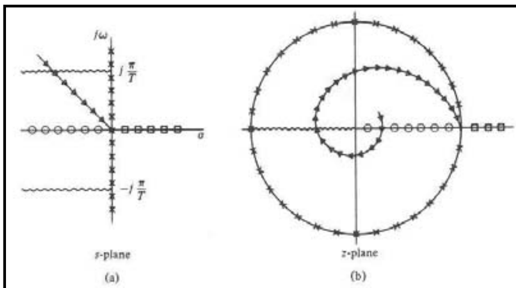


$$\xi\omega_n \geq \frac{4.6}{Ts}$$

$$\omega_n \geq \frac{1.8}{Tr}$$

$$\xi \geq \sqrt{\frac{\left(\frac{\ln Mp}{\pi}\right)^2}{1+\left(\frac{\ln Mp}{\pi}\right)^2}}$$

- zeta = Damping ratio
- Wn = Natural frequency **(rad/sec)**
- Ts = Settling time
- Tr = Rise time
- Mp = Maximum overshoot

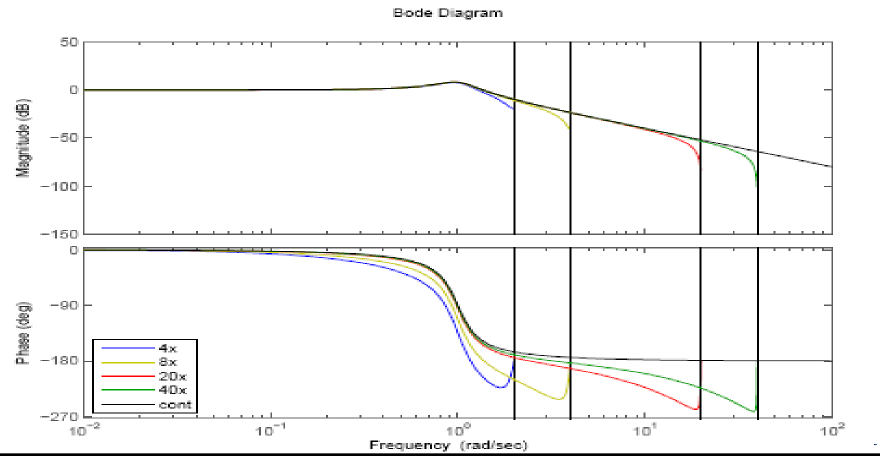


$Z=e^{sT}$

s-plane	Symbol	z-plane
$s = j\omega$ Real frequency axis	× × ×	$\lvert z\rvert = 1$ Unit circle
$s = \sigma \geq 0$	□□□	$z = r \geq 1$
$s = \sigma \leq 0$	○○○	$z = r, 0 \leq r \leq 1$
$s = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$ $= -a + jb$ Constant damping ratio if ζ is fixed and ω_n varies	△△△	$z = re^{j\theta}$ where $r = \exp(-\zeta\omega_n T) = e^{-aT}$, $\theta = \omega_n T\sqrt{1-\zeta^2} = bT$ Logarithmic spiral
$s = \pm j(\pi/T) + \sigma$,	$\sigma \leq 0$	$z = -r$

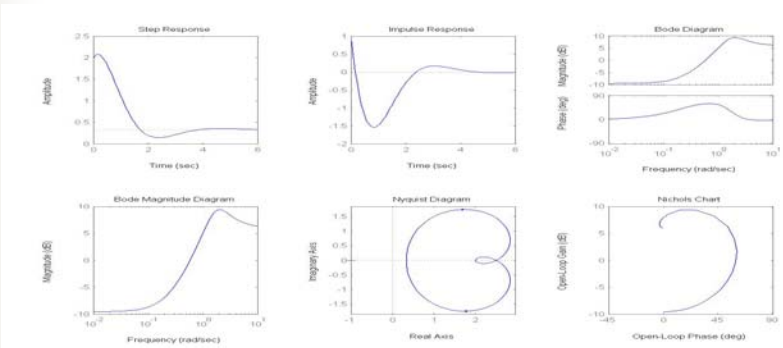
Frequency response...

- Bode plot
- Bandwidth and resonant peak



Matlab: LTI Viewer

- The LTI Viewer is a graphical user interface (GUI) that supports ten plot responses, including step, impulse, Bode, Nyquist, Nichols, zero/pole, sigma (singular values), lsim, and initial plots. The latter two are only available at the initialization of the LTI



Digital control systems – **Design**

Digital controller can be obtained using:

- Emulation, which finds the discrete equivalent of a continuous controller
- Direct discrete design (next lecture)

Emulation Method

1. A *continuous* controller is designed
2. Sample time is selected
3. Discrete equivalent is computed
4. Evaluation of design

Case Study: Antenna Control

General System Model:

$$J\ddot{\theta} + B\dot{\theta} = T_c + T_d$$

Discarding the disturbances T_d gives the transfer function:

$$\frac{\Theta(s)}{U(s)} = \frac{1}{s\left(\frac{s}{a}+1\right)}$$

where $a = \frac{B}{J} = 0.1$ and $u(t) = \frac{T_c(t)}{B}$.

Design Specifications:

- Overshoot to a step input less than 16% (PM $\approx$ 55)
- Settling time to 1% in less than 10s
- Tracking error to ramp of slope $0.01\frac{\text{rad}}{\text{sec}}$ less than 0.01rad
- Sampling time to give at least 10 samples in a rise-time

Step 6

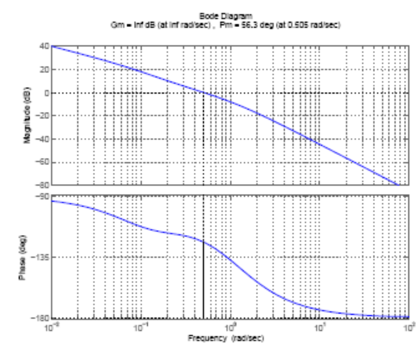
Draw the compensated frequency response, check PM

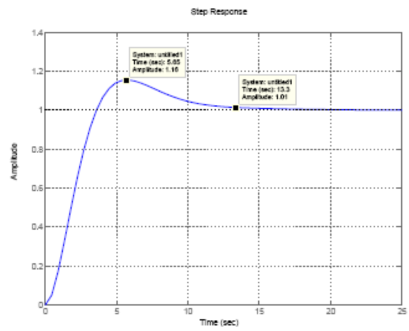
Using the formulation:

$$D(s) = \frac{Ts+1}{\alpha Ts+1}$$

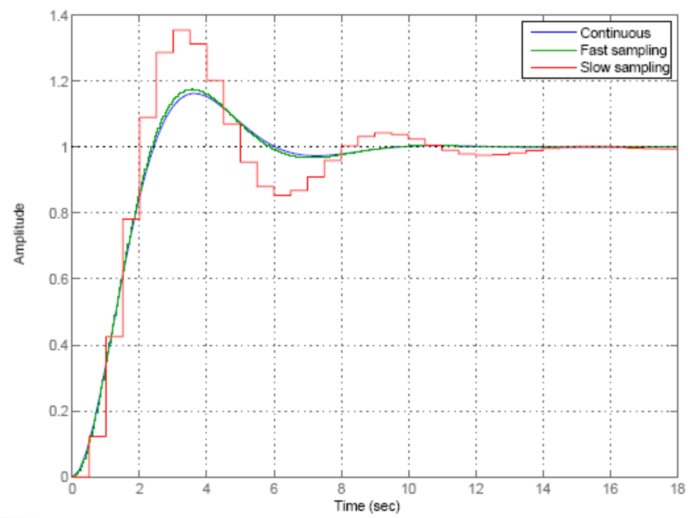
Continuous lead controller

$$D(s) = \frac{10s+1}{s+1}$$





Effect of Sample Time on Step Response



Direct Digital Control Design

Continuous Systems

For a minimum-phase transfer function, the phase is uniquely determined by the magnitude curve:

$$\angle G(j\omega) \approx n \times 90^\circ$$

where n is the slope of $G(j\omega)$ in units of decade of amplitude

Discrete Systems

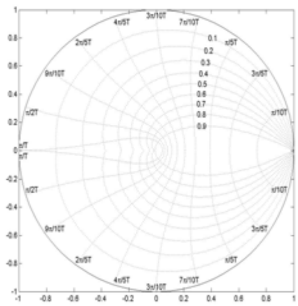
The amplitude and phase relationship is lost!
The prediction of stability from the amplitude curve alone for minimum-phase systems is lost
It is typically necessary to determine both magnitude and phase for discrete systems

Root Locus in the Z-Plane

- Evan's root locus method can be used for the direct digital design after the performance specifications have beentranslated into the z-plane
- **Z-plane specifications:**

  get acceptable pole location in the z-plane

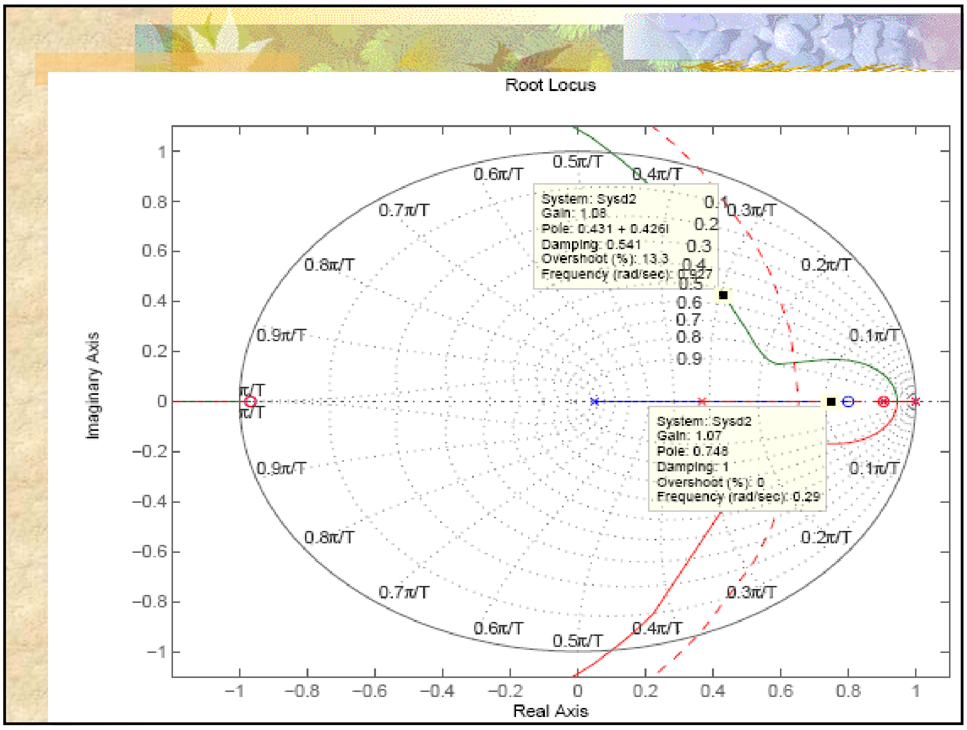
  - Natural frequency
  - Damping ratio
  - Overshoot and real part
  - Steady-state errors



$$K_p = \lim_{z\to 1} D(z)G_o(z) \qquad e_{ss} = \frac{1}{1+K_p}$$

$$K_v = \lim_{z\to 1} \frac{(z-1)(1+D(z)G_o(z))}{Tz} \qquad e_{ss} = \frac{1}{K_v}$$

  - **Example 7.5 p.224(DC)**



Root Locus
System: Sysd2
Gain: 1.08
Pole: 0.431 + 0.426i
Damping: 0.541
Overshoot (%): 13.3
Frequency (rad/sec): 0.927
System: Sysd2
Gain: 1.07
Pole: 0.748
Damping: 1
Overshoot (%): 0
Frequency (rad/sec): 0.29
Imaginary Axis
Real Axis