

# Classical Control

## Lecture 1



# Outline

- 1 Introduction
- 2 Control Specifications



# Prerequisites

- Block diagram for system modeling
- Modeling
  - Mechanical
  - Electrical



# Outline

- 1 Introduction
  - Background
  - Basic Systems
  - Models/Transfers functions
- 2 Control Specifications
  - Closed Loop Stability
  - System Performance
  - Effect of Poles
  - Effect of Additional Poles/Zeros



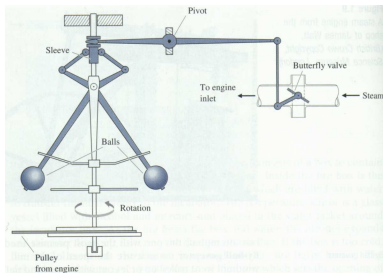
# Goal of Control Engineering

Graham C. Goodwin:

The fundamental goal of CE is to find technically, environmentally, and economically feasible ways of acting on systems to control their outputs to a desired level of performance in the face of uncertainty of the process and in the presence of uncontrollable external disturbances acting on the process.



# The Fly-Ball Governor



## Historical Periods

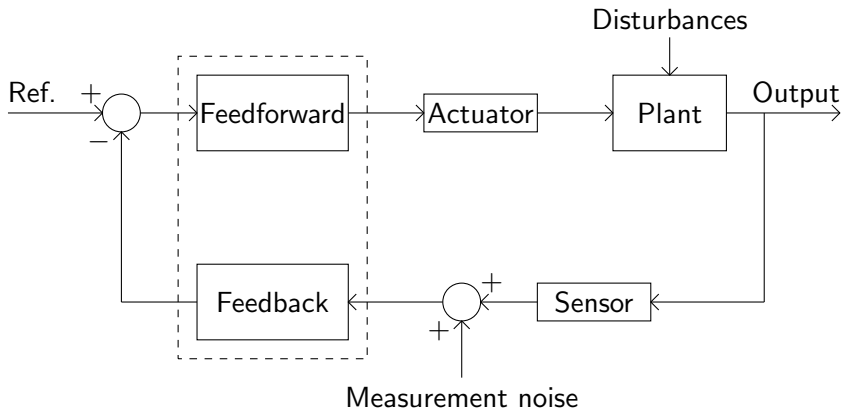
- The Industrial Revolution (1860s)
- World War II (1940-1945)
- The Space Race (1960s,1970s)
- Economic Globalization (1980s)

# Evolution of Control

- 1940 Classical Control
  - Frequency Domain
  - Mainly useful for SISO systems
  - A fundamental tool for many practicing engineers
- 1960 Modern Control
  - State-space approach to linear control theory
  - Suitable for both SISO and MIMO systems
  - No explicit definition of performance and robustness
- 1970 Optimal Control
  - Minimize a given objective function (fuel, time)
  - Suitable for both open-loop and closed-loop control
- 1980 Robust Control
  - Generalization of classical control ideas into MIMO context
  - Based on operator theory which can be interpreted into frequency domain
- Nonlinear control, Adaptive Control, Hybrid Control

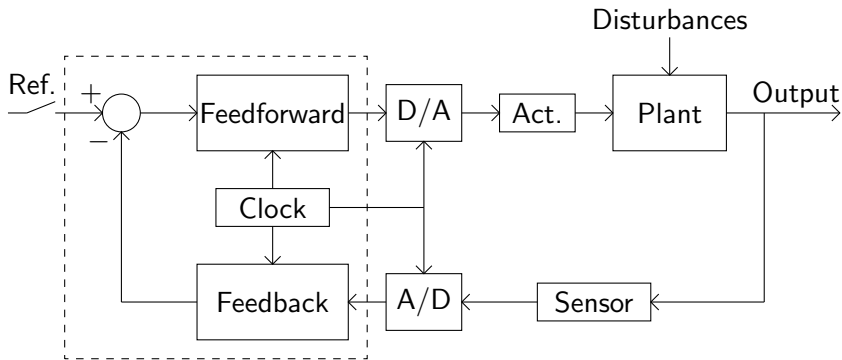


# Basic Continuous Control System





# Basic Digital Control System



# Continuous System Models

## Numerator and denominator

- General formula

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_m}{a_0 s^n + a_1 s^{n-1} + \dots + a_n}$$

- Second order system (special case)

$$G(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- MATLAB

```
sys=tf(num,den)
```



# Continuous System Models

## Zeros and poles

- General formula

$$G(s) = \frac{Y(s)}{U(s)} = K \frac{\prod_{k=1}^m (s - z_k)}{\prod_{i=1}^n (s - p_i)}$$

- Second order system (special case)

$$G(s) = \frac{K}{(s - p_1)(s - p_2)} \quad p_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2}$$

- MATLAB

```
sys=zpk(z,p,k)
```



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# Bounded Input Bounded Output

- Impulse response
- Roots of characteristic equation
- Gain and phase margins
- Nyquist stability criterion



# Stability - Impulse Response

## Continuous systems

The system is BIBO stable if and only if the impulse response  $h(t)$  is absolutely integrable

## Discrete systems

The system is BIBO stable if and only if the impulse response  $h[n]$  is absolutely summable



# Stability - Characteristic Roots

## Asymptotic internal stability

### Continuous systems

All poles of the system are strictly in the LHP of the  $s$ -plane

### Discrete systems

All poles of the system are strictly inside the unit circle of the  $z$ -plane



# Analyzing System Performance

## System types

- Continuous system
- Discrete system

## Analysis domain

- Time-domain specifications
- Frequency-domain specifications

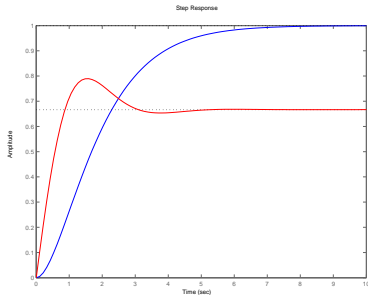
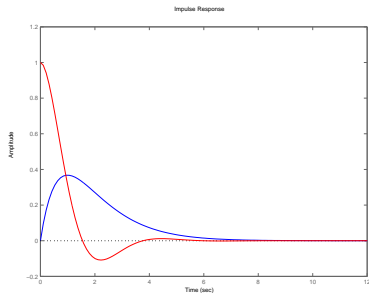
## Different periods

- Dynamic (transient) responses
- Steady-state responses



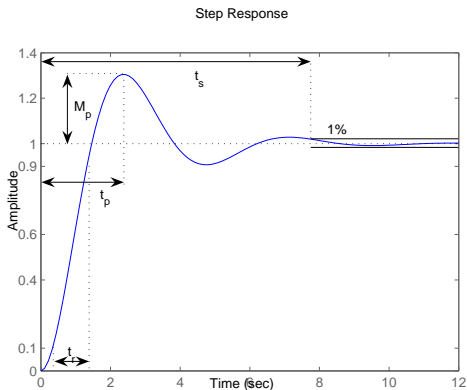


## Continuous Time - Dynamic Response



```
num1=[1];  
den1=[1 2 1];  
num2=[1 2];  
den2=[1 2 3];  
impulse(tf(num1,den1),'b',tf(num2,den2),'r')  
step(tf(num1,den1),'b',tf(num2,den2),'r')
```

# Continuous Time - Dynamic Response

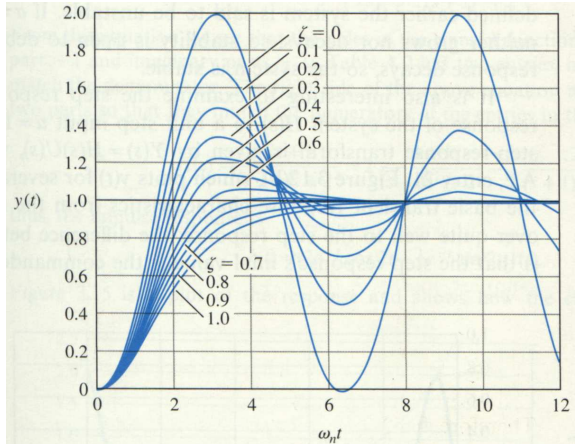
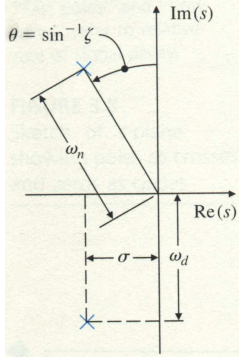


- Overshoot ( $M_p$ )
- Rise time ( $t_r$ )
- Settling time ( $t_s$ )
- Peak time ( $t_p$ )

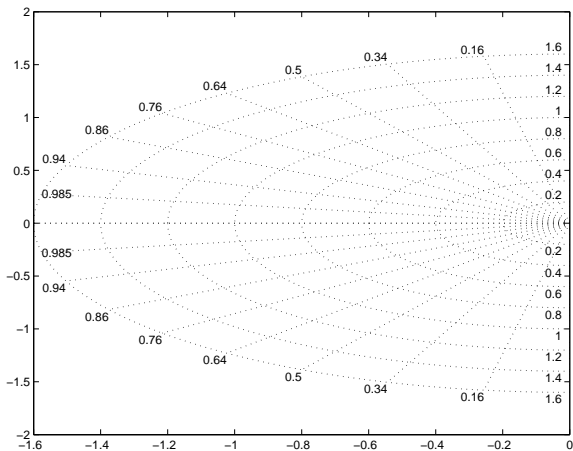


# Natural Frequency and Damping

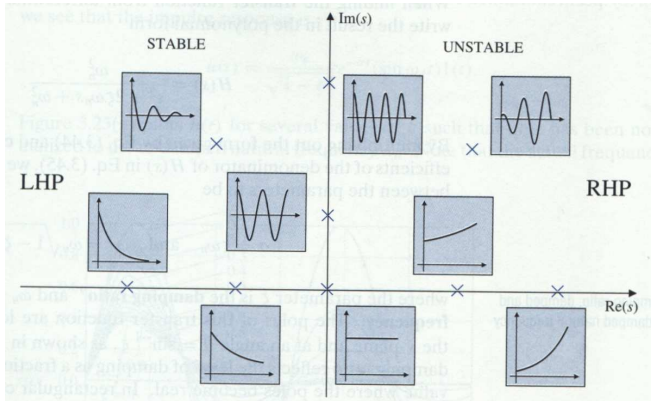
**FIGURE 3.11**  
**s-plane plot for a pair of complex poles**



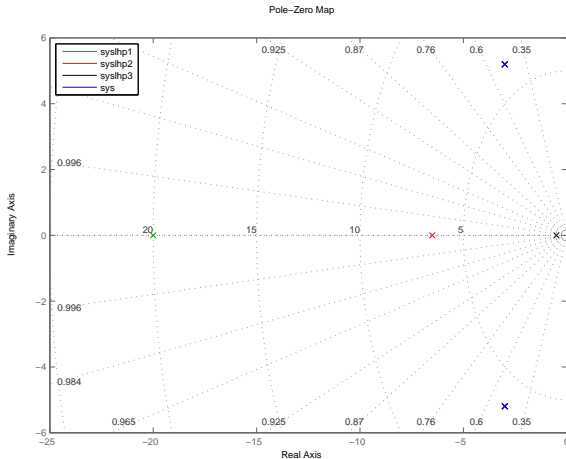
# Poles in the Continuous s-plane



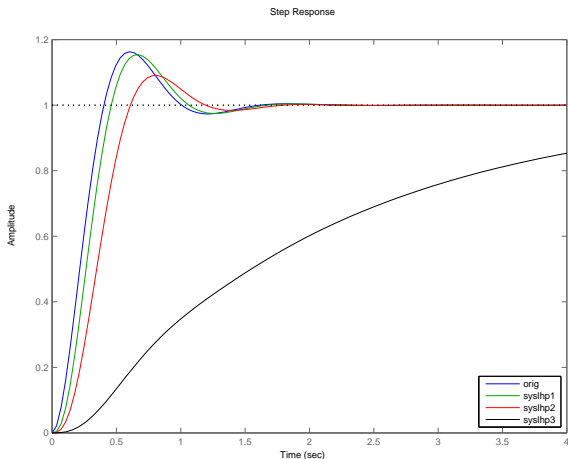
# Poles in the Continuous s-plane



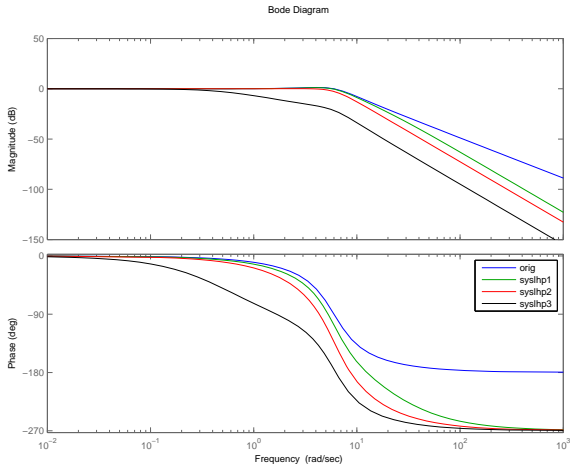
# Systems with Different Poles



# Step Response of Different Systems

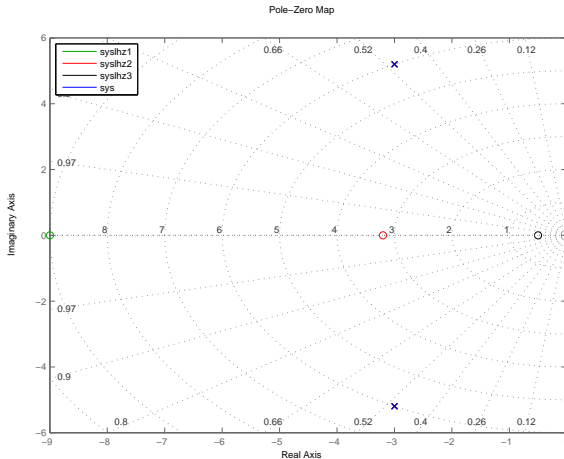


# Bode Plot of Different Systems

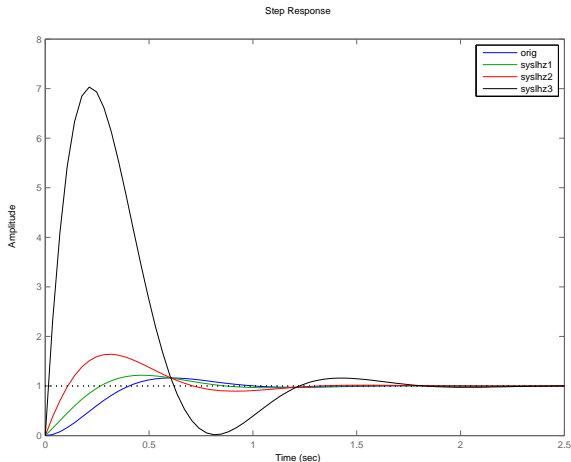




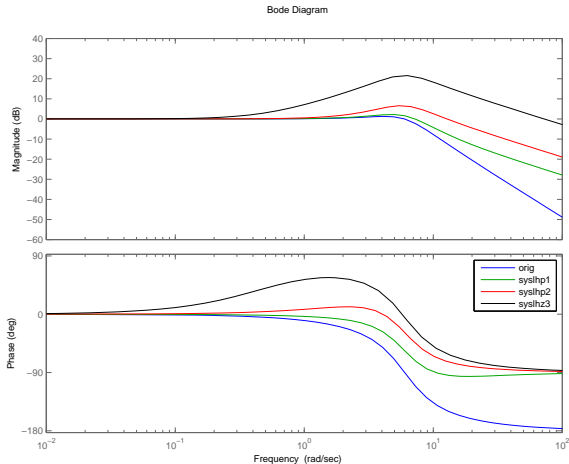
# Systems with Different Zeros



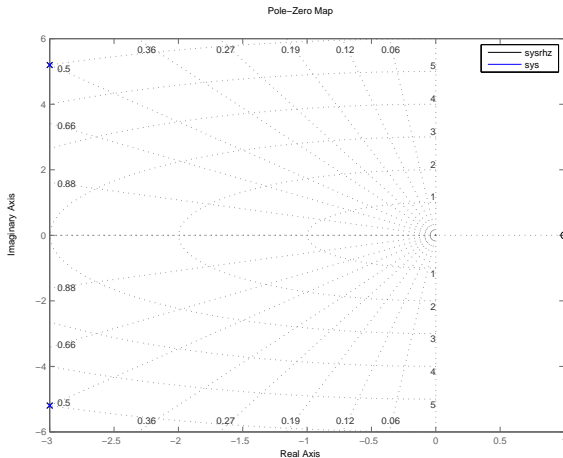
# Step Response of Different Systems



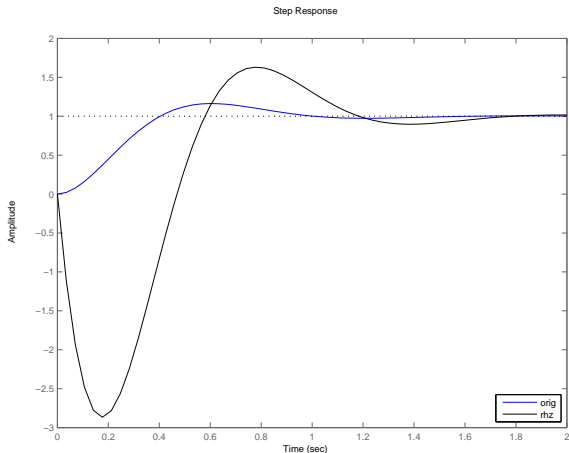
# Bode Plot of Different Systems



# System with Zero in RHP



# Step Response



# Bode Plot of System with RHP

