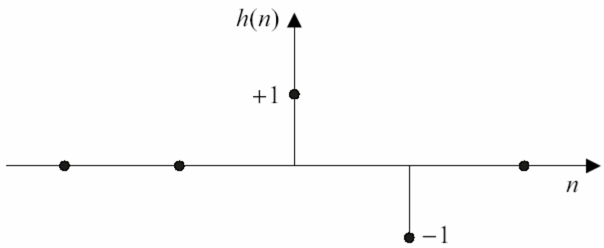
Input-output relationship:

$$Y(n) = X(n) - X(n-1)$$

Impulse response:

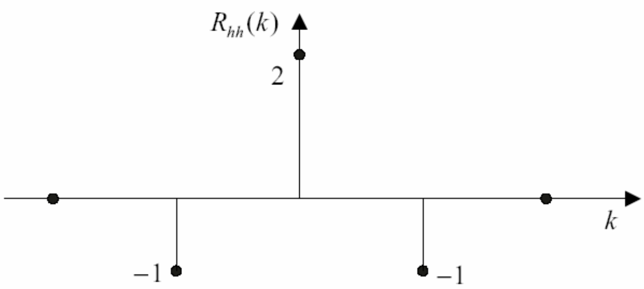
$$h(n) = \delta(n) - \delta(n-1)$$



$$h(n) = \delta(n) - \delta(n-1)$$

Autocorrelation function:

$$R_{hh}(k) = -\delta(k-1) + 2\delta(k) - \delta(k+1)$$



$$Y(n) = X(n) - X(n-1)$$

Transfer function:

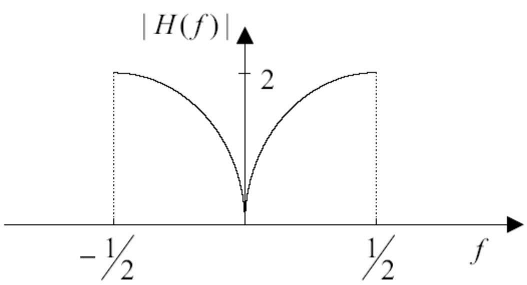
$$\begin{aligned} Y(f) &= X(f) - \exp(-j2\pi f)X(f) \\ &= [1 - \exp(-j2\pi f)]X(f) \end{aligned}$$

$$\begin{aligned} H(f) &= Y(f)/X(f) \\ &= [1 - \exp(-j2\pi f)] \\ &= \exp(-j\pi f)[\exp(j\pi f) - \exp(-j\pi f)] \end{aligned}$$

$$H(f) = 2j\exp(-j\pi f)\sin(\pi f)$$

$$H(f) = 2j\exp(-j\pi f)\sin(\pi f)$$

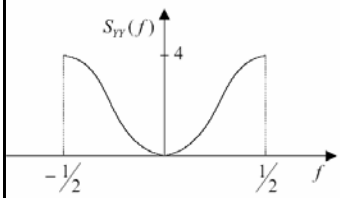
$$|H(f)| = 2|\sin(\pi f)|$$



Power spectrum of $Y(n)$:

$X(n)$ is a white process with unit variance

- $\diamond$ $E(X(n)) = 0 \quad \Rightarrow \quad E[Y(n)] = 0$
- $\diamond$ $R_{XX}(k) = \delta(k) \quad \Rightarrow \quad S_{XX}(f) = 1$



$$\begin{aligned} S_{YY}(f) &= |H(f)|^2 S_{XX}(f) \\ &= 4\sin^2(\pi f) \end{aligned}$$

$$S_{YY}(f) = 2[1 - \cos(2\pi f)]$$

Autocorrelation of $Y(n)$:

$$\begin{aligned} R_{YY}(k) &= R_{hh}(k) * R_{XX}(k) \\ &= R_{hh}(k) * \delta(k) \end{aligned}$$

$$R_{YY}(k) = R_{hh}(k).$$

Variance of $Y(n)$:

$$\begin{aligned} E[Y^2(n)] &= R_{YY}(0) \\ &= (R_{hh}(k) * R_{XX}(k))|_{k=0} \\ &= (R_{hh}(k) * \delta(k))|_{k=0} \\ &= R_{hh}(k)|_{k=0} \\ &= \sum_{m=-\infty}^{+\infty} h^2(m) \\ &= 1 + 1 \end{aligned}$$

$$E[Y^2(n)] = 2$$

Another method to compute the variance:

$$E[Y(n)^2] = R_{YY}(0) = \int S_{YY}(f)df = 2$$