Fault Tolerant Control (Part Two)

PE course, 1 ECTS

Zhenyu Yang

Objective

Investigate how to use some advanced control techniques for Fault Tolerant Control (FTC) design and analysis

Content

- **Robust control techniques for FTC analysis and design (MM1-3)**
- **Adaptive control techniques for FTC analysis and design (MM4)**
- **Model predictive control techniques for FTC design (MM5)**

Software

Matlab/simulink with robust control toolbox

From Youmin Zhang's lecture – FTC part one:

What is Fault-Tolerant Control System (FTCS)?

Definition: A FTCS is a control system that possesses the ability to accommodate system component faults/failures *automatically* and is capable of maintaining overall system stability and acceptable performance in the event of such failures.

Objectives: Increase reliability, safety and automation level of modern technological/engineering systems.

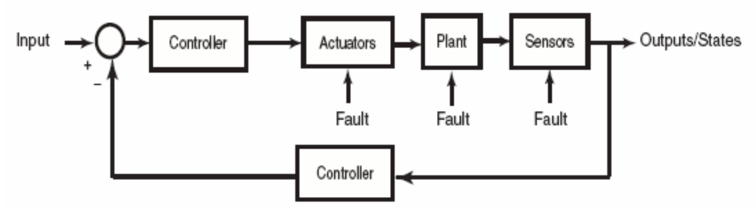


Figure 2.1: Location of potential faults in a control system.

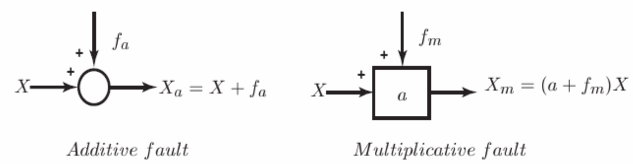
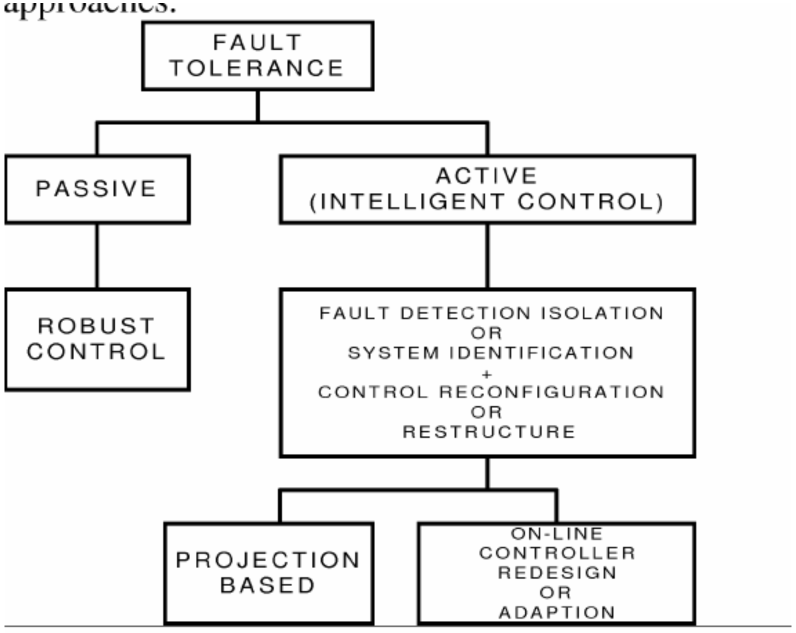


Figure 2.2: Different fault-induced changes.

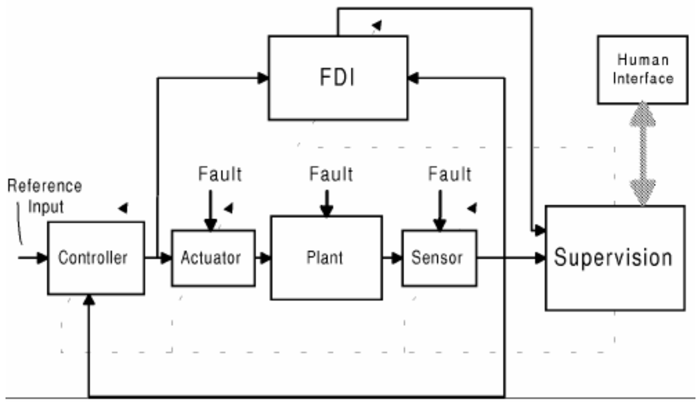
FAULT-TOLERANT CONTROL SYSTEMS: THE 1997 SITUATION

Ron J Patton
The University of Hull,
School of Engineering,
Hull HU6 7RX, UK



FAULT-TOLERANT CONTROL SYSTEMS: THE 1997 SITUATION

Ron J Patton
The University of Hull,
School of Engineering,
Hull HU6 7RX, UK



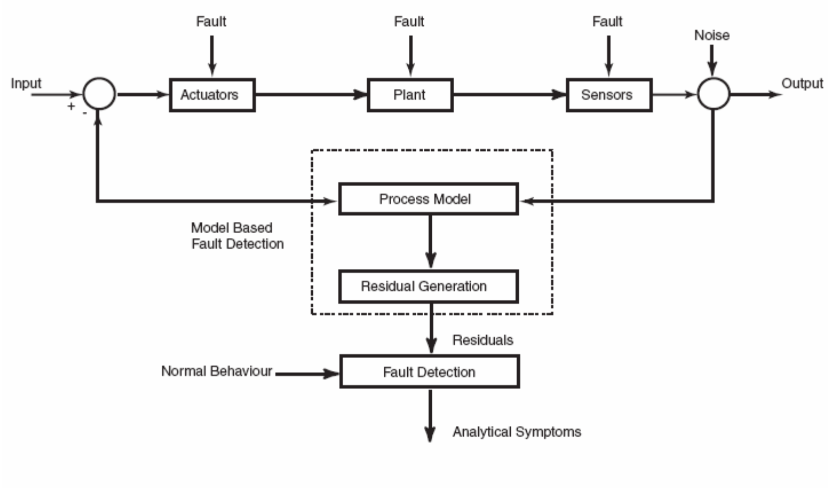
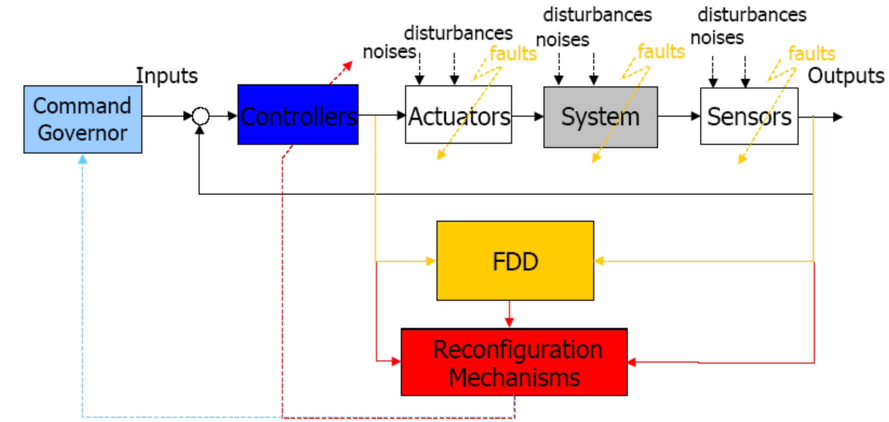


Figure 2.4: General scheme of model-based fault detection.

MM1: Robust control for FTC

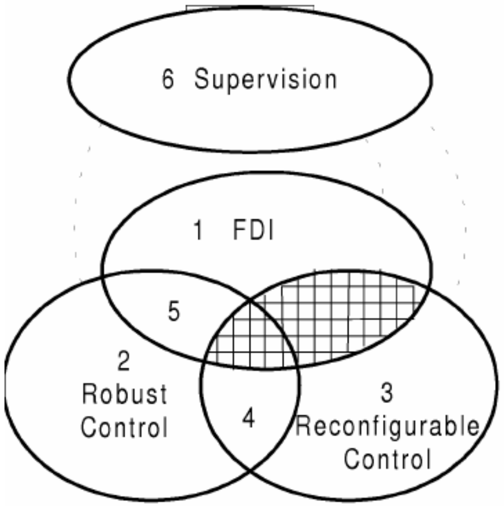
From Youmin Zhang's lecture – FTC part one:



One scheme of active fault tolerant control

FAULT-TOLERANT CONTROL SYSTEMS: THE 1997 SITUATION

Ron J Patton
The University of Hull,
School of Engineering,
Hull HU6 7RX, UK

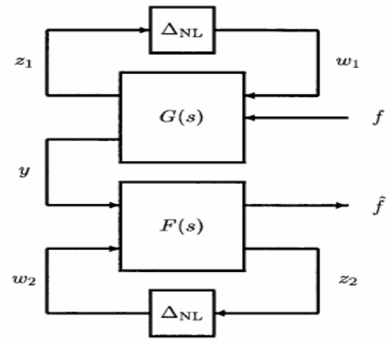


The scattered areas of fault-tolerant control research

Fault Detection for Nonlinear Systems - A Standard Problem Approach

Jakob Stoustrup
Dept. of Control Engineering
Aalborg University
DK-9220 Aalborg, Denmark
E-mail: jakob@control.auc.dk
URL: www.control.auc.dk/~jakob

Henrik Niemann*
Department of Automation
Technical University of Denmark
Building 326, DK-2800 Lyngby, Denmark
E-mail: hhn@iau.dtu.dk



e 2: Fault detection for a nonlinear system

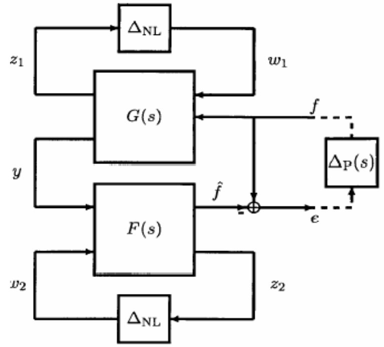


Figure 3: Introduction of a performance block

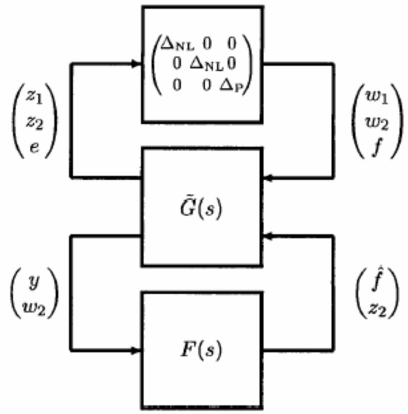


Figure 4: Standard problem formulation

Proceedings of the American Control Conference
Chicago, Illinois • June 2000

The Robust Control Mixer Module Method for Control Reconfiguration

Zhenyu Yang *and* Mogens Blanke

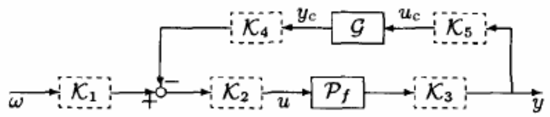


Figure 1: Considered System Configuration with Possible Control Mixer Modules (dash-boxes)

tem. Then the robust control mixer module (RCMM) design problem can be defined as: *(1) Selecting a subset of the compensating systems* $\mathcal{K}_i$ $(i = 1, \cdots, 5)$ *from Fig. 1, which can be different from the identity matrix, and denoting the selected subset by* $\mathcal{K} \hat{=} \{\mathcal{K}_i\}$; *Furthermore, (2) Designing the selected modules* $\mathcal{K}_i$ *by solving the optimization problem*

$$\min_{\mathcal{K}} \|(\mathcal{F} - \mathcal{F}_f(\mathcal{K}))\mathcal{W}\|_\infty \tag{6}$$

under the condition that the reconfigured closed-loop system is internally stable, where $\mathcal{W}$ *is a weighting function.*

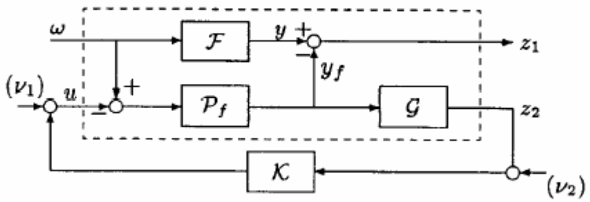
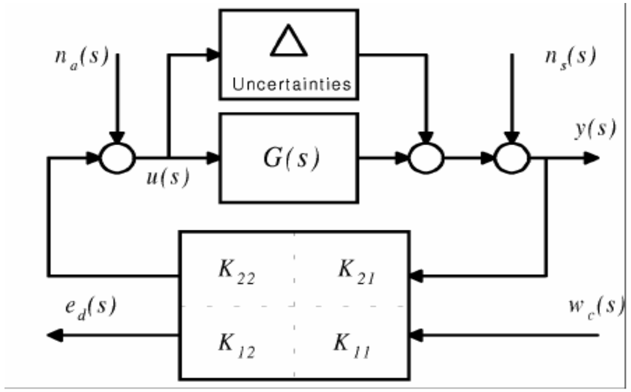


Figure 4: The Augmented Control System with Controller $\mathcal{K}$ $(\mathcal{K}_4)$

FAULT-TOLERANT CONTROL SYSTEMS: THE 1997 SITUATION

Ron J Patton
The University of Hull,
School of Engineering,
Hull HU6 7RX, UK



Four parameter scheme for FDI (Robust control based design)

The H_∞ optimisation approach is employed to achieve the following properties:

1) Plant output signal tracks reference commands and is insensitive to actuator faults;
2) Diagnostic output signal tracks actuator faults (abrupt and slow ramp-type faults);
3) Properties (1) and (2) hold in the presence of a bounded uncertainty.

Proceedings of the American Control Conference
Anchorage, AK May 8-10, 2002

Multiple Objective Robust Control Mixer Method for Synthesis of Reconfigurable Control

Zhenyu Yang *and* David L. Hicks

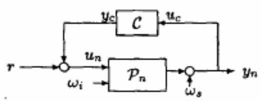


Figure 1: The Considered LTI Control System

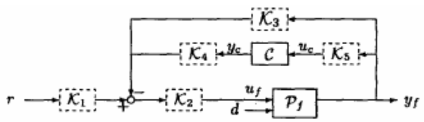


Figure 2: Possible Control Mixer Modules in Considered Configuration

Find a minimum-element set of non-identity (or different with previous values) compensating modules from a permitted set with N elements, denoted as $\{\mathcal{K}_i\}_{i=1}^{l^*}$, *where* $\mathcal{K}_i$ *are real-rational and proper for* $i = 1, \cdots, l^*$, *denoted as* $\mathcal{K}_i \in \mathcal{R}_{ss}$, *such that*

$$\min_{\mathcal{K}_i, i=1,\cdots,l} \|\mathcal{W}\begin{pmatrix} \mathcal{P}_{nc} - \mathcal{P}_{fc}(\mathcal{K}_1,\cdots,\mathcal{K}_l) \\ \overline{\mathcal{P}}_{perf} - \overline{\mathcal{P}}_{fc}^{perf}(\mathcal{K}_1,\cdots,\mathcal{K}_l)\end{pmatrix}\|_\infty \tag{5}$$

under the condition that the reconfigured system is internally stable. Here

$$l^* = arg \min_{l \leq N} \text{(equation (5))}, \tag{6}$$

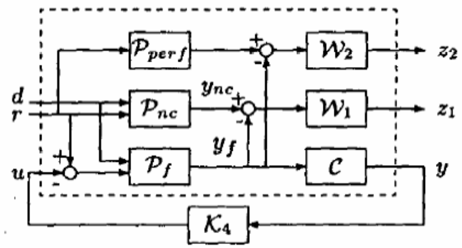


Figure 5: Augmented System for MORCM $\mathcal{K}_4$ Synthesis

Proceedings of the 39th IEEE
Conference on Decision and Control
Sydney, Australia • December, 2000

Robust Reconfigurable Control for Parametric and Additive Faults with FDI Uncertainties

Zhenyu Yang *and* Jakob Stoustrup

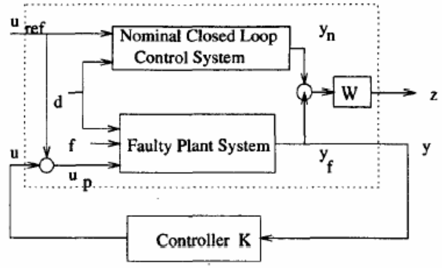


Figure 1: The Fictitious Augmented Control System

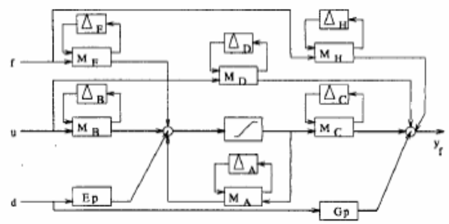


Figure 2: Faulty Plant with FDI Uncertainties

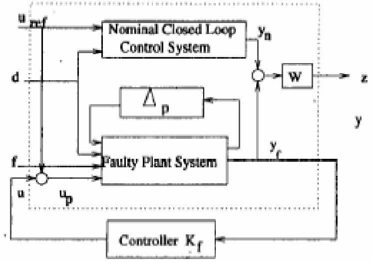


Figure 3: The Augmented Control System

Starting:

What does robustness mean?

Insensitive to what – uncertainty

e.g., person is sensitive to change of temperature: a little bit high or low – uncomfortable, reduce/add clothes -→ take care of him, otherwise becomes sick!

From engineering point of view, we are dealing with physical systems, systems run good or bad, depend on the features of the system – are they sensitive to the changing environmental situations? Especially unknown (at least cannot be known in advance) situations. (1) Unknown things are always all the time existing with us! --called it uncertainty!

Then, if the system operation is insensitive to the existing uncertainty, we say the system is robust – robustness (wrt. Some kind of uncertainty) is one feature of the system

Why robustness?

- System keeps a stable satisfactory operation if it is designed properly
- System is reliable – especially safe-critical systems, e.g., aircrafts, nuclear reaction system,…

What's the specific connection with FTC?

FT system definitely needs robustness feature.

- Faults may be detectable or not. Once the possible failures can be considered into the uncertainty tolerance of the system, then robustness of the system implies the fault-tolerance. → **passive FTC methodology**
- Even though the faults can be detected through FDD, and then some reconfiguration will be taken to get the faulty system back normal or degraded performance, like the active FTC methodology, robustness of the whole procedure *(robustness of system, robustness of method/procedure)* is also unavoided: Robust FDD/FDI, robust (structure/control) reconfiguration due to the fact that the uncertainty exists anywhere and anytime. Within limited time (even though infinity time), we can't get 100% accuracy of the system knowledge, fault estimation, changing environment etc.

Robust active FTC is required!!

How to achieve/improve robustness of the system?

First step: need to know how to do robustness analysis –

- How to describe the uncertainty, especially in a quantitative way;
- How to analyze the uncertainty influence of the system stability;
- How to analyze the uncertainty influence of the system performance;
- are there any available tools to help this kind of analysis.

Second step: need to know some systematic and feasible way to achieve robustness –

- Back the product design stage…
- **Through feedback control!**

ROBUST CONTROL THEORY AND ROBUST CONTROL TOOLBOX/MATLAB

Sem8 robust control course,

here two tasks:

1. **review the necessary knowledge of robust control and be able to run robust control toolbox for some robust analysis and synthesis**
2. **how to use robust control theory in the FTC analysis and design**.

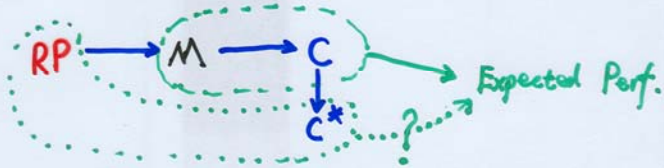
MM1: Robust control for FTC

①

Robust Control (Sem8)

Design a control C for a math. model M and what the corresponding real process RP to behave well

- Problems:
  - ★ RP ≠ M ?
  - ★ Even RP = M there is always a round-off etc errors in controller implementation?



- Robustness Philosophy.

The controller C is called robust if

$$\left.\begin{array}{l} RP \approx M \\ C^* \approx C \end{array}\right\} \Rightarrow (RP, C^*) \approx (M, C)$$

- Q. (1) What's the meaning "≈"?
  (2) How to check it? – Analysis
  (3) How to find the controller? – Design

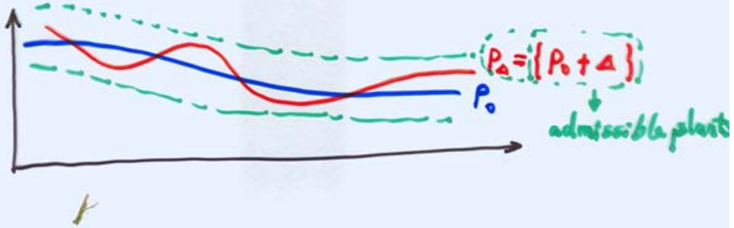
2

Uncertainty in the System

- Modelling $\begin{cases} \text{As external inputs} \\ \text{As perturbations to the nominal plants} \end{cases}$

e.g.. $y = (P_0 + \Delta)u + n$ → unknown noise/disturbance

output, nominal, unknown plant perturbation, input



Connection to FTC ?

4

Metric Spaces

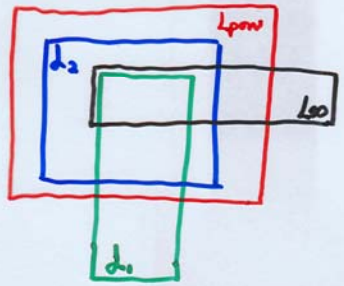
- Norms for signals

1-norm: $u \in \mathcal{L}_1$, $\|u\|_1 = \int_{-\infty}^{\infty} |u(t)|\, dt$ $\geq \mathcal{L}^2$

2-norm: $u \in \mathcal{L}_2 . \{u \mid \|u\|_2 = \left(\int_{-\infty}^{\infty} |u(t)|^2 dt \right)^{\frac{1}{2}} < \infty\}$

∞-norm: $u \in \mathcal{L}_\infty$, $\|u\|_\infty = \sup_t |u(t)|$

Power-norm: $u \in \mathcal{L}_{pow}$, $\|u\|_{pow} = \left(\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} u^2(t)\, dt \right)^{\frac{1}{2}}$



Relationship:

- If $u \in \mathcal{L}_2$, $\Rightarrow \|u\|_{pow} = 0$
- If $u \in \mathcal{L}_{pow}$ and $u \in \mathcal{L}_\infty \Rightarrow \|u\|_{pow} \leq \|u\|_\infty$
- If $u \in \mathcal{L}_\infty$, $u \in \mathcal{L}_1 \Rightarrow \|u\|_2 \leq (\|u\|_\infty \|u\|_1)^{\frac{1}{2}}$ and $u \in \mathcal{L}_2$

Metric Spaces

5

- Norms for systems "space of systems"
- The space of linear systems $\forall G, G_2 \in \mathcal{L}$

(*) $y_1 = G_1 u$, $y_2 = G_2 u$ $\Rightarrow (G_1 + G_2)u = y_1 + y_2$

(*) $y = Gu \Rightarrow \lambda G u = \lambda y \quad \lambda \in R^n$

what does it mean $G_1 \approx G_2$?

- System can be considered as an operator:

$$G: U \to Y$$

$$\|G\| = \overset{\lim}{\sup_{\|u\|_U \le 1}} \|GU\|_Y$$

$\frac{n}{n+1} \xrightarrow{n\to\infty} 1$

$\sup_n \{\frac{n}{n+1}\}$

$\lim_{n\to\infty} \{\frac{n}{n+1}\} = 1$

output \ input	$\|U\|_2$	$\|U\|_\infty$	$\|u\|_{pow}$
$\|Y\|_2$	$\|G\|_\infty$	∞	∞
$\|Y\|_\infty$	$\|G\|_2$	$\|G\|_1$	∞
$\|Y\|_{pow}$	0	$\le \|G\|_\infty$	$\|G\|_\infty$

System Spaces: $\mathcal{L}_\infty$, $\mathcal{L}_2$, $\mathcal{L}_1$, (RH_∞)

MM1: Robust control for FTC

Metric Spaces

6

- Norms for TFs

$$\|G\|_2 = \left(\frac{1}{2\pi}\int_{-\infty}^{\infty} |G(jw)|^2 dw\right)^{\frac{1}{2}}$$

$$\|G\|_\infty = \sup_\omega |G(jw)|$$

- Norms for SS $\begin{cases} \dot{X} = AX + BU \\ Y = CX + DU \end{cases}$

Theorem1: Let $G(s) = C(sI - A)^{-1}B$ and A is stable

$$\|G\|_2^2 = tr(B^T L_o B) = tr(C L_c C^T)$$

where

L_o is the observability Gramiane, L_c is the Controllability Gramian, i.e.,

$$A L_c + L_c A^T + BB^T = 0$$

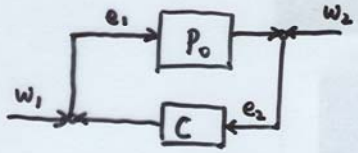
$$A^T L_o + L_o A + C^T C = 0$$

Four Classical Problems

8

To find a controller C such that

• NS: (P_0, C) is internally stable (Youla Parameterization



$$e_1 = Ce_2 + w_1$$
$$e_2 = P_0 e_1 + w_2$$
$$\Updownarrow$$
$$\begin{bmatrix} I & -C \\ -P_0 & I \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

• The closed-loop system is internally stable if

$$\begin{bmatrix} I & -C \\ -P_0 & I \end{bmatrix}^{-1} \in RH_\infty$$

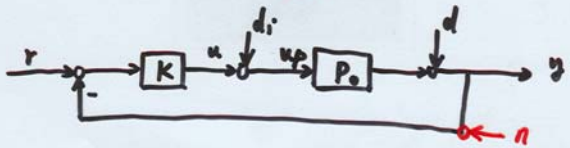
↳ Exists for almost all s and is a proper function

⇓

well-posedness (solvability)

e.g. $P_0(s) = \frac{s+1}{s+2}$, $C(s) = 1$, then

$$\text{closed-loop} = \frac{\frac{s+1}{s+2}}{1 - \frac{s+1}{s+2}} = s+1 \quad ?$$



MM1: Robust control for FTC

Four Classical Problems

9

• NP: NS + (P_0, C) has a good performance (LQG, H_∞, etc)

• RS: (P_Δ, C) is internally stable for all $P_\Delta \in \mathcal{P}_\Delta$

$$\mathcal{P}_\Delta = \{ P_0 + \Delta P \}$$

(Small Gain Theorem, μ-analysis)

RS for different uncertainty models are different.

• RP: RS + (P_Δ, C) has a good performance for all $P_\Delta \in \mathcal{P}_\Delta$

(Robust Control from Sem 8, Palle Andersen)

☺ $\boxed{\text{connection to FTC}}$

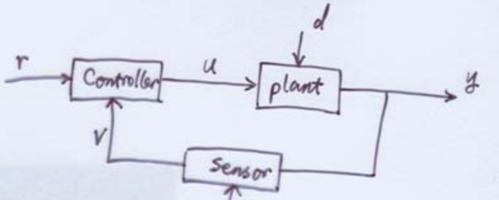
• passive FTC ✓

• FDD Active FTC ?

Internal Stability (I)

- Motivation:

$\boxed{\text{BIBO criterion is not enough to analyze the stability.}}$



r: reference / command input
v: sensor output
u: actuating signal, plant input
d: external disturbance
y: plant output
n: sensor noise

Example,

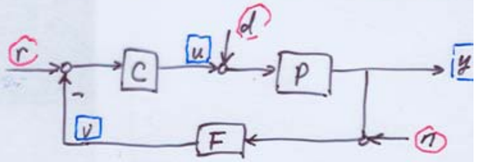
$C(s) = \frac{s-1}{s+1}$

$P(s) = \frac{1}{s^2-1}$

$F(s) = 1$

G_{ry} is stable ☺

How about G_{dy} ?



- Definition:

If all the nine transfer functions are stable (BIBO), then the feedback system is said to be $\boxed{\text{internally stable}}$

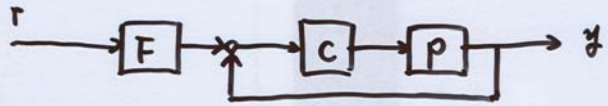
- Characteristics:

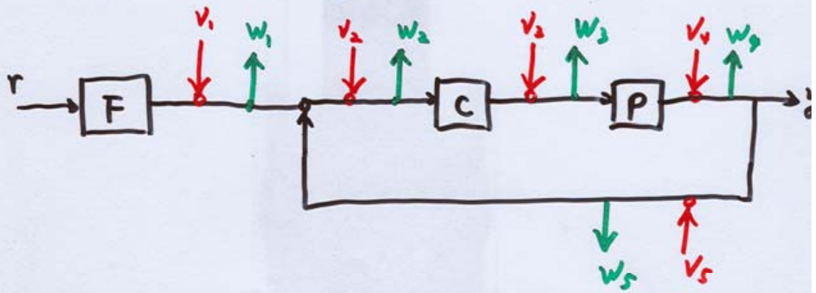
Internal stability guarantees bounded internal signals for all bounded exogenous signals.

Internal Stability

- Inject "internal" signals into each "exposed interconne[ction]" of the system. and define additional "internal" output signals after each injection point

Example:





- If each component is stabilizable and detectab[le] (no hidden unstable modes) then

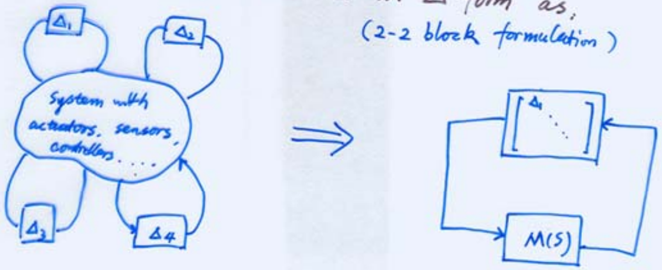
input-output stability $\Longleftrightarrow$ internal stability

Robust Control Analysis

- Objective: How big can Δ be before instability occurs.
- (Multiple-variable Stability Margin)
- Procedure:

(a) Define the uncertainty model

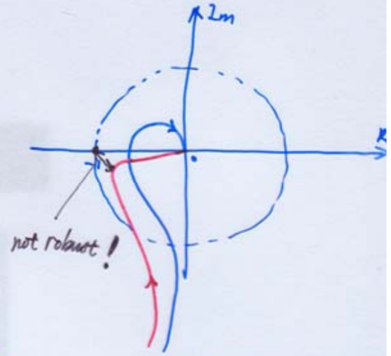
(b) Pull out the uncertainty channels (structured or unstructured) into a M-Δ form as, (2-2 block formulation)



(C) Small-gain Theorem (standard)

Robust Analysis – Classical Approach

- SISO stability margins
  - * Gain margin
  - * Phase margin
  - * Nyquist plot



- MIMO Stability – Generalized Nyquist Stability Theorem

A MIMO system is stable iff the Nyquist loci of the eigenvalues of the loop transfer function L(S) encircle "-1" once counterclockwise for each unstable pole of L(S)

* Computation of eigenstructure is a numerically sensitive process in that a small variation in one matrix element can result huge changes in eigenvectors.

* GNST is not sufficient in robust analysis! Give too optimistic result!

Robust Analysis - Modern Approach

[J.C. Doyle et al, 1981, M.G. Safonov et al. 1981] started to realize that using the singular value and its related robustness tests, some difficulties associated with the classical methods can be substantially overcomed

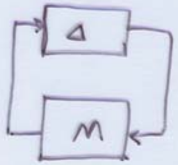
- Singular-value stability Robustness Theorem

The M-Δ system is stable for all stable $\Delta(s)$ satisfying

$$\bar{\sigma}(\Delta(j\omega)) < \frac{1}{\bar{\sigma}(M(j\omega))}$$

or $\quad \|\Delta\|_\infty < \frac{1}{\|M\|_\infty}$

for all $\omega \in R$.



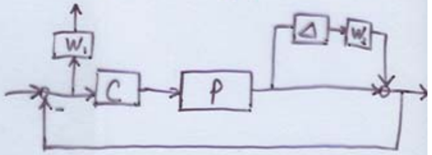
? Exercise!

- Multiplicative uncertainty model.

The controller C provides Robust Stability iff

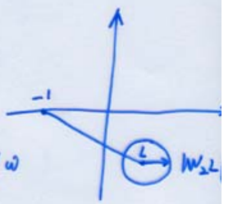
$$\|W_2 T\|_\infty < 1$$

for all stable Δ with $\|\Delta\|_\infty < 1$



$$\|W_2 T\|_\infty < 1 \iff \left| \frac{W_2(j\omega) L(j\omega)}{1 + L(j\omega)} \right| < 1 \quad \forall \omega$$

$$\iff |W_2(j\omega) L(j\omega)| < |1 + L(j\omega)|, \ \forall \omega$$



Singular Values & H_2, H_∞ Norms (7-1)

- The singular values of a rank r matrix $A \in C^{m \times n}$, denoted as σ_i are the non-negative square-roots of the eigenvalues of A^*A ordered such that $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \cdots \sigma_p$, $p = \min\{m, n\}$

- Singular-value decomposition (SVD)

$\exists$ two unitary matrices $U \in C^{m \times m}$, $V \in C^{n \times n}$ and a diagonal matrix $\Sigma \in R^{m \times n}$ such that

$$A = U \Sigma V^* = U \begin{bmatrix} \Sigma_r & 0 \\ 0 & 0 \end{bmatrix} V^*$$

- The greatest singular value $\sigma_1 \triangleq \bar{\sigma}(A)$

The least singular value $\sigma_n \triangleq \underline{\sigma}(A)$

- Some useful properties:

(1) $\bar{\sigma}(A) = \max_{x \in C^n} \frac{\|Ax\|}{\|x\|}$

(2) $\underline{\sigma}(A) = \min_{x \in C^n} \frac{\|Ax\|}{\|x\|}$

(3) $\underline{\sigma}(A) \leq |\lambda_i(A)| \leq \bar{\sigma}(A)$. λ_i eigenvalue of A

(4) If A^{-1} exists, $\underline{\sigma}(A) = \frac{1}{\bar{\sigma}(A^{-1})}$, $\bar{\sigma}(A) = \frac{1}{\underline{\sigma}(A^{-1})}$

(5) $\bar{\sigma}(\alpha A) = |\alpha| \bar{\sigma}(A)$

(6) $\bar{\sigma}(A+B) \leq \bar{\sigma}(A) + \bar{\sigma}(B)$, $\bar{\sigma}(AB) \leq \bar{\sigma}(A)\bar{\sigma}(B)$

(7) $\underline{\sigma}(A) - \bar{\sigma}(E) \leq \underline{\sigma}(A+E) \leq \underline{\sigma}(A) + \bar{\sigma}(E)$

(8) $\sum_{i=1}^{p} \sigma_i^2 = \text{Trace}(A^*A)$

(7-2)

- For stable transform matrices $G(s) \in C^{m\times n}$, $p = \min\{m, n\}$
The H_2-norm and H_∞-norm can be defined in terms of the frequency dependent singular values $\sigma_i(j\omega)$ as:

H_2-norm: $$\|G\|_2 \triangleq \left[\int_{-\infty}^{\infty} \sum_{i=1}^{p} (\sigma_i(j\omega))^2 \, d\omega \right]^{\frac{1}{2}}$$

H_∞-norm: $$\|G\|_\infty \triangleq \sup_{\omega} \bar{\sigma}(G(j\omega))$$

(the least upper bound)

- Intepretation of SVD:
Any real matrix A geometrically maps a hyper-sphere of unit radius into a hyper-ellipsoid. The singular values $\sigma_i(A)$ specify the length of the main axis for the ellipsoid. The singular vectors u_i specify the orthogonal directions of these main axes, and the singular vectors v_i are mapped to the u_i vectors with a gain of σ_i, i.e., $Av_i = \sigma_i u_i$