Stochastic Processes II (FP-7.5)
Problem set 7

Problem 1

Let us consider the signal model of the first-order Gauss-Markov process

$$Y(n) = 0.5Y(n-1) + Z(n) \qquad n > 0.$$

where

- $Y(0) \sim \mathcal{N}(0, 1)$
- $Z(n)$ in a zero-mean white Gaussian process with variance $\sigma_Z^2 = 2$.

We assume that the signal $Y(n)$ is observed in white Gaussian noise, i.e. the observation equation reads

$$X(n) = Y(n) + W(n)$$

with $W(n)$ denoting non-stationary zero-mean white Gaussian noise with variance $\sigma_W^2(n) = (1/2)^{n-1}$.

- Derive the equations of the Kalman filter.
- Plot the block diagram of the Kalman filter.
- Calculate the estimates $\hat{Y}(n|n)$, the mean squared estimation error $R(n|n)$ and the Kalman gain $b(n)$ for $n = 1, 2$.
- Implement the Kalman filter in Matlab and investigate the behavior of the mean squared estimation error as a function of n.