Stochastic Processes II (FP-7.5) Problem Set 6

Problem 6.1 (Problem 7.15 in Shanmugan)

Given that

- $Y(n)$: zero-mean WSS unobservable sequence with autocorrelation function $R_{YY}(k)$;

- $X(n)$: zero-mean WSS observable sequence with autocorrelation-function $R_{XX}(k)$.

The "dependency" between both sequences is described by the cross-correlation function $R_{XY}(k)$.

Find the LMMSEE of $Y(n)$ based on the observation of $X(-M), ..., X(M)$:

$$\hat{Y}(n) \;=\; \sum_{m=-M}^{+M} h(m)X(n-m). \tag{1}$$

Problem 6.2

LMMSEE of a noisy WSS sequence with finite memory:

Let $Y(n)$ denote a zero-mean WSS sequence with autocorrelation function:

$$R_{YY}(0) = \frac{3}{4}, R_{YY}(-1) = R_{YY}(1) = \frac{1}{4}, R_{YY}(k) = 0 \quad \text{for } |k| > 1.$$

The observable sequence is of the form

$$X(n) = Y(n) + W(n)$$

where $W(n)$ is white noise with variance $\frac{1}{4}$.

a) Find the Wiener-Hopf equations for the coefficients of the noncausal Wiener filter of length $2M + 1$ and the causal Wiener filter of length $M + 1$ estimating $Y(n)$ based on the observation of $X(n)$;

b) Calculate the filter coefficients for $M = 1$. Compute the mean-square estimation errors resulting when using both filters.