Stochastic Processes II (FP-7.5)
Problem Set 4

Problem 4.1 (Problem 6.13 in Shanmugan)

The conditional pdfs corresponding to two hypothesis are given:

$$f_{Y|H_0}(y|H_0) = \frac{1}{2}\exp\left(-\frac{y}{2}\right), \quad 0 < y$$
$$f_{Y|H_1}(y|H_1) = \frac{1}{4}\exp\left(-\frac{y}{4}\right), \quad 0 < y$$

Suppose we want to test these hypothesis based on two independent samples Y_1 and Y_2. Assume equally likely priors.

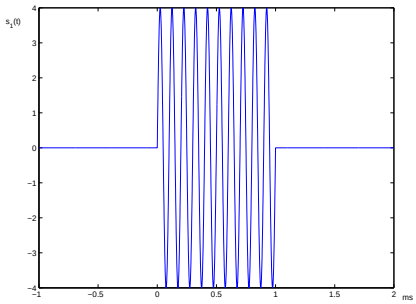
a. Derive the MAP decision rule for the test.

b. Calculate $P_M = P(D_0|H_1)$ and $P_F = P(D_1|H_0)$.

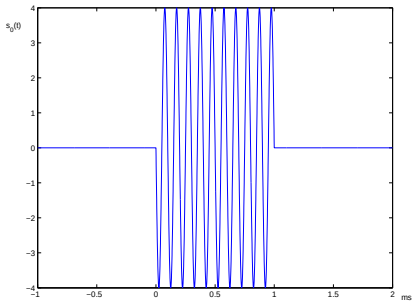
Problem 4.2 (Problem 6.14 in Shanmugan)

The signaling waveforms used in a binary communication system are

$$\begin{aligned} s_1(t) &= \begin{cases} 4\sin(2\pi f_0 t) & , \quad 0 \le t \le T \\ 0 & , \quad \text{elsewhere} \end{cases} \\ s_0(t) &= -s_1(t), \end{aligned}$$



$s_1(t)$



$s_0(t)$

$$P[x(t) = s_1(t)] = P[x(t) = s_0(t)] = \frac{1}{2}$$

where $T = 1$ms is the signal duration and $f_0 = 10/T$.

Assume that the signal is corrupted by zero-mean additive white Gaussian noise with power spectral density

$$S_{WW}(f) = 10^{-3} \text{ W/Hz}$$

a. Find the decision rule that minimizes the average probability of error P_e.

b. Find the value of P_e.