SE Course: Stochastic Analysis for Engineers

http://www.cs.aue.auc.dk/~yang/course/stoc04.htm
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MM3: Signal Detection (Part One)

September 22, 2004

1 Reading

Chapt 6, pp.341-352 of K. Sam Shanmugan and Arthur M. Breipohl: "Random Signals - Detection, Estimation and Data Analysis ", John Wiley Sons, Inc., 1988.

2 Content

- Hypothesis testing;
- Decision rules;
- Binary detection.

3 Exercise

1. (Problem 6.2 in Textbook, page 371.)
   Suppose that we want to decide whether or not a coin is fair by tossing it eight times and observing the number of heads showing up. Assume that we have to decide in favor of one of the following two hypotheses:

   $$H_0: \quad \text{Fair coin}, P(head) = 0.5$$

   $$H_1: \quad \text{Unfair coin}, P(head) = 0.4$$

   (a) Derive the MAP decision rule assuming $P(H_0) = 0.5$;

   (b) Calculate the average probability of error.

2. (Problem 6.8 in Textbook, page 373.)
   In a detection problem, $f_{Y|H_1} \sim N(-1, 1)$, and $f_{Y|H_0} \sim N(1, 1)$. The notation $N(\mu, \sigma^2)$ denotes a Gaussian distribution with mean μ and variance σ^2.

   (a) Assume $P(H_0) = 1/3$ and $P(H_1) = 2/3$, find the MAP decision rule.

   (b) With $C_{00} = C_{11} = 0,\ C_{01} = 6$, and $C_{10} = 1,\ P(H_0) = 1/3,\ P(H_1) = 2/3$, find the decision rule that minimizes $\overline{C}$ and the value of $\overline{C}_{min}$.

3. Prepare your self-study materials and presentation.