Stochastic Processes II (FP-7.5)
Problem Set 8

Problem 8.1 (Example 7.17 in Shanmugan)

Apply the Kalman filter to estimate the position and the velocity of a particle, using the system and channel model given in Example 7.17 in Shanmugan.

Remark: The Kalman filter to be derived is different from that described in the Example 7.17. The former filter estimates the present value of the position and the velocity while the latter performs a one-step prediction of these parameters.

1. Sketch the block diagram of the Kalman filter for the above example.
2. Calculate the prediction step and the updating step of the Kalman filter for the time indices $n = 1, 2$.
3. Implement the Kalman filter in Matlab and experimentally investigate its tracking ability.

Problem 8.2

Let us consider the following wide-sense stationary model:

- System Model
  $Y(n) = 0.9Y(n-1) + Z(n)$
  with $E[Z(n)Z(n+k)] = \delta(k)$,
- Channel Model
  $X(n) = Y(n) + W(n)$
  with $E[W(n)W(n+k)] = \delta(k)$.

Verify by means of Monte Carlo simulations the steady state of the Kalman filter estimating $Y(n)$:

$$\begin{aligned} R(n|n) &\rightarrow R(\infty] = 0.5974 \\ R(n+1|n) &\rightarrow R_p(\infty] = 1.4839 \\ B(n) &\rightarrow b(\infty] = 0.5974 \end{aligned}$$

as $n \rightarrow \infty$