SE Course: Numerical Methods

http://www.cs.aaue.dk/~yang/course/NMbasis/NM2010.htm
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MM3: Iterative Solution of Equations

1 kl.8:15-9:00, Review of MM2 and Some Examples

- What we talked in MM2;
- Examples of using Taylor expansions;
- Matlab implementations.

2 kl.9:10-10:40, Exercises for MM2

Question One:

Regarding to the function $f(x) = cos(x)$,

- Derive the Taylor expansion of it up to 5th order at the point $x = 0$;
- Use the above polynomial to approximate $cos(-0.2)$ and
- Evaluate the approximation error using Taylor's Theorem.

Question Two:

(Exercise 3.2.2 and 3.2.2, page 60) Function $ln(1 + x)$ can be approximated by a power series as

$$ln(1+x) = -\sum_{k=0}^{\infty} \frac{(-1)^{k+1}x^{k+1}}{k+1} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \cdots \quad (1)$$

- Write a Matlab m-file to approximate $ln(1.25)$ using the first 6 terms of equation (1);
- How many terms of the series (1) are needed to approximate $ln(1.25)$ with error smaller than 10^{-6}?
- Use Matlab's built-in function $log()$ to verify that the error of the above second analysis is indeed within the tolerance.

Question Three:

The function $erf(x)$ is defined as

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt. \quad (2)$$

This function is often used in the probabilistic analysis of normalized stochastic variable.

- Derive the series approximation of function e^x up to 9th order at point $x = 0$;
- Use the series approximation obtained in last step to approximate e^{t^2} and thereby prove that the series approximation of function erf looks like

$$\widehat{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{9} \frac{(-1)^k x^{2k+1}}{(2k+1)k!}. \quad (3)$$

- Write a Matlab m-file to realize the approximation $\widehat{erf}(x)$ and plot this approximation within interval $[-2, 2]$ and $[0, 4]$, respectively;
- Use the Matlab's built-in function $erf()$ and plot the difference (errors)between this function and approximation $\widehat{erf}(x)$ within interval $[-2, 2]$ and $[0, 4]$, respectively;
- How to evaluate the approximation errors? Use the Matlab function $norm(x, p)$ to evaluate the L_1-norm, L_2-norm and L_∞-norm of the errors within interval $[-2, 2]$ and $[0, 4]$, respectively.

3 kl.10:50-11:30, Iterative solution of equations

- Reading material: Subsection 2.1, 2.2, 2.3 in Textbook.