Signal Processing

(Part One: Digital Filter Design)

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Contents

- **Digital filtering**
  - Synthesis of FIR, IIR filters
  - Filter property analsis (frequency analysis)
  - Realization/implementations
- **Spectral analysis**
  - Discrete Fourier transformation(DFT)
  - Fast Fourier Transform (FFT) algorithms
  - Effects of Windows, zeropadding, resolution

Motivation Examples

- **Filtering wanted signal**

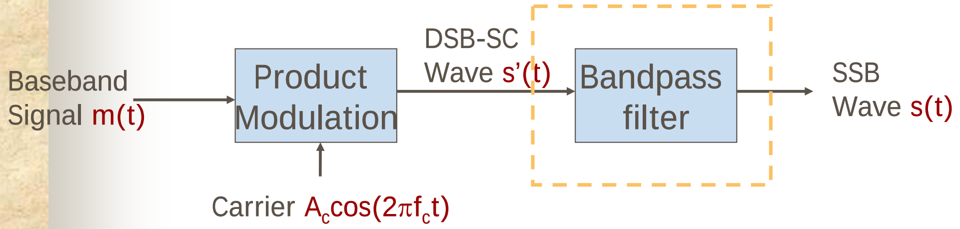
  Matlab demo: signal processing – filtering...
- **Digital feedback control**

  Robust control of a mechanic system
- **Noise cancellation**

  Active Noise Control (ANC)

SSB Modulation: **Principle**

- Frequency-discrimination method
  - Step 1: generate a DSB-SC wave
  - Step 2: get a bandpass filter – one sideband pass



SSB Modulation: **Spectrum**

- Precondition:

  message signal should have an energy gap centered at the origin

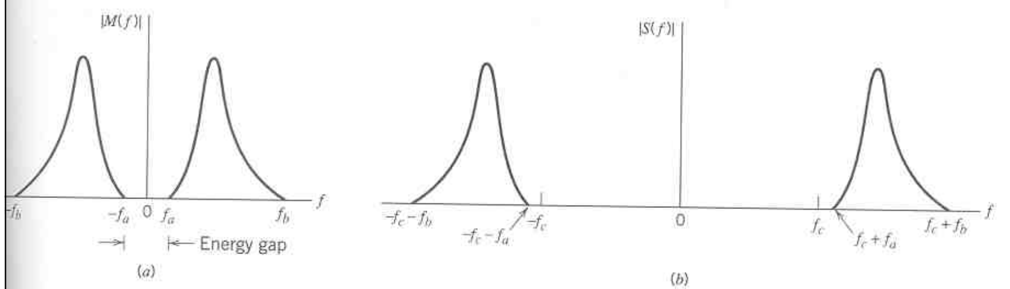
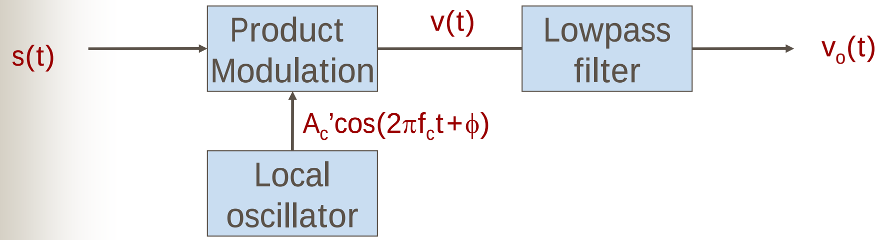


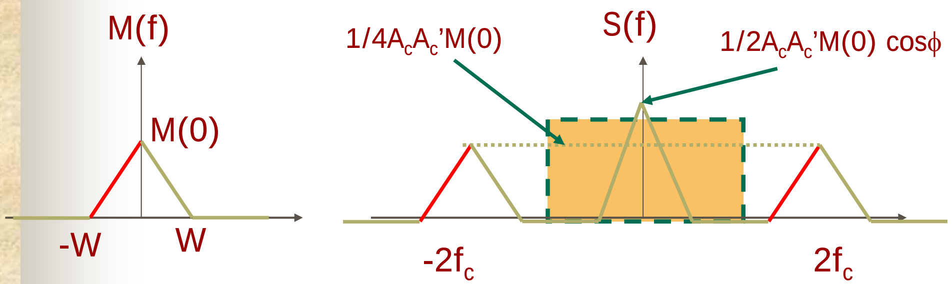
FIGURE 2.11 (a) Spectrum of a message signal $m(t)$ with an energy gap of width $2f_a$ centered on the origin. (b) Spectrum of corresponding SSB signal containing the upper sideband.

DSB-SC Modulation: **Coherent Detection (I)**



- $v(t) = 1/2 A_c A_c' \cos(4\pi f_c t + \phi) m(t) + 1/2 A_c A_c' \cos\phi \, m(t)$
- $v_o(t) = 1/2 A_c A_c' \cos\phi \, m(t)$

DSB-SC Modulation: **Coherent Detection (II)**



- **Quadrature null effect**

Part One. Digital Filter Design

http://www.cs.aaue.auc.dk/~yang/course/filter08.html

Contents (1)

- MM1: Introduction to digital filter techniques
  - Brief review of discrete-time processes and systems
  - Frequency response for LTI systems
  - Filter design problem and examples
- MM2: Synthesis of IIR discrete-time filters
  - Synthesis of continuous-time filters
  - Impulse-invariance method
  - Bilinear transformation method
- MM3: Algebraic transformation of Low-pass IIR filters and Linear phase systems

Contents (2)

- MM4: Synthesis of FIR by windowing
  - Window methods
  - Frequency response (Pole-Zero Diagram)
- MM5: Implementation of digital filters
  - Block diagram and signal flow graph
  - Structures of IIR and FIR systems
  - Round-off noise in digital filters
- Next Step: Spectral analysis
  - DFT, FFT….

M1. Introduction to Digital Filter Techniques

(Reading: p.16-40 and 240-270)

Objectives:

- Brief review of discrete-time processes and systems (DE4)
- Frequency response for LTI systems
- Filter design problem and examples

Signals

- A **signal** can be mathematically modeled as a function, like

$$U(\bullet): T \rightarrow R, \text{ i.e., } U(t)=r_i$$

- **Discrete-time signals** are defined at discrete times and represented as sequences of numbers.

e.g., **r[1],r[2],r[3],….**

Signal Processing Systems

- **Signal processing** concerns with representation, transform and manipulation of signals and the information they contain
- **Discrete-time systems** are those for which both the input and output are discrete-time signals,

  e.g., $y[n]=T\{x[n]\}$

Linear and Time-Invariance

- The system is ***linear system*** if and only if

$$T\{ a\, x_1[n]+b\, x_2[n] \} = a\, y_1[n] + b\, y_2[n]$$

- A **time-invariant system** (shift-invariant) is a system that a time shift or delay of the input causes a corresponding shift in the output sequence, i.e.,

$$T\{\}: x[n] \rightarrow y[n], \quad \Rightarrow$$
$$T\{\}: x[n-n_d] \rightarrow y[n-n_d],\ \underline{\text{for any}}\ n_d$$

Linear-Time-Invariant Systems

- LTI system: $y[n]=\sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$
- Cascade connection $h[n]=h1[n]*h2[n]=h2[n]*h1[n]$
- Parallel connection $h[n]=h1[n]+h2[n]$
- A LTI system is **(BIBO) stable** if and only if the impulse response is absolutely summable, i.e.,

$$S=\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

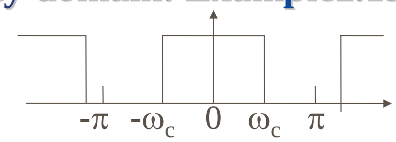
- A LTI system is causal if and only if

$$h[n]=0, \quad n< 0$$

- **FIR**: Finite-duration impulse response systems
- **IIR**: Infinite-duration impulse response systems

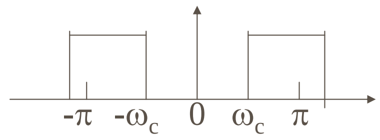
Practical desires in frequency domain: Example2.19

- Ideal lowpass filter



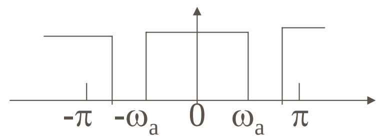
$$H_{lp}(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \omega c \\ 0, & others \end{cases}$$

- Ideal highpass filter



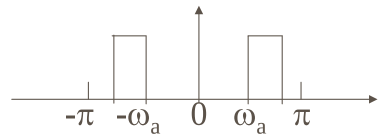
$$H_{hp}(e^{j\omega}) = \begin{cases} 1, & |\omega| \ge \omega c \\ 0, & others \end{cases}$$

- Ideal bandstop filter



$$H_{bs}(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \omega_a \text{ and } |\omega| \ge \omega_{ba} \\ 0, & \omega_a \le |\omega| \le b_a \end{cases}$$

- Ideal bandpass filter



$$H_{bp}(e^{j\omega}) = \begin{cases} 0, & |\omega| \le \omega_a \text{ and } |\omega| \ge \omega_{ba} \\ 1, & \omega_a \le |\omega| \le b_a \end{cases}$$

Fourier Transform for Discrete Systems

- The **frequency response** $H(e^{j\omega})$ of the LTI system is the Fourier transform of **impulse response** $h[n]$:

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n}$$

$$h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega$$

  - $H(e^{j\omega}) = |H(e^{j\omega})| e^{\sphericalangle H(e^{j\omega})}$

Frequency-Domain Representation

- The system input and output have
  $Y(e^{j\omega}) = H(e^{j\omega}) X(e^{j\omega})$, i.e.,
  - Magnitude relation:
    $|Y(e^{j\omega})| = |H(e^{j\omega})| |X(e^{j\omega})|$
  - Phase relation:
    $\sphericalangle Y(e^{j\omega}) = \sphericalangle H(e^{j\omega}) + \sphericalangle X(e^{j\omega})$
- **Magnitude and phase distortions**

Linear Phase and Group Delay

$H(e^{j\omega}) = |H(e^{j\omega})| e^{\sphericalangle H(e^{j\omega})}$

- The LTI system is **linear phase** system if $\sphericalangle H(e^{j\omega}) = a\omega$, where a is a real constant;
- The system is **generalized linear phase** system if $\sphericalangle H(e^{j\omega}) = a\omega + b$, where a,b are real constants
- The **group delay** of a LTI system is defined as

$$\tau(\omega) = grd[H(e^{j\omega})] = -\frac{d}{d\omega}\{\arg[H(e^{j\omega})]\}$$

Example 5.1 Effects of Group Delay (p.244)

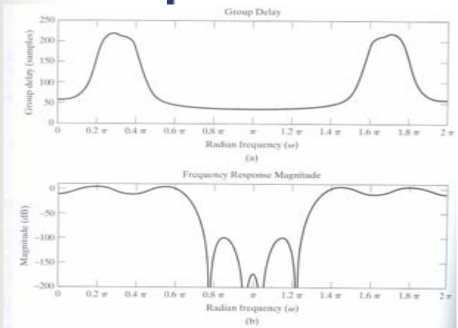


Figure 5.1 Frequency response magnitude and group delay for the filter in Example 5.1.

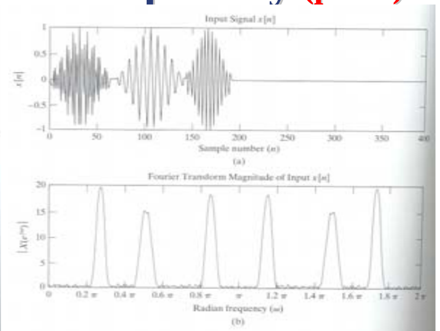


Figure 5.2 Input signal and associated Fourier transform magnitude for Example 5.1.

- The filter has considerable attenuation at $\omega=0.85\pi$. The group delay at $\omega=0.25\pi$ is about **200** steps, while at $\omega=0.5\pi$, the group delay is about **50** steps

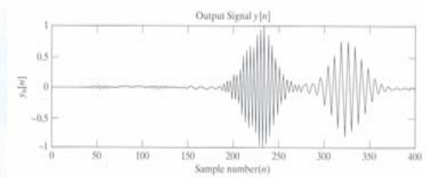


Figure 5.3 Output signal for Example 5.1.

Frequency Response–log Magnitude

- From the pole-zero form, there is

$$H(z) = \frac{b_0}{a_0} \frac{\prod_{k=1}^{M}(1 - c_k z^{-1})}{\prod_{k=1}^{N}(1 - d_k z^{-1})} \qquad |H(e^{j\omega})| = \left|\frac{b_0}{a_0}\right| \frac{\prod_{k=0}^{M}|1 - c_k e^{-j\omega}|}{\prod_{k=0}^{N}|1 - d_k e^{-j\omega}|}$$

- Log magnitude of **$H(e^{j\omega})$** is **$20\log_{10}|H(e^{j\omega})|$**

$$20\log_{10}|H(e^{j\omega})| = 20\log_{10}\left|\frac{b_0}{a_0}\right| + \sum_{k=1}^{M} 20\log_{10}|1 - c_k e^{-j\omega}| - \sum_{k=1}^{N} 20\log_{10}|1 - d_k e^{-j\omega}|$$

- The gain in dB (decibels):

Gain in dB = $20\log_{10}|H(e^{j\omega})|$

Frequency Response – Phase and GRD

- The phase response has the form

$$\sphericalangle H(e^{j\omega}) = \sphericalangle\left[\frac{b_0}{a_0}\right] + \sum_{k=1}^{M} \sphericalangle[1 - c_k e^{-j\omega}] - \sum_{k=1}^{N} \sphericalangle[1 - d_k e^{-j\omega}]$$

- The principal phase of **$H(e^{j\omega})$** is

$-\pi < ARG[H(e^{j\omega})] \le \pi$

- The corresponding group delay is

$$grd[H(e^{j\omega})] = -\sum_{k=1}^{M} \frac{d}{d\omega}(\arg[1 - c_k e^{-j\omega}]) + \sum_{k=1}^{N} \frac{d}{d\omega}(\arg[1 - d_k e^{-j\omega}])$$

Frequency Response of a Single Factor

- A single factor **$(1-re^{j\theta}e^{-j\omega})$** can represent a pole factor **$(1-d_k z^{-1})$** or a zero factor **$(1-c_k z^{-1})$**
- Log magnitude in dB is

$10\log_{10}[1+r^2-2r\cos(\omega-\theta)]$

- The principal phase is

$$\arctan\left[\frac{r\sin(\omega-\theta)}{1 - r\cos(\omega-\theta)}\right]$$

- The group delay is

$$\frac{r^2 - r\cos(\omega-\theta)}{|1 - re^{j\theta}e^{-j\omega}|^2}]$$

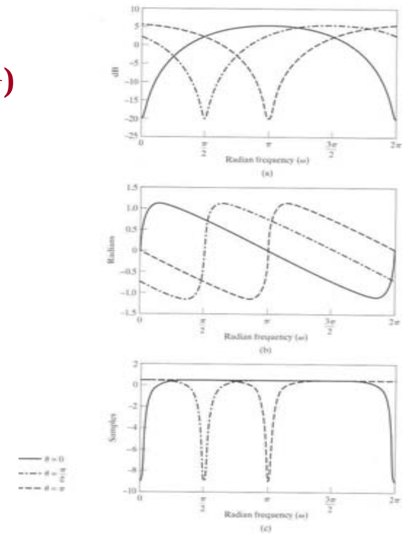


Figure 5.8 Frequency response for a single zero, with $r = 0.9$ and the three values of θ shown. (a) Log magnitude. (b) Phase. (c) Group delay.

Geometric Explanation of a Single Zero

- Consider the first-order system

$H(z) = (1-re^{j\theta}z^{-1}) = (z-re^{j\theta})/z, \quad r<1$

- This system has a pole at **$z=0$** and a zero at **$z= re^{j\theta}$**
- The magnitude is

$|H(e^{j\omega})| = |e^{j\omega} - re^{j\theta}| / |e^{j\omega}| = |v_3|/|v_1|$

- The phase is

$\sphericalangle H(e^{j\omega}) = \sphericalangle(v_3) - \sphericalangle(v_1) = \phi_3 - \omega$

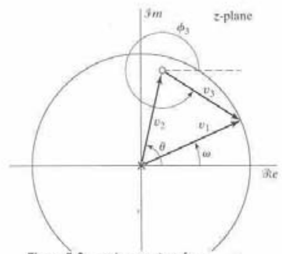


Figure 5.9 z-plane vectors for a first-order system function evaluated on the unit circle, with $r < 1$.

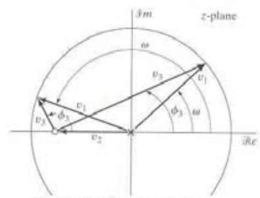


Figure 5.10 z-plane vectors for a first-order system function evaluated on the unit circle, with $\theta = \pi$, $r < 1$. The pole vector v_1 and the zero vector v_3 are shown for two different values of ω.

Second-Order IIR and FIR Examples

Example5.8 Second order IIR system

- System function

$$H(z) = \frac{1}{(1 - re^{j\theta}z^{-1})(1 - re^{-j\theta}z^{-1})}$$

- Description by difference equation.
- Pole-zero plot…
- Impulse response h[n]…
- Log magnitude …
- Phase …
- Group delay …
- Geometric explanation…
- **Example5.9** a FIR system…

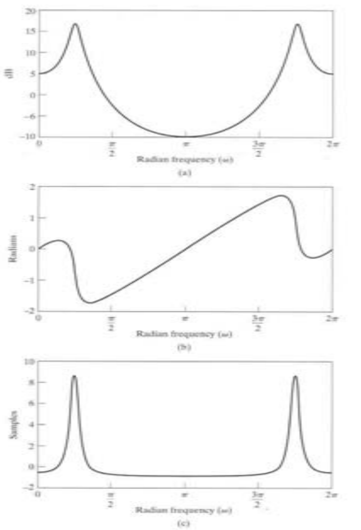
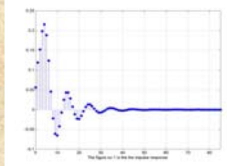
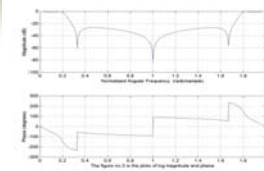


Figure 5.16 Frequency response for a complex-conjugate pair of poles as in Example 5.8, with $r = 0.9$, $\theta = \pi/4$. (a) Log magnitude. (b) Phase. (c) Group delay.

Exercise One





- **See the opgave on the distributed paper;**
- **Run program IIR.m, which can be download**