

Stochastic Processes II (FP-7.5)

Problem set 9

Problem 9.1 Variance of \overline{X}

Compute the variance of the arithmetic mean

$$\overline{X} = \frac{1}{N} \sum_{n=0}^{N-1} X_n.$$

(see bottom of P.6-2 of the lecture notes).

Problem 9.2 Periodogram

Show that the periodogram can be obtained in the following two ways:

$$\hat{S}_{XX}(f) = \mathcal{F}\{\hat{R}_{XX}(k)\} = \frac{1}{N}|X(f)|^2$$

Hint: Write $\hat{R}_{XX}(k)$ as a convolution and then apply the property of the Fourier transform.

Problem 9.3 Wölfer sunspots numbers

Use Matlab to compute the following functions from the "Wölfer sunspots numbers" series:

- The sample (biased) autocorrelation and sample (biased) autocovariance function.
- The periodogram of this series. Identify the fundamental cycles of the series.
- The Blackman-Tukey estimator of the series using the spectral window given in the lecture notes (bottom Figure on Page 6.13).

The series can be downloaded from the course webpage.