Reading material: **p.291 – 311 and the given document**

# MM3: Frequency Transformation of LP-IIR Filters and Linear phase systems

1. Explanation of Exercise Two
2. Frequency transformations
3. Linear phase systems



# Explanation of Exercise Two



# DT Lowpass IIR Filter Design

**Discrete filters**
- •Butterworth...
- •Chebyshev...
- •Elliptic ...

**Analog filters**
- •Butterworth...
- •Chebyshev...
- •Elliptic ...



# CT Butterworth lowpass Filters

- **Characteristics**
  - The magnitude response is maximally flat in the passband
  - The magnitude response is monotonic in the passband and stopband
- **Magnitude-squared function**

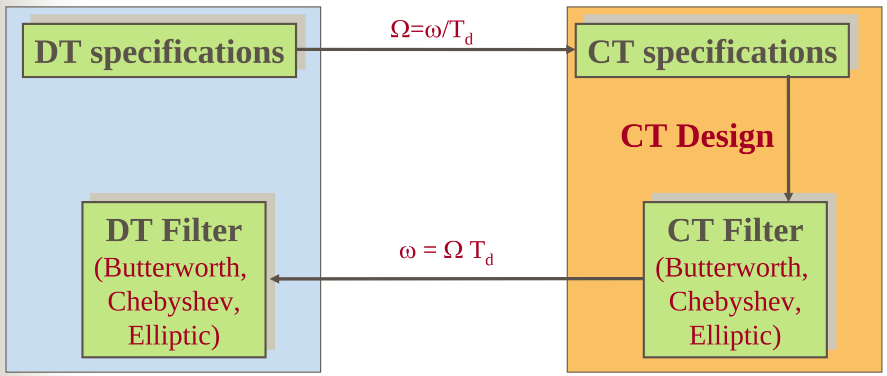
$$|H_c(j\Omega)|^2 = \frac{1}{1+(j\Omega/j\Omega_c)^{2N}}$$

- **Filter Construction**
  - System function

$$H_c(s)H_c(-s) = \frac{1}{1+(j\Omega/j\Omega_c)^{2N}}$$

  - Roots of denominator polynomial
  - Stable and causal system $H_c(s)$

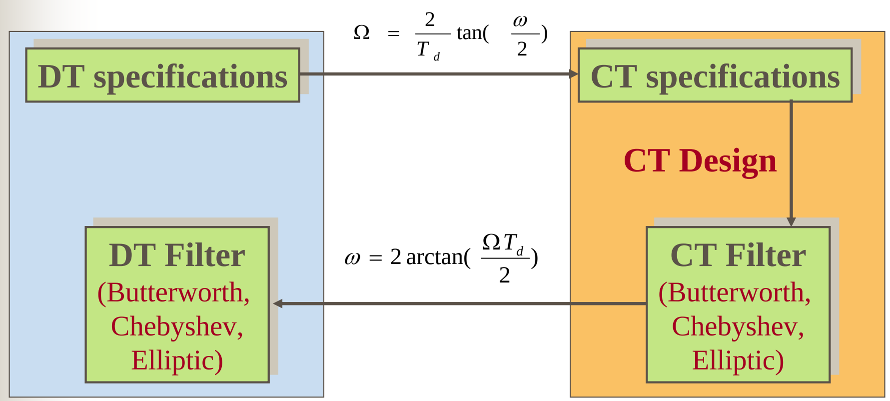
# DT IIR Filter by Impulse Invariance




$$H(e^{j\omega}) = H_c\left(j\frac{\omega}{T_d}\right) \quad |\omega| \le \pi \qquad H_c(s) = \sum_{k=1}^{N} \frac{A_k}{s - s_k} \Rightarrow H(z) = \sum_{k=1}^{N} \frac{T_d A_k}{1 - e^{s_k T_d} z^{-1}}$$



# DT IIR Filter Design by Bilinear Transform




$$s = \frac{2}{T_d}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right), \quad H(z) = H_c\left(\frac{2}{T_d}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)\right)$$



**?**

**How to design some other frequency-selective DT IIR filters besides lowpass ones?**

**Discrete filters**
- •Lowpass ...
- •Highpass...
- •Bandpass ...
- •Bandstop....

**Algebraic transform** ← **A prototype DT lowpass filter**



# Transform from Lowpass to Another Lowpass

- **Preknowledge**

  **a prototype lowpass filter** $H_{lp}(Z)$ with cutoff frequency $\theta_p$

- **Transform** $$Z^{-1} = \frac{z^{-1} - \alpha}{1 - \alpha z^{-1}}$$

- **Design Parameter** (Desired cutoff freq. $\omega_p$)

$$\alpha = \frac{\sin\left(\frac{\theta_p - \omega_p}{2}\right)}{\sin\left(\frac{\theta_p + \omega_p}{2}\right)}$$

- **Wanted lowpass** $$H(z) = H_{lp}(Z)\Big|_{Z^{-1} = \frac{z^{-1} - \alpha}{1 - \alpha z^{-1}}}$$

## Transform from Lowpass to a Highpass

- **Preknowledge**

  **a prototype lowpass filter** $H_{lp}(Z)$ with cutoff frequency $\theta_p$

- **Transform** $Z^{-1} = -\dfrac{z^{-1} + \alpha}{1 + \alpha z^{-1}}$

  Desired cutoff freq.

- **Design Parameter** $\alpha = -\dfrac{\cos(\frac{\theta_p + \omega_p}{2})}{\cos(\frac{\theta_p - \omega_p}{2})}$

- **Wanted highpass** $H(z) = H_{lp}(Z)\,|_{Z^{-1} = -\frac{z^{-1}+\alpha}{1+\alpha z^{-1}}}$



**Prototype filter**
Open matlab:
>>freqz([0.125],[1 -0.8825])
Then, Bode plot ....

## Exercise: Design a Highpass Filter

**(see distributed paper)**

**Designed filter**
Open matlab:
>> freqz([0.125 -0.0939],[0.3373 0.1316])
Then, Bode plot ....



## Discrete-Time Linear Phase Filters




## Effect of Filtering

- System frequency response:

  $H(e^{j\omega}) = |H(e^{j\omega})|\, e^{\sphericalangle H(e^{j\omega})}$

- Input and output relationship

  $|Y(e^{j\omega})| = |H(e^{j\omega})|\,|X(e^{j\omega})|$

  $\sphericalangle Y(e^{j\omega}) = \sphericalangle H(e^{j\omega}) + \sphericalangle X(e^{j\omega})$