

# PE Course: Fault Tolerant Control (Part Two)

<http://www.cs.aue.auc.dk/~yang/DE9/FTC/FTC2005.htm>

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## MM1: Robust Control for FTC Analysis and Design -I

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### 1 Reading

- Lecture notes provided by Palle Andersen for 8th Semester course - Robust Control (<http://www.control.auc.dk/pa/kurser/robust/index.html>)
- Chapter 3-5, J.C. Doyle, B.A. Francis and A.R. Tannenbaum, "Feedback Control Theory", Macmillan Publishing Company, 1992.
- *User's Guard/Getting started* for Robust Control Toolbox in Matlab (<http://www.mathworks.com/access/helpdesk/help/toolbox/robust/>)

### 2 Content

- Why the robustness is required in fault tolerant control analysis and design?
  - Robustness of FDD;
  - Robustness of passive FTC;
  - Robustness of active FTC.
- Review of robust control techniques(I)
  - Fundamental principles and Basic concepts: robustness, uncertainty, norms, internal stability, robust stability, robust performance, ...
  - Robust analysis: uncertainty modelling, singular value, small gain theorem,  $\mu$ -analysis, ...
  - Connection with FTC, ...
- Introduction to Robust Control Toolbox/Robust Analysis.

### 3 Exercise

Consider a LTI system described by the state space form:

$$\dot{X} = AX + Bu, \quad y = CX, \quad (1)$$

where the system matrices are

$$A = \begin{pmatrix} -1 & 0 \\ 4 & -3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 1 \end{pmatrix}. \quad (2)$$

A simple P controller is developed for this system in order to reach some specific performance, where the controller is  $K = 1.5$ .

1. Is the controlled closed-loop system stable or not? If stable, what's the Gain Margin and Phase Margin (you can use Matlab function);
2. Due to some changing environment or some possible failures of the components, system matrices have some deviations comparing with the nominal values given in (2). We denote the perturbed matrices as

$$\tilde{A} = A + \Delta A, \quad \tilde{B} = B + \Delta B, \quad \tilde{C} = C + \Delta C. \quad (3)$$

In order to analyze the robustness of the P controller towards these deviations  $\Delta A$ ,  $\Delta B$ ,  $\Delta C$ , i.e., to find out the largest boundary of these deviation such that the P controller can still guarantee the stability of the perturbed closed-loop system, formalize the above robust analysis into a standard " $\Delta$ -M" formulation;

3. What's the allowed largest boundary of these deviations under the stability constraint? Write your Matlab codes for this analysis, and give the comments about the result, e.g., is it conservative or not?